Proton-Neutron Interactions and the New Atomic Masses

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Double differences of masses can be used to isolate specific nucleonic interactions. With the new 2003 mass tabulation a significant increase in the number of empirical average proton-neutron interactions of the last nucleons can be extracted. It is shown that they exhibit dramatic and distinctive patterns, especially near doubly magic nuclei, that these patterns can be interpreted with a simple ansatz based on overlaps of proton and neutron orbits, and that the trends in *p*-*n* interactions across entire shells can be understood if they are correlated with the fractional shell filling. It is shown how these empirical interactions can be sensitive to changes in shell structure in exotic nuclei. Finally, these results are used to suggest criteria for future mass measurements with new exotic beam facilities.

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Binding energies reflect the interactions that give rise to correlations in many-body systems. Nuclear binding energies (obtained from masses) represent the sum of all the nucleonic interactions; differences of masses give separation energies, providing clues to shell structure and phase transitions; double differences of masses can be designed so as to isolate specific classes of interactions. One of these double differences which has proved very useful in the past [1–3] is the average interaction of the last proton(s) with the last neutron(s) and is called δV_{pn}. Proton-neutron interactions are fundamentally important to nuclear structure, and its evolution with N and Z. They play a key role in the development of configuration mixing and in the onset of collectivity and deformation in nuclei [4], in changes to the underlying shell structure (single particle energies and magic numbers) [5], and in the microscopic origins of phase transitional behavior in nuclei [5,6]. As such, insight into their magnitudes is critical for understanding nuclear structural evolution, and may give guidance in interpreting other many-body systems that undergo phase transitional behavior.

δV_{pn} values were extensively studied about 15 years ago and showed striking singularities for nuclei with $N = Z$, reflecting the $T = 0$ interaction, and smaller irregularities near closed shells, revealing insights into the microscopic basis for nuclear structure.

Recently, a new mass evaluation has been published [7] and shows a dramatic increase in the number of known masses and their precision. This, in turn, leads to a corresponding growth in the number of δV_{pn} values that can be extracted. These values form the basis for the present analysis.

It is the purpose of the present Letter to show that these δV_{pn} values distinctively highlight the variations of the *p*-*n* interaction, and, in particular, that regions straddling doubly magic nuclei show striking patterns of *p*-*n* interaction strengths that reflect their orbit dependence. Close to closed shells, we will discuss an interpretation of the very characteristic behavior observed using simple intuitive arguments based on the overlaps of proton and neutron wave functions. (Far from magic nuclei, detailed calculations will clearly be necessary, although, even in such cases, we will show a correlation of δV_{pn} values with shell filling.) It will be seen that δV_{pn} values can help in delineating shell structure and orbit occupations near the Fermi surface. This analysis also points to the importance of new mass measurements far from stability (e.g., with the new GSI project), and gives guidelines on the required accuracy of such measurements, aiding in their design and implementation.

For even-even nuclei, the average *p*-*n* interaction of the last nucleons can be defined [1,2,8] by

$$\delta V_{pn}(Z, N) = \frac{1}{4}[\{B(Z, N) - B(Z, N-2)\} - \{B(Z-2, N) - B(Z-2, N-2)\}], \quad (1)$$

where B stands for the binding energy of the nucleus. As discussed in Refs. [1,8], assuming that the core is not significantly altered, δV_{pn}, by construction, largely cancels out the interactions of the last nucleons with the core. A given δV_{pn} (Z, N) value for even-even nuclei refers to the interaction of the $(Z-1)$ and Zth protons with the $(N-1)$ and Nth neutrons. Similar formulas applicable to odd-A nuclei are found in Ref. [3] in which δV_{pn} (Z, N) refers, for example, for odd Z nuclei, to the interaction of the Zth proton with the Nth and $(N-1)$th neutrons. To understand the phenomenology we will see, we review briefly the origins of shell structure in nuclei and the typical sequences of independent particle levels (nlj) in potentials relevant to nuclei that are currently accessible.

A fundamental feature of shell structure is that the normal parity orbits within a major shell vary from those with high j and low n early in a shell to low j, high n near the end. (The specific location of the unique parity orbit will vary from shell to shell.) This systematic behavior arises from three well-known effects: the clustering of levels of different principal and orbital angular momentum quantum numbers in any reasonable potential, the spreading of such clusters due to the squaring off of the mean field potential, and the spin-orbit force. The first effect arises because single particle energies increase with both n (more nodes, shorter effective wavelength) and l (higher kinetic energy) so that pairs of orbits with different n values and l values differing in the opposite direction can be degenerate or nearly so (e.g., harmonic oscillator levels are degenerate for $\Delta l = -2\Delta n$, such as $2s$-$1d$). The second effect lowers the energies of the higher l orbits whose effective potentials are squeezed to larger radii by the centrifugal force. The third effect splits the energies of the $j = l \pm 1/2$ levels lowering the higher j orbit. The general trend from high j–low n to low j–high n within a shell, and its relation to valence p-n interactions, is the conceptual underpinning of this Letter.

Figure 1 (top) shows the δV_{pn} values for even-even nuclei near ^{208}Pb. At the time of Ref. [2], only a few data points existed near ^{208}Pb. The presently available data reveal a striking pattern. Starting near $N \sim 100$, going towards larger neutron numbers, the δV_{pn} values bifurcate into two trends, one downsloping, one upsloping. Just beyond $N = 126$, the δV_{pn} values for the lower group suddenly jump to large values with a magnitude comparable to the δV_{pn} values of the higher group. We will discuss how the behavior of δV_{pn} may be interpreted in terms of shell structure, at least near doubly magic nuclei where correlations and collective effects are less important.

For $Z \lesssim 82$, the last protons are filling the last orbits in the 50–82 shell. For $N \lesssim 126$, the last neutrons occupy orbits near the end of the 82–126 shell. Both types of nucleons occupy low j–high n orbits that are highly overlapping. Therefore, for a short range residual interaction, δV_{pn} values should be high and increase as Z and N increase towards $Z = 82$ and $N = 126$.

The lower trend line in Fig. 1 (top), corresponding to smaller p-n interactions, involves Z values just *above* 82 and N values *below* 126. Thus the protons are in high j–low n orbits and overlap poorly with the neutrons in low j–high n orbits. As soon as N becomes larger than 126, though, the neutrons also occupy high j–low n orbits, like the protons, and δV_{pn} abruptly jumps.

Figure 1 (bottom) shows the same results in a Z-N chart with δV_{pn} indicated by color coding that vividly highlights the above simple point and also shows where new data are needed. If we think of this region in terms of the magic numbers $Z = 82$ and $N = 126$, the plot splits into four quadrants, according as protons and neutrons are above or below these magic numbers. Recalling the overlap argument, we see that large δV_{pn} values occur in "symmetric" regions that can be called (with respect to the proton and neutron magic numbers $Z = 82$, $N = 126$) "below-below" and "above-above," while the upper left quadrant, with weak p-n interactions, is an asymmetric "above-below" region. The figure focuses attention on the complete lack of data in the "below-above" ($Z \leq 82, N > 126$) region. Even a few data here would be useful, such

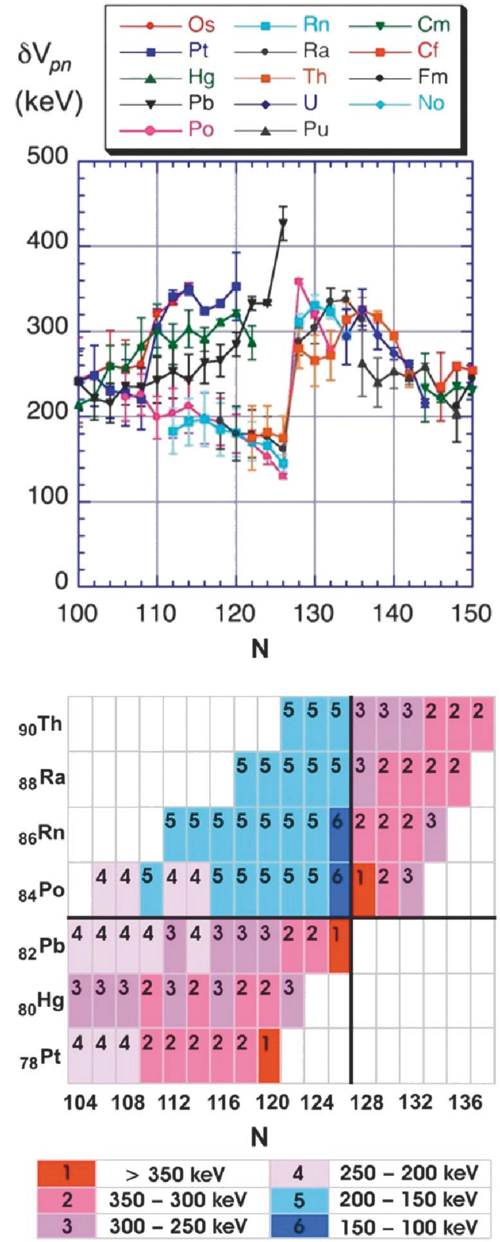

FIG. 1 (color). Top: δV_{pn} values near ^{208}Pb. Empirical δV_{pn} values are negative. (The p-n interaction is predominantly attractive.) For simplicity in the Letter, we show $|\delta V_{pn}|$. The upward trending points at the left have $Z \leq 82$, the downward trend has $Z > 82$, and the higher points beyond $N = 126$ are also for $Z > 82$. Values extracted from the tabulations in Ref. [7]. Only δV_{pn} values with errors <50 keV are shown. Bottom: Similar to top panel with color coded δV_{pn} values in a Z-N chart, highlighting the symmetry of δV_{pn} with respect to shell closures and the need for data in the lower right quadrant.

as δV_{pn} (82, 128). Of the four masses needed for this point, only the ^{208}Hg value is unknown.

If we look at the $Z = 50$ region, we see similar effects around $N = 82$. However, in contrast to the Pb region, which is near the valley of stability, these nuclei lie near neutron rich ^{132}Sn where fewer data exist and where the error bars are much larger. It would be useful to have more accurate values in this region.

It is interesting to see if an extraordinarily simple, yet semiquantitative, interpretation of δV_{pn} values is possible. We restrict ourselves to regions very close to doubly magic nuclei where one can, to some approximation, interpret the ground state configurations in terms of the independent particle model. To do this, we identify the orbits of the last proton and neutron. For example, for δV_{pn} (Z, N) for even-even nuclei, we use the ground state spins and parities of the neighboring odd-A nuclei $(Z - 1, N)$ and $(Z, N - 1)$. Figure 2 illustrates the dependence of the proton-neutron interaction for a δ force on Δn and Δl (the differences in the n and l quantum numbers for a pair of interacting protons and neutrons). Note that the calculation is specifically for the orbits $j - 1s_{1/2}$. For other orbit pairs, the detailed dependence on Δn and Δl will be different, but one expects that the basic trend is quite general, namely, the p-n orbit overlaps, and hence the interactions for a short range force, are largest for similar orbits such that Δn and Δl are minimized.

We now use Fig. 2 to estimate the p-n interactions in two mass regions. The results are illustrated in Fig. 3, again for the Pb region where the best and most extensive data exist, as well as for the ^{132}Sn region. The different panels of the figure show sequences of δV_{pn} values for even-even nuclei against N [Figs. 3(a) and 3(c)], against Z [Fig. 3(b)], and for an odd-even isotopic chain [Fig. 3(d)]. The numbers near each point are the estimated δV_{pn} values (in relative units from Fig. 2).

Despite the simplicity of this approach, the striking changes in δV_{pn} are qualitatively reproduced. Of course,

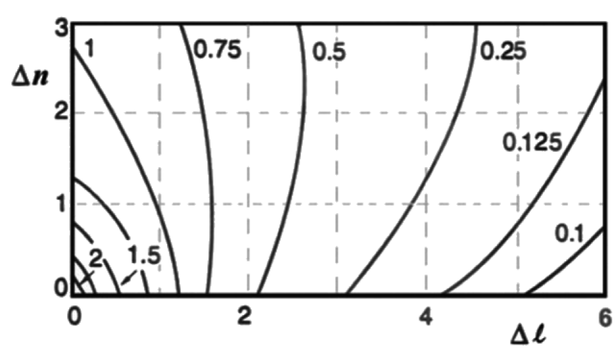

FIG. 2. Relative proton and neutron interactions for a δ force with one particle in the $1s_{1/2}$ orbit and the other in any other orbit. Note that, of course, these overlaps are defined only for integer values of Δn and Δl (at the crossings of the grid lines). Figure based on Ref. [9].

configuration mixing will alter the predictions and a more accurate treatment would entail consideration of the specific orbit occupations. Moreover, in regions far from closed shells, where extensive correlations occur, a more rigorous approach with detailed microscopic calculations will be needed. While our approach is simple and qualitative, the fact that it produces trends similar to those observed empirically, that similar patterns of empirical δV_{pn} values are seen for even-even, odd-even, and even-odd nuclei near closed shells, and in several mass regions (Fig. 3 shows two such mass regions, and panel (d) shows results for odd-even nuclei) suggests that the interpretation in terms of short range p-n interactions in particular orbits captures the essence of the physics.

We can generalize these results even to regions where such simple quantitative estimates as in Fig. 3 cannot be made. Combining the idea that δV_{pn} values are largest for smaller values of Δn and Δl, with the generic trend of orbits in each major shell from high j–low n to low j–high n, we expect that orbits with large overlap will tend to occur at roughly similar positions in the respective proton and neutron shells. Hence δV_{pn} values should be largest for such cases. To test this idea in a general and simple way we look at the *correlation* of δV_{pn} values with the *fractional filling* of a pair of proton and neutron shells, defined as the number of valence nucleons of a particular type (protons or neutrons) divided by the respective shell sizes. For example, the proton and neutron fractional fillings for the $Z = 50$–82 and $N = 82$–126 shells are given by

$$f_p = \frac{N_p}{32} \quad \text{and} \quad f_n = \frac{N_n}{44}, \tag{2}$$

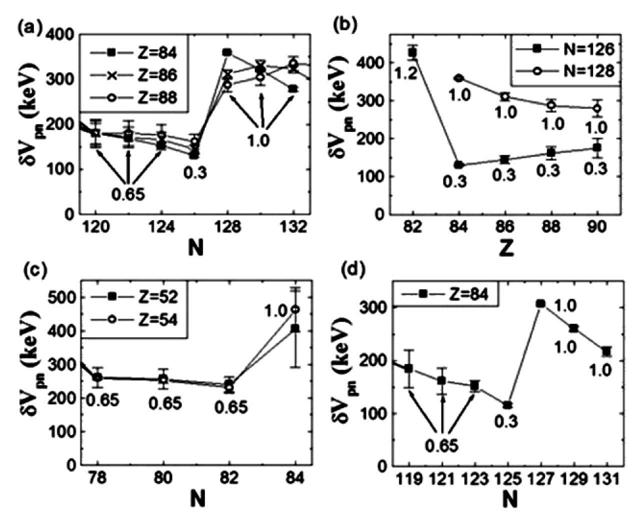

FIG. 3. Selection of δV_{pn} values for even-even and odd-even nuclei near ^{208}Pb and ^{132}Sn, along with estimates (relative scale using Fig. 2) based on the differences in principal and orbital angular momentum quantum numbers, for the orbits occupied by the last nucleons.

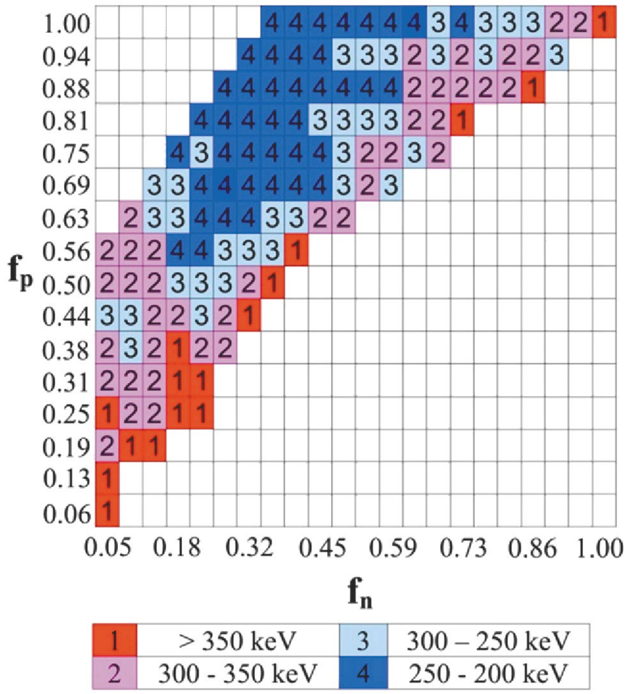

FIG. 4 (color). δV_{pn} values (colors) as functions of f_p and f_n, the fractional fillings of the $Z = 50$–82 and the $N = 82$–126 shells. δV_{pn} values with errors less than 125 keV are shown. All entries have errors less than $1/3$ of the δV_{pn} value.

where N_p and N_n are the valence nucleon numbers. The correlation of δV_{pn} with f_p and f_n is shown in Fig. 4. The color coding (red for largest values) clearly shows a preponderance of the largest δV_{pn} values near the diagonal where $f_p \sim f_n$. We also note a grouping of large δV_{pn} values near $Z = 64$ ($f_p \sim 0.44$) and N values below 90 ($f_n \sim 0.18$) that needs further investigation. Finally, the figure clearly illustrates the need for δV_{pn} values (i.e., mass measurements) for *neutron rich* nuclei, where essentially no data are available. This alone is a strong motivation for additional mass measurements in exotic neutron rich nuclei.

There is another motivation. Far from the valley of stability, we expect changes in single particle energies and changes in magic numbers. Such effects are already known in light nuclei (see, e.g., Refs. [10,11]) and are expected in heavier species. Should they occur, the expected patterns in δV_{pn} may be altered. For example, in extremely neutron rich nuclei, the shell structure may change for the loosely bound neutrons but not for the protons. If the neutron potential becomes more diffuse, magic numbers may change, and the sequence of orbits might more closely resemble "nested" values, with the highest l orbits at the top and bottom of a shell, middle l orbits closer to the center, surrounding in turn the lowest l. If this occurred, it could be reflected in a multiply peaked pattern of δV_{pn} values in an f_p-f_n plot rather than a single ridge along the diagonal. Thus new mass data in exotic nuclei can give sensitive clues to shell structure and to the fragility of magicity, even prior to the availability of spectroscopic data.

Finally, we look at the magnitudes of the variations in δV_{pn} to estimate criteria for future experiments. Except for the $N = Z$ nuclei, changes in δV_{pn} typically span about 150–250 keV in a given region. Given that four masses enter in Eq. (1), and that meaningful trends can only be distinguished if the errors on δV_{pn} are considerably less than the local variations, it appears that individual mass values need to be measured to $\lesssim \pm 25$ keV. This has implications for the design of future experiments (e.g., at the new GSI facility) for the number of nuclei of a given (Z, N) that need to be produced, for the length of time they need to be observed, and whether measurements with storage rings or traps are most suitable.

To summarize, we have shown that the new 2003 mass table reveals a number of striking patterns in δV_{pn} values, the average interaction of the last proton with the last neutron, especially near doubly magic nuclei. We showed that it is possible to understand this systematic behavior, at least near closed shells, with simple arguments concerning the overlaps of proton and neutron orbits as a function of their l and n values. We also showed that generic patterns of single particle energy sequences from high j–low n to low j–high n across major shells results in systematic trends in average proton-neutron interactions, and provides signatures of changes in shell structure that may be experimentally accessible prior to other data on exotic nuclei. Finally, these results give guidance for future mass experiments, in particular, for the required levels of precision.

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