

## Suppressing Proton Decay in the Minimal SO(10) Model

Bhaskar Dutta,<sup>1</sup> Yukihiro Mimura,<sup>1</sup> and R. N. Mohapatra<sup>2</sup>

<sup>1</sup>*Department of Physics, University of Regina, Regina, Saskatchewan S4S 0A2, Canada*

<sup>2</sup>*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

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We show that in a class of minimal supersymmetric SO(10) models which have been found to be quite successful in predicting neutrino mixings, all proton decay modes can be suppressed by a particular choice of Yukawa textures. The required texture not only fits all lepton and quark masses as well as Cabibbo-Kobayashi-Maskawa parameters, but it also predicts neutrino mixing parameter  $U_{e3}$  and Dirac  $CP$  phase  $\sin|\delta_{MNS}|$  to be 0.07–0.09 and 0.3–0.7, respectively.

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The seesaw mechanism [1] for understanding small neutrino masses observed in recent experiments seems to suggest a grand unified theory (GUT) based on the SO(10) group. Since all quarks and leptons including right-handed neutrinos are unified under SO(10) group into one **16** dimensional spinor multiplet, this raises the hope that the masses and mixings of quarks and leptons can be understood in terms of a smaller number of parameters than in the standard model. Various recent works in a class of minimal SO(10) models with a single **10** and a single **126** Higgs multiplets [2–4] have substantiated this point of view and have led to predictions for neutrino mixings. The predictions for solar and atmospheric mixing angles are in agreement with present observations [3,4] and that for  $U_{e3}$  is not far below the present upper limits, making the model testable in planned experiments. The simplest way to accommodate Cabibbo-Kobayashi-Maskawa (CKM)  $CP$  violation in these models is to include an additional Higgs field belonging to the **120** dimensional representation [5] which still remains predictive [5,6] and leads to a solution to the supersymmetric (SUSY)  $CP$  problem.

Proton decay provides important constraints on GUT models [7,8]. In most generic SUSY GUTs the dimension five operators induced by colored Higgsino [9] provide the dominant contribution to the proton decay amplitudes. Since they arise from diagrams involving Yukawa couplings, predictions for proton lifetime get related to fermion mass textures. The  $\Delta B = 1$  interactions in generic SO(10) models [10] involve several GUT scale symmetry breaking parameters and therefore the situation for proton decay is less restrictive compared to the minimal SU(5) model. Nonetheless, since there are experimental bounds on 16 nucleon decay modes, it is not *a priori* obvious that the model will be consistent. In particular, in the minimal SO(10) models of the type discussed in Ref. [3], there are four free parameters [11] and it was shown through a numerical analysis [11,12] (without including  $RRRR$  operators) that there exists a very small region in these parameter spaces for  $LLLL$  operators, where all the present experimental constraints are satisfied for lower

$\tan\beta \leq 3$ . The inclusion of the  $RRRR$  operator creates a tension since we will see later that the  $LLLL$  and  $RRRR$  operators involve the same terms with opposite sign (due to  $D$  parity) and thereby it becomes impossible to suppress both of them by the cancellation of the contributing terms.

In this Letter we study the proton decay constraints on SO(10) models with **10** + **126** + **120** Higgs fields, and present proton decay constraints combined with fermion masses and mixings imply a specific relation among SO(10) breaking vacuum expectation values (VEVs) and a very specific form for the Yukawa textures. Roughly, they imply that the proton decay operators are proportional to the product of two up-type quark Yukawa couplings, i.e.,  $Y_{u,ij}Y_{u,kl}$  instead of  $Y_dY_u$ , as in the minimal SU(5) model. As a consequence, the suppression of both  $LLLL$  and  $RRRR$  contributions for proton decay works (without any fine-tuning) even for large  $\tan\beta$ , which makes this model easily distinguishable from other simple GUT models. In addition, it leads to definite predictions for the neutrino mixing parameter  $U_{e3}$  and the Dirac phase  $\delta_{MNS}$ .

The field content of our SO(10) model is as follows: three **16**<sub>*i*</sub>( $\psi_i$ ) spinors for three generations of matter, and Higgs fields in **10** ( $H$ ), **120** ( $D$ ), and **126**( $\Delta$ )  $\oplus$  **126**( $\bar{\Delta}$ ), and a **210** ( $\Phi$ ) multiplet [13] for GUT symmetry breaking and for the minimal supersymmetric standard model (MSSM) vacua. The Yukawa superpotential of the model is

$$W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \bar{\Delta} + \frac{1}{2} h'_{ij} \psi_i \psi_j D. \quad (1)$$

The SO(10) invariance implies that the coupling matrices  $h$  and  $f$  are symmetric and  $h'$  is antisymmetric. Altogether we have six pairs of Higgs doublets:  $\varphi_d = (H_d^{10}, D_d^1, D_d^2, \bar{\Delta}_d, \Delta_d, \Phi_d)$ ,  $\varphi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \bar{\Delta}_u, \Phi_u)$ , where the superscripts 1, 2 of  $D_{u,d}$  stand for SU(4) singlet and adjoint pieces under the  $G_{422} = \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$  decomposition. The mass term of the Higgs doublets is given as  $(\varphi_d)_a (M_D)_{ab} (\varphi_u)_b$ , and  $M_D$  is given in Ref. [14]. We diagonalize this mass matrix by  $UM_D V^T = M_D^{\text{diag}}$  and assume that  $(M_D^{\text{diag}})_{11}$  is much smaller than the GUT scale. The MSSM Higgs doublets are given as linear combinations:  $H_d = U_{1a}^* (\varphi_d)_a$ ,  $H_u = V_{1a}^* (\varphi_u)_a$ .

We use the SU(5) basis to describe the standard model decomposition of the SO(10) representation [14,15]. The Yukawa interactions are written ( $G_{422}$  decomposition is given in Ref. [16]) as

$$\begin{aligned} W_Y^{\text{doub}} = & hH_d^{10}(qd^c + \ell e^c) + hH_u^{10}(qu^c + \ell \nu^c) \\ & + \frac{1}{\sqrt{3}}f\bar{\Delta}_d(qd^c - 3\ell e^c) + \frac{1}{\sqrt{3}}f\bar{\Delta}_u(qu^c - 3\ell \nu^c) \\ & + h'D_d^1(qd^c + \ell e^c) + h'D_u^1(qu^c + \ell \nu^c) \\ & + \frac{1}{\sqrt{3}}h'D_d^2(qd^c - 3\ell e^c) - \frac{1}{\sqrt{3}}h'D_u^2(qu^c - 3\ell \nu^c), \end{aligned} \quad (2)$$

and Yukawa matrices for fermions are obtained as

$$\begin{aligned} Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}', \quad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}'), \quad (3) \\ Y_e = r_1(\bar{h} - 3\bar{f} + c_e\bar{h}'), \quad Y_\nu = \bar{h} - 3r_2\bar{f} + c_\nu\bar{h}', \quad (4) \end{aligned}$$

where the subscripts  $u, d, e, \nu$  denote up-type quark, down-type quark, charged-lepton, and Dirac neutrino Yukawa couplings, respectively, and  $\bar{h} = V_{11}h$ ,  $r_1 = U_{11}/V_{11}$ ,  $r_2 = r_1V_{15}/U_{14}$ ,  $r_3 = r_1(V_{12} - V_{13}/\sqrt{3})/(U_{12} + U_{13}/\sqrt{3})$ ,  $\bar{f} = U_{14}/(\sqrt{3}r_1)f$ ,  $\bar{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1h'$ ,  $c_e = (U_{12} - \sqrt{3}U_{13})/(U_{12} + U_{13}/\sqrt{3})$ ,  $c_\nu = r_1(V_{12} + \sqrt{3}V_{13})/(U_{12} + U_{13}/\sqrt{3})$ . The Majorana mass matrices for both left- and right-handed neutrinos are proportional to the coupling  $f$ . In this Letter we will be using type II seesaw [17].

The dimension five operators ( $LLLL$  and  $RRRR$  operators) induced by Higgs triplets,

$$-W_5 = \frac{1}{2}C_L^{ijkl}q_kq_lq_i\ell_j + C_R^{ijkl}e_k^c u_l^c u_i^c d_j^c, \quad (5)$$

are obtained by integrating out the triplet Higgs fields,  $\varphi_{\bar{T}} = (H_{\bar{T}}, D_{\bar{T}}, D_{\bar{T}}', \bar{\Delta}_{\bar{T}}, \Delta_{\bar{T}}, \Delta_{\bar{T}}', \Phi_{\bar{T}})$  and  $\varphi_T = (H_T, D_T, D_T', \Delta_T, \bar{\Delta}_T, \bar{\Delta}_T', \Phi_T)$ , whose hypercharges are  $Y = \pm 1/3$ . In the expression the fields with “ $'$ ” are decuplet, and the others are sextet under SU(4) decomposition. The  $C_R$  operator is also generated by other triplets with  $Y = \pm 4/3$ ,  $\varphi_{\bar{C}} = (D_{\bar{C}}, \Delta_{\bar{C}})$ , and  $\varphi_C = (D_C, \bar{\Delta}_C)$ . The mass term of the Higgs triplets are given as  $(\varphi_{\bar{T}})_a(M_T)_{ab}(\varphi_T)_b + (\varphi_{\bar{C}})_a(M_C)_{ab}(\varphi_C)_b$ . The mass matrices,  $M_T$  and  $M_C$ , are  $7 \times 7$  and  $2 \times 2$  matrices, respectively [14]. The Yukawa couplings which cause proton decay are written as

$$\begin{aligned} W_Y^{\text{trip}} = & hH_{\bar{T}}(q\ell + u^c d^c) + hH_T\left(\frac{1}{2}qq + e^c u^c\right) \\ & + f\bar{\Delta}_{\bar{T}}(q\ell - u^c d^c) + f\bar{\Delta}_T\left(\frac{1}{2}qq - e^c u^c\right) + \sqrt{2}f\bar{\Delta}_T' e^c u^c \\ & + \sqrt{2}h'(D_{\bar{T}}u^c d^c + D_{\bar{T}}' q\ell - D_T e^c u^c + D_T' e^c u^c) \\ & + 2f\bar{\Delta}_C d^c e^c + 2h'D_{\bar{C}} u^c e^c + 2h'D_C d^c e^c. \end{aligned} \quad (6)$$

The dimension five operators are written by the Yukawa couplings  $h, f$ , and  $h'$  as follows:

$$\begin{aligned} C_L^{ijkl} = & ch_{ij}h_{kl} + x_1f_{ij}f_{kl} + x_2h_{ij}f_{kl} + x_3f_{ij}h_{kl} \\ & + x_4h'_{ij}h_{kl} + x_5h'_{ij}f_{kl}, \end{aligned} \quad (7)$$

$$\begin{aligned} C_R^{ijkl} = & ch_{ij}h_{kl} + y_1f_{ij}f_{kl} + y_2h_{ij}f_{kl} + y_3f_{ij}h_{kl} \\ & + y_4h'_{ij}h_{kl} + y_5h'_{ij}f_{kl} + y_6h_{ij}h'_{kl} + y_7f_{ij}h'_{kl} \\ & + y_8h'_{ij}h'_{kl} + y_9h'_{il}f_{jk} + y_{10}h'_{il}h'_{jk}. \end{aligned} \quad (8)$$

The coefficient  $c$  is given as  $c = (M_{\bar{T}}^{-1})_{11}$ , and the other coefficients  $x_i, y_i$  are also given by the components of  $M_{\bar{T}}^{-1}$  or  $M_C^{-1}$ . Note that  $y_3 = -x_3$ , due to the fact that  $H_T$  and  $\bar{\Delta}_T$  have opposite  $D$  parity.

A more convenient form of the proton decay operators can be given by diagonalizing the Higgs triplet mass matrix  $M_T$  by two unitary matrices,  $X$  and  $Y$ , as  $XM_T Y^T = \text{diag}(M_1, M_2, \dots, M_7)$ ,

$$\begin{aligned} C_L^{ijkl} = & \sum_a \frac{1}{M_a} (X_{a1}h + X_{a4}f + \sqrt{2}X_{a3}h')_{ij} (Y_{a1}h + Y_{a5}f)_{kl}, \\ C_R^{ijkl} = & \sum_a \frac{1}{M_a} (X_{a1}h - X_{a4}f + \sqrt{2}X_{a2}h')_{ij} (Y_{a1}h - (Y_{a5} \\ & - \sqrt{2}Y_{a6})f + \sqrt{2}(Y_{a3} - Y_{a2})h')_{kl} + (y_9, y_{10} \text{ terms}). \end{aligned} \quad (9)$$

A consistency check of the formula is obtained by considering a specific vacua. For example, in the SU(5) limit, only one of the colored triplets is much lighter than the others, i.e.,  $M_1 \ll M_a$  ( $a \neq 1$ ), and we can obtain the following relations for the diagonalizing matrices from the explicit form of the Higgs mass matrices in Refs. [14,15]:  $U_{11} = X_{11}$ ,  $V_{11} = Y_{11}$ ,  $U_{14} = X_{14} = 0$ ,  $V_{15} : Y_{15} : Y_{16} = \sqrt{3} : 1 : \sqrt{2}$ ,  $U_{12} : U_{13} : X_{12} : X_{13} = V_{12} : V_{13} : Y_{12} : Y_{13} = 1 : \sqrt{3} : \sqrt{2} : \sqrt{2}$ . As a result,  $r_2 \rightarrow \infty$  with  $\bar{f} \rightarrow 0$ ,

$$r_3 = 0, \quad c_e = -1 \quad (10)$$

for the Yukawa matrices in Eqs. (3) and (4) and thus, as expected, we get the SU(5) relations,  $Y_u = Y_u^T$ ,  $Y_d = Y_d^T$ , and the dimension five proton decay operators can be written in terms of the Yukawa couplings as  $C_L^{ijkl} \simeq C_R^{ijkl} \simeq (Y_d)_{ij}(Y_u)_{kl}/M_1$ .

Let us now investigate the conditions required to suppress the proton decay rate in this model. From the experimental inputs of quark and lepton masses and mixings, the coupling  $f$  is almost determined to have the form

$$\bar{f} \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} m_s/m_b,$$

where  $\lambda \sim 0.2$ . A naive implication of this is that since up-type quark masses are more hierarchical than down-type ones (i.e.,  $m_u/m_t \ll m_d/m_b$ ,  $m_c/m_t \ll m_s/m_b$ ), the expression for the up-type Yukawa matrix requires the following two typical choices: (a) there is cancellation among

$h$ ,  $f$ , and  $h'$ , or (b)  $h$  itself has a hierarchical form similarly to the up-type quark masses. Choice (a) corresponds to the cases where  $[1, 2]$  block of  $h_{ij}$  is not far smaller than  $f_{ij}$ , but  $r_{2,3}$  are chosen such that  $m_u, m_c$  are hierarchically small. Choice (b) corresponds to  $r_{2,3} \sim 0$  and is required to suppress the proton decay naturally. To suppress the decay rate we need small couplings for first and second generations in the  $C_{L,R}$  operators. Clearly this would also require a cancellation among  $h$ ,  $f$ , and  $h'$  in case (a). Since in general the coefficients  $r_{2,3}$  in  $Y_u$  and  $x_i, y_i$  in  $C_{L,R}$  are unrelated, one must find a situation where both cancellations can be achieved in a satisfactory manner. However, since the **126** Higgs contribution has an opposite signature ( $y_3 = -x_3$ ) for one of the coefficients in  $C_{L,R}$ , the cancellation required to obtain small Yukawa coupling for  $Y_u$  by tuning  $r_2 \bar{f}$  cannot simultaneously suppress both  $C_L$  and  $C_R$  operators by tuning  $X_{14}$ . Moreover, since the  $kl$  part of  $C_L$  is symmetric because of  $SU(3) \times SU(2)$  contraction, the **120** Higgs contribution vanishes due to the antisymmetry of  $h'$ . Thus, if the cancellation in  $Y_u$  happens by tuning  $r_3 h'$ , such cancellation will not help in suppressing the  $C_L$  contribution. Thus the decay rate cannot be suppressed in a natural way if we take choice (a).

We show how the proton decay rate in choice (b) is suppressed compared to the minimal  $SU(5)$  model. If  $r_{2,3} \approx 0$ , the  $C_L$  can be written as  $C_L^{ijkl} \propto (Y_u + \gamma h')_{ij}(Y_u)_{kl}$  and, in the operator  $C_R^{ijkl}$ , the  $ij$  part is also related to  $Y_u$ . This will correspond to the case where  $X_{14}, Y_{15} \sim 0$ . We will give an example later of when this can happen. The  $C_R$  contribution now to the  $p \rightarrow K \bar{\nu}_\tau$  mode is suppressed giving a suppression factor  $\lambda_u/\lambda_d \sim 1/100$  for  $\tan\beta \sim 50$  compared to the minimal  $SU(5)$  model. Similarly, since the  $kl$  part of  $C_L$  is also related to  $Y_u$  instead of  $Y_d$ , the  $C_L$  contribution to  $p \rightarrow K \bar{\nu}$  is also suppressed even for  $\tan\beta \sim 50$ , compared to the  $SU(5)$  model (since  $\lambda_c/\lambda_s \sim 1/5$ ). However, as it turns out, these suppressions are not enough to satisfy the current experimental bound. Rewriting the proton decay amplitude as  $A = \alpha_2 \beta_p / (4\pi M_T m_{\text{SUSY}}) \tilde{A}$ , we can write  $\tilde{A} = c \tilde{A}_{hh} + x_1 \tilde{A}_{ff} + x_2 \tilde{A}_{hf} + x_3 \tilde{A}_{fh} + \dots$ . The coefficients  $c$  and  $x_i$  are given in Eq. (7), and there are also similar  $C_R$  contributions. To satisfy the current nucleon decay bounds, we need  $|\tilde{A}_{p \rightarrow K \bar{\nu}}| \lesssim 10^{-8}$ ,  $|\tilde{A}_{n \rightarrow \pi \bar{\nu}}| \lesssim 2 \times 10^{-8}$ , and  $|\tilde{A}_{n \rightarrow K \bar{\nu}}| \lesssim 5 \times 10^{-8}$  if the colored Higgsino mass is  $2 \times 10^{16}$  GeV, and squark and wino masses are around 1 TeV and 250 GeV, respectively. In order to satisfy those bounds naturally, we need  $\tilde{A}_{hh} \lesssim 5 \times 10^{-8}$ . If  $\tilde{A}_{hh} \gtrsim 10^{-7}$ , we need to tune  $x_i$  and  $y_i$  for every decay mode to cancel  $\tilde{A}_{hh}$ , which is unnatural. (Further, assuming  $c \rightarrow 0$  cannot make successful suppression of the decay amplitude since in that case we need to suppress  $\tilde{A}_{ff}$  which can only be suppressed for  $\tan\beta \lesssim 3$ .) Note that the  $\tilde{A}_{hh}$  depends on the magnitudes of the elements from the  $[1, 2]$  block of  $\tilde{h}$  which is determined from the fit to the up-type Yukawa coupling as a function of  $r_2$  and  $r_3$ . The  $\tilde{A}_{hh}$  is calculated in the basis where  $Y_u = \text{diag}(\lambda_u, \lambda_c, \lambda_t)$ . For

the case where we invoke large  $r_{2,3}$  and cancellation to fit fermion masses,  $\tilde{A}_{hh}$  can be  $\sim 10^{-4}$ , thereby requiring a very high level of fine-tuning for all the decay modes. Proton decay suppression therefore prefers the range where  $r_2$  and  $r_3$  are small. However, even in the case  $r_2 = r_3 = 0$ ,  $\tilde{A}_{hh} \sim 10^{-7}$ , cancellations among the coefficients  $x_i$  and  $y_i$  are needed to satisfy the data and we need to specify the Yukawa texture to suppress the proton decay rate further. We find the necessary Yukawa texture to be  $\tilde{h} \approx \text{diag}[0, 0, O(1)]$ ,

$$\tilde{f} \approx \begin{pmatrix} \sim 0 & \sim 0 & \lambda^3 \\ \sim 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad \tilde{h}' \approx i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix},$$

where  $\lambda \sim 0.2$ . With  $r_3 = 0$ , and  $r_2$  given by  $r_2 m_s/m_b \approx \lambda_c$  ( $r_2 \approx 0.1$ ), we can then generate the correct charm mass;  $h'_{12}$  generates the down-quark mass and Cabibbo angle  $\theta_c$  with  $m_d/m_s \approx \sin^2 \theta_c$ . The up-quark Yukawa coupling is found to be  $\lambda_u \sim (r_2 \lambda^3)^2$ . We also have a relation  $m_d m_s m_b \approx c_e^2 m_e m_\mu m_\tau$ , where  $c_e^2 \approx 1$  in the preferred vacuum for the **120** Higgs coupling, Eq. (10). In the basis where  $Y_u$  is diagonal,  $\tilde{A}_{hh}$  in this texture is not completely zero but can become much smaller than  $10^{-8}$ .

After we suppress the  $\tilde{A}_{hh}$ , we also need to examine the contribution of the other components, e.g.,  $\tilde{A}_{ff, hf, fh, h'h, \dots}$ . They involve the colored Higgs mixings, which can be suppressed by our choice of the VEVs and the Higgs couplings. According to our numerical studies, some of the mixing angles must be about a few percent in the case of  $\tan\beta \sim 50$  to suppress the decay. The mixing angles can become larger as  $\tan\beta$  becomes smaller.

The proton lifetime for  $p \rightarrow K \bar{\nu}$  for this choice of texture can be larger than the current experimental bound,  $\tau_p \gtrsim 2 \times 10^{33}$  years for any  $\tan\beta$  (using the lightest colored Higgsino mass to be  $2 \times 10^{16}$  GeV and squark mass scale around 1 TeV). All other nucleon decay modes are suppressed as well. In our calculation, we use long- and short-distance renormalization factors,  $A_L = 1.43$  and typically  $A_S = 1.8$ , similar to Ref. [18]. It is important to emphasize that, without the particular choice of texture, as mentioned above, the proton decay cannot be suppressed naturally in these models unless the  $\tan\beta$  is very small. The presence of  $h'$  is a necessity to suppress proton decay (suppress  $\tilde{A}_{hh}$ ) and fit the fermion masses. This  $h'$  also helps to explain CKM  $CP$  violation [5].

Given the above texture for Yukawa couplings, it is very interesting to see that the current neutrino data can be fit and  $U_{e3}$  is restricted to a range. In Fig. 1, we plot  $U_{e3}$  as a function of  $r_2$  with  $f_{11,12} \rightarrow 0$  (as required in the most preferred texture) and the value at  $|r_2| \approx 0.1$  is the most important. We find that  $U_{e3}$  is in the range 0.07–0.09. For this fit,  $\sin^2 2\theta_{23}$  is maximal ( $> 0.9$ ),  $\tan^2 \theta_{\text{solar}} \sim 0.4$ , and  $\Delta m_{\text{sol}}^2 / \Delta m_A^2: 0.02\text{--}0.07$  (lower values are more preferred). We also predict the Dirac  $CP$  phase as  $\sin|\delta_{MNS}|$  to be 0.3

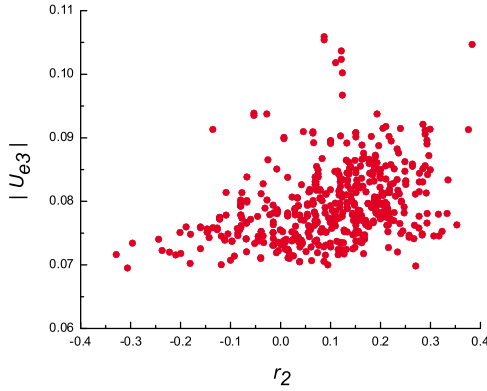


FIG. 1 (color online).  $|U_{e3}|$  is plotted as a function of  $r_2$ .

to 0.7. The Yukawa matrices are assumed to be Hermitian to keep the model free from SUSY  $CP$  problem.

Finally, we show how the above proton decay suppression arises by an adjustment among different VEVs. As noted, we need to have  $r_3 \approx 0$  in Eq. (3). Since this is satisfied in Eq. (10) for the SU(5) condition, it may be a hint that we stay close to the SU(5) symmetric vacuum. Secondly, we need  $r_2 \approx 0$  and suppression of the colored Higgs contributions from  $\mathbf{126}$  submultiplets, namely,

$$U_{14} \gg V_{15}, X_{14}, Y_{15}. \quad (11)$$

We denote the VEVs of the submultiplets in  $\mathbf{210}$  multiplet as follows:  $\Phi_1 : (\mathbf{1}, \mathbf{1}, \mathbf{1})$ ,  $\Phi_2 : (\mathbf{15}, \mathbf{1}, \mathbf{1})$ ,  $\Phi_3 : (\mathbf{15}, \mathbf{1}, \mathbf{3})$  (where numbers in the parentheses denote  $G_{422}$  quantum numbers). Recall that in the SU(5) symmetric vacua [14,15], the  $\Phi_i$ 's satisfy the relation  $\sqrt{6}\Phi_1 = \sqrt{2}\Phi_2 = \Phi_3$  (using the same normalization as in Ref. [14]). Perturbing the Higgs potential with a small coupling,  $\lambda_2 H \Delta \Phi$ , we obtain  $r_3 \propto \lambda_1 (\sqrt{6}\Phi_1 - \Phi_3)$  (where  $\lambda_1$  is associated with the  $\lambda_1 H D \Phi$  term). If  $\sqrt{6}\Phi_1 = \Phi_3$ , we have  $r_3 \approx 0$ . We also obtain the Higgs mixings  $U_{14} \approx -6\sqrt{5}\lambda_2/\eta \frac{\sqrt{2}\Phi_2 - \Phi_3}{\sqrt{6}\Phi_1 + \sqrt{2}\Phi_2 + 8\Phi_3} + \dots$ ,  $X_{14} \approx -2\sqrt{15}\lambda_2/\eta \times \frac{\sqrt{6}\Phi_1 - \sqrt{2}\Phi_2}{\sqrt{6}\Phi_1 + 3\sqrt{2}\Phi_2 + 6\Phi_3} + \dots$ , where  $\eta$  is a coupling of the  $\Phi \Delta \Delta$  term, and similar terms for  $V_{15}$  and  $Y_{15,16}$ . All these terms have different denominators. All the Higgs mixing angles tend to zero in the limit  $\lambda_2 \rightarrow 0$ . However, suppose that  $\sqrt{6}\Phi_1 + \sqrt{2}\Phi_2 + 8\Phi_3 \sim 0$  is satisfied, only  $U_{14}$  can be of finite value, and Eq. (11) is satisfied. This is just an example, and in our detailed quantitative work we keep all other terms in the Higgs potential, and we satisfy Eqs. (10) and (11) to suppress the proton decay rate.

In conclusion, we have analyzed the fermion masses and dimension five  $\Delta B = 1$  operators in the minimal SO(10) model with  $\mathbf{10}$ ,  $\mathbf{126}$ , and  $\mathbf{120}$  Higgs fields coupling to matter. We show that by a choice of suitable textures, one can not only get correct fermion masses and mixings but also suppress the contributions to proton decay from both the  $LLLL$  and  $RRRR$  operators for the entire range of allowed  $\tan\beta$  parameter of MSSM. This choice of textures

requires a suitable SO(10) breaking vacuum condition which is close to SU(5) invariant vacua. In the favorable region of parameter space  $U_{e3}$  is predicted to be 0.07–0.09 and  $\sin|\delta_{MNS}|$  to be 0.3 to 0.7.

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