Multiphoton Path Entanglement by Nonlocal Bunching

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Multiphoton path entanglement is created without applying postselection, by manipulating the state of stimulated parametric down-conversion. A specific measurement on one of the two output spatial modes leads to the *nonlocal bunching* of the photons of the other mode, forming the desired multiphoton path entangled state. We present experimental results for the case of a heralded two-photon path entangled state and show how to extend this scheme to higher photon numbers.

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Multiphoton path entangled states are superpositions of n photons in one out of two or more paths. Such states can be used to exceed the limitations imposed by the light wavelength. One example is quantum photolithography where the multiphoton interference of the different paths is used to define details on a special photoresist film which are $1/n$ finer than the diffraction limit [1,2]. The photoresist should respond only to n photons or more, which is still an open challenge. Other uses are the enhancement of the resolution of interferometric measurements [3–5] and atomic spectroscopy [6,7]. In the context of interferometry, path entangled states are a subset of a more general group of usable photon-number correlated states [3,8].

Previously suggested methods to produce path entangled states either require large nonlinearities, nonunitary operations, nondetection or include large statistical bottlenecks [9–12]. Nondetection can be replaced by postselection, which actually destroys the state. Furthermore, the schemes where $n > 2$ rely on the availability of various Fock states, which are difficult to produce. Two photons from parametric down-conversion can bunch to form a path entangled state [13], but this source is not expandable to larger photon numbers. Recently, a state of three path entangled photons was observed with postselection through a bottleneck [14].

In this Letter we present a way to create multiphoton path entangled states without applying postselection. The scheme relies on two unique quantum-mechanical phenomena: bunching of bosons and nonlocality. The former reflects the discreteness and symmetries of the quantum world. For example, it leads to the Hong-Ou-Mandel effect that two indistinguishable photons entering a beam splitter simultaneously from both sides will always exit at the same output port [15]. The latter implies that two (or more) distant particles can occupy a single quantum state and possess correlations which no classical theory can explain [16].

The scheme addressed in this Letter is based on the manipulation of multiphoton entangled states that originate from stimulated parametric down-conversion (PDC) [17,18]. By a specific measurement of one of the PDC output spatial modes, the photons of the other mode *nonlocally bunch* and form the desired multiphoton path entangled state. It should be emphasized that because no detection is needed at the second PDC mode, the desired state is prepared by preselection. The detection in the first mode is used as a heralding signal that announces the creation of the path entangled state in the second mode.

We used noncollinear type-II parametric down-conversion with spatial and spectral filtering to create the following bipartite state [19]

$$|\psi\rangle = \frac{1}{\cosh^2\tau}\sum_{n=0}^{\infty}\sqrt{n+1}\tanh^n\tau|\psi_n^-\rangle, \tag{1a}$$

$$|\psi_n^-\rangle = \frac{1}{\sqrt{n+1}}\sum_{m=0}^{n}(-1)^m|n-m,m\rangle_a|m,n-m\rangle_b, \tag{1b}$$

where $|m,n\rangle_i$ represents m horizontally and n vertically polarized photons in mode i. The magnitude of the interaction parameter τ depends on the nonlinear coefficient of the crystal, its length, and the intensity of the pump pulse. The state ψ, as well as its individual terms ψ_n^- of different photon-pair number n, are invariant under mutual rotations of the polarization basis of both spatial modes. The one-pair term ($n = 1$) is the familiar ψ^- Bell state. We concentrate on the case when two indistinguishable photon pairs are produced ($n = 2$). It will subsequently be shown how the presented method extends to larger numbers of photons.

The two-pair normalized term contains three equally weighted elements:

$$|\psi_2^-\rangle = \frac{1}{\sqrt{3}}(|2,0\rangle_a|0,2\rangle_b - |1,1\rangle_a|1,1\rangle_b + |0,2\rangle_a|2,0\rangle_b). \tag{2}$$

The goal is to perform a measurement on mode a that will prepare mode b in a path entangled state. To achieve this, consider the setup presented in Fig. 1(a). The middle term of Eq. (2) has one horizontally and one vertically polarized photon in mode a as well as in mode b. The photons in mode a are separated by a polarizing beam splitter (PBS). A $\lambda/2$ wave plate in one of the output arms of the PBS rotates the polarization in that arm to the polarization of the other. The two photons from the middle term, now indis-

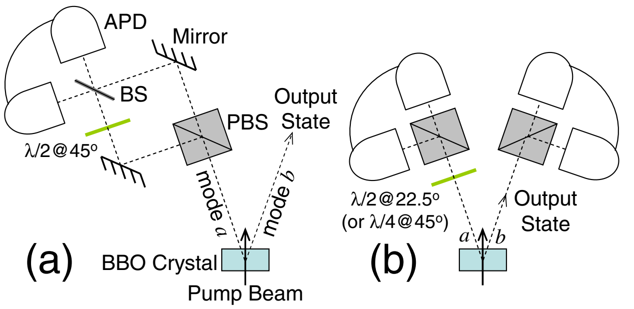

FIG. 1 (color online). (a) A schematic setup for nonlocal bunching of two photons. The polarized photons from one of the spatial modes of PDC (mode a) are separated on a polarization beam splitter (PBS), the polarization of one arm is rotated by a $\lambda/2$ wave plate at 45°, and the two arms are combined on a beam splitter (BS). Coincidence detection is marked by connected single-photon avalanche photodiodes (APD). (b) A simpler but equivalent scheme is to measure coincidence between the polarizations in a rotated polarization basis ($\lambda/2$ at 22.5° or $\lambda/4$ at 45°).

tinguishable in polarization, bunch on a 50/50 beam splitter (BS), and therefore they cannot give rise to a coincidence detection between the two detectors [15]. Therefore, when such a coincidence is observed between the two detectors in mode a, it could only have originated from the first and third terms of Eq. (2). The coherence between the two terms is preserved by the coincidence measurement, thus projecting the state of mode b to $|\psi_b\rangle = |0,2\rangle_b + e^{i\phi}|2,0\rangle_b$ (we drop further normalization for simplicity). The phase ϕ is determined by the difference between the lengths of the two arms after the PBS in mode a. The coincidence detection heralds the successful production of a path entangled state of two photons in mode b. Entanglement in photon numbers is created between two polarization modes rather than two paths. A polarization beam splitter and a $\lambda/2$ wave plate can translate between the two representations.

Using the equivalence between the operation of beam splitters on two spatial modes and the operation of wave plates on two polarization modes [20], it is possible to considerably simplify the required coincidence measurement. As shown in Fig. 1(b), the two modes a_h and a_v bunch when the polarization is rotated from the horizontal-vertical linear polarization basis (HV) to either the plus-minus 45° linear (PM) or right-left circular (RL). With the same argument as above, it can be seen that coincidence at mode a results in the desired state ψ_b at mode b. Actually, because of the rotational invariance of the state of Eq. (2), coincidence detection in mode a at any polarization state implies bunching of mode b in the other two polarization bases. The difference between the bunched states in the two bases is the sign between their two terms.

The nonlocal bunching result can be understood from another point of view—starting from the detectors and propagating backwards along the photon paths through the optical elements. The detection operation is represented by annihilation operators, e.g., a_h for a detection of a horizontally polarized photon in mode a. The two-photon coincidence detection operator in mode a is the product $a_h a_v$. This operator is transformed at a $\lambda/2$ wave plate to $a_h^2 - a_v^2$ and at a $\lambda/4$ wave plate to $a_h^2 + a_v^2$. Applying the transformed detection operator to mode a of Eq. (2) nonlocally collapses mode b to the bunched state ψ_b with an efficiency of 1/3:

$$|\psi\rangle = (a_h^2 + e^{i\theta}a_v^2)|\psi_2^-\rangle = |0,0\rangle_a \otimes (|2,0\rangle_b + e^{i\theta}|0,2\rangle_b), \tag{3}$$

where θ is a birefringent angle in the polarization representation that equals 0 or π, depending on the choice of measurement basis of mode a (RL or PM, respectively).

In order to demonstrate nonlocal bunching, we down-converted 200 fs pulses at 390 nm in a beta-barium borate (BBO) crystal with a double-pass configuration [17]. From the measured rates, the interaction parameter τ was evaluated to be about 0.1; thus the production ratio of three-to-two pairs was less than 2%. The state of mode b was analyzed by two single-photon detectors b_h and b_v behind the horizontally and vertically polarized output modes of a polarization beam splitter [Fig. 1(b)]. To prove path entanglement we first observed the absence of coincidences in mode b, implying that photons travel in pairs, and then examined the state coherence by interfering its two components.

Visibility measurements were taken by fixing the polarization basis of mode a and recording various coincidences while rotating the polarization basis of mode b. When the polarization bases are different in the two modes, the twofold coincidence on mode a bunches the photons in mode b. The bunching prevents the $a_h a_v b_h b_v$ fourfold coincidence and corresponds to the minima points (dashed lines) in Fig. 2. Tsujino *et al.* [21] showed that the visibility of this fourfold coincidence is related to the content α of indistinguishable two-photon pairs, defined as

$$|\psi\rangle = \sqrt{\alpha}|\psi_2^-\rangle + \sqrt{1-\alpha}|\psi_{1,\mathrm{I}}^-\rangle \otimes |\psi_{1,\mathrm{II}}^-\rangle, \tag{4}$$

where roman digits mark a distinguishing quantum number. In their experiment they evaluated the content of the indistinguishable state to be 37%. Figure 2 presents visibility curves for two polarization settings. From the measured fourfold visibility of 79% ± 2% we calculate α to be 83% ± 1%.

The visibility measurements indicate that a two-photon path entangled state was produced in one of the two down-conversion modes. In order to observe the presence of the two terms of the state and their coherence, we interfered them on a beam splitter. We used again the analogy between the two spatial modes of a beam splitter and the two polarization modes and interfered the b_h and b_v modes with a $\lambda/2$ wave plate at 22.5°. Before the wave plate, a phase θ_b between the two polarization modes was introduced by tilting a birefringent crystal. As the birefringent phase is scanned, θ of Eq. (3) varies as $2\theta_b$ and the state

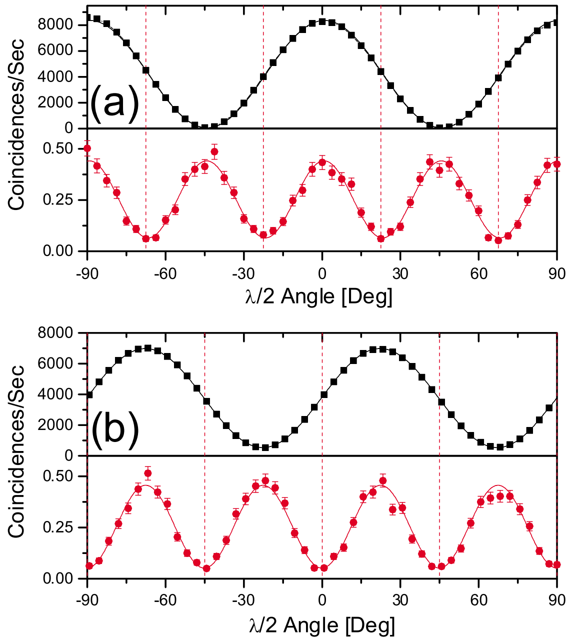

FIG. 2 (color online). Twofold (squares) and fourfold (circles) visibilities and their fits (solid lines). (a) Mode a at the HV basis and mode b is scanned between HV and PM. (b) Mode a at the PM basis and mode b is scanned between HV and PM. Path entangled states are created at $\pm 22.5°$ and $\pm 67.5°$ for (a) and at $0°, \pm 45°$ and $\pm 90°$ for (b) (dashed lines).

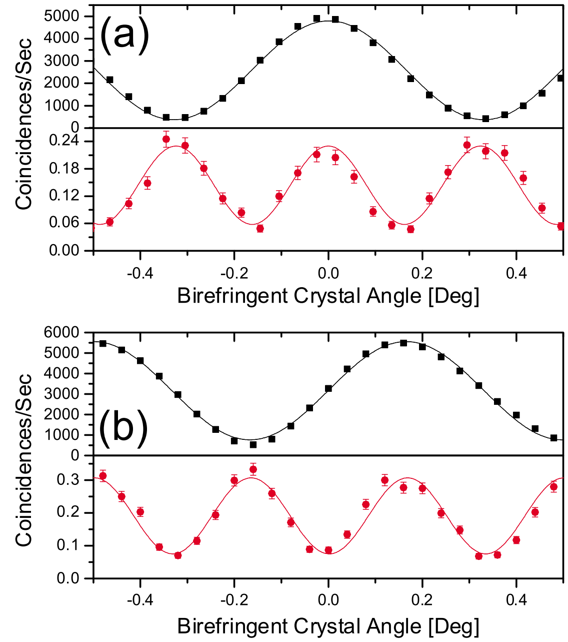

FIG. 3 (color online). Twofold (squares) and fourfold (circles) coincidences as a function of the birefringent phase. Fits are presented as solid lines. (a) Mode b is bunched at the HV basis by detecting mode a in the PM basis. (b) Mode b is bunched at the HV basis by detecting mode a in the RL basis.

behind the $\lambda/2$ wave plate oscillates between $|2,0\rangle_b + |0,2\rangle_b$ and $|1,1\rangle_b$ at twice the induced phase. These oscillations were observed by detecting the fourfold coincidences of $b_h b_v$ conditioned on $a_h a_v$. They are compared in Fig. 3 to the oscillations of the $a_h b_v$ twofold coincidence which follows θ_b. The state of mode b was bunched in the HV basis, once by coincidence detection of mode a in the PM basis (resulting in $\theta = \pi$) and once in the RL basis ($\theta = 0$). Thus, at zero birefringent phase the fourfold detection has a maximum in the first case and a minimum in the second.

In order to extend the scheme to higher photon numbers, operations that bunch more detection operators should be used. This is a generalization of the idea presented in Refs. [10–12], but it does not require nondetection or postselection. In order to bunch n photons at a certain polarization basis p (defined by an arbitrary axis crossing the Poincaré sphere through its center; see Fig. 4), one should combine n polarized photons q_m, residing equidistantly on the great circle whose plane is perpendicular to that axis. The product of the n linear annihilation (or creation) operators q_m is a different representation for the bunched state in the p basis:

$$q_m = p_+ + e^{i[(2\pi m+\theta)/n]} p_-,$$
$$\prod_{m=0}^{n-1} q_m = p_+^n - e^{i(n\pi+\theta)} p_-^n, \qquad (5)$$

where $p_\pm$ are the annihilation operators at the two polarizations defined by the axis. Notice how only two photons q_0 and q_1 of orthogonal polarizations in an arbitrary basis can have many great circles passing through them; thus their bunching can be observed in any basis p on the equatorial between them.

An example of a setup for four-photon path entanglement generation is shown in Fig. 5(a). The two output modes of the beam splitter are marked as a' and a''. The fourfold coincidence operator is $a'_h a'_v a''_h a''_v$. It is transformed by the two wave plates to $(a'^2_h - a'^2_v)(a''^2_h + a''^2_v)$. The beam splitter combines the two mode operators to one

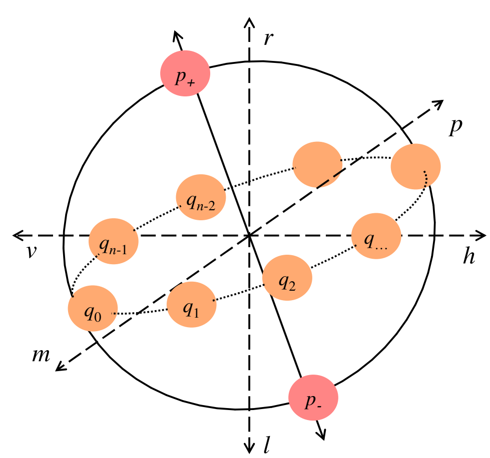

FIG. 4 (color online). Poincaré sphere representation of bunching of n photons. The n equidistant photons q_m reside on a great circle and bunch at the $p_\pm$ polarizations.

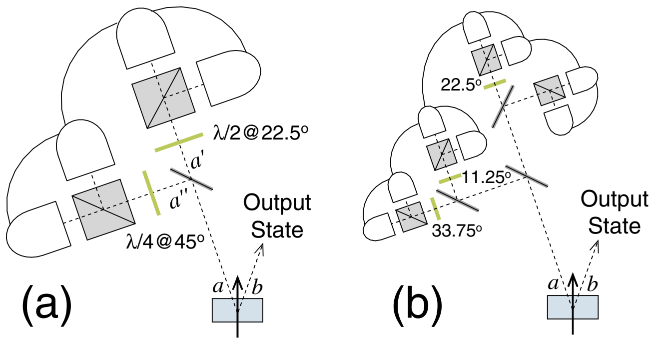

FIG. 5 (color online). A schematic setup for extension of the scheme for nonlocal bunching. Outputs are (a) a path entangled state of four photons in the HV basis and (b) eight photons in the RL basis. All wave plates in (b) are $\lambda/2$.

$(a_h^4 - a_v^4)$ with an efficiency of $\binom{4}{2} = \frac{6}{16}$. Applying this operator to the five terms of ψ_4^- [Eq. (1b)] results in the heralded four-photon path entangled state with an efficiency of 3/80:

$$|\psi\rangle = (a_a^4 - a_v^4)|\psi_4^-\rangle = |0, 0\rangle_a \otimes (|4, 0\rangle_b - |0, 4\rangle_b). \quad (6)$$

Using a similar argument of rotational invariance, it is clear that bunching of the photons in mode b will occur in any polarization basis as long as modes a' and a'' are detected in the other two complementary bases. A schematic for the generation of an eight photon state is also shown [Fig. 5(b)].

In conclusion, we demonstrated a scheme for a heralded source of path entangled photon states by nonlocal bunching. The photon resource is stimulated parametric down-conversion which is relatively easy to produce compared to pure Fock states as demanded by other proposals. The scheme is generally expandable to higher photon numbers by using beam splitters to combine detection in different polarization bases. This heralding detection signal can be used to open a polarization insensitive switch in mode b, filtering out the lower photon-number content. The generation of a heralded two-photon path entangled state was detected by observing interference at half the photon wavelength.

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