Transport Spectroscopy of Kondo Quantum Dots Coupled by RKKY Interaction

Maxim G. Vavilov[1,*] and Leonid I. Glazman[2]

[1]*Center for Materials Sciences and Engineering, MIT, Cambridge, MA 02139, USA*

[2]*Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455, USA*

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We develop the theory of conductance of a quantum dot which carries a spin and is coupled via RKKY interaction to another spin-carrying quantum dot. The found dependence of the differential conductance on the bias and magnetic field at a fixed RKKY interaction strength may allow one to distinguish between the possible ground states of the system. Transitions between the ground states are achieved by tuning the RKKY interaction, and the nature of these transitions can be extracted from the temperature dependence of the linear conductance. The feasibility of the corresponding measurements is evidenced by recent experiments by Craig *et al*.

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The exchange interaction between a localized electron and itinerant electrons of a Fermi sea leads to the Kondo effect [1]. Recently, the Kondo effect was observed in the quantum dot setting, where it causes an anomalously high conductance at low temperatures [2]. The itinerant electrons not only screen an impurity spin, leading to the Kondo effect, but also give rise to the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between localized spins [3]. The interplay between the Kondo screening and RKKY interaction remains at the focus of the investigation of strongly correlated electron systems and may play an important role in the heavy fermion metals [4]. This interplay is not trivial even in the minimal system allowing it, which consists of two localized spins "imbedded" into an electron Fermi sea [5].

Until recently, such a two-spin system was the subject of theoretical investigations only. The quest for practical implementation of quantum computing ideas has led to interest in the physics of spin devices. In this context, transport properties of two-spin-carrying quantum dots, coupled with each other by RKKY interaction, were studied experimentally [6] in a device schematically shown in Fig. 1. The authors of Ref. [6] were able to see the effect of RKKY interaction between the spins by monitoring the Kondo-enhanced conductance through one of the dots. Sufficiently strong RKKY interaction locks the localized spins into a singlet or triplet state and destroys or weakens the Kondo effect, thus liquidating the enhancement of the conductance. The experiments[6] demonstrated the fact of spin coupling and detailed quantitative measurements seem to be within the reach of experimental capabilities. Further experiments will also bring an exciting prospect of an experimental investigation of the interplay between RKKY interaction and Kondo effect in a controllable setting.

In this Letter we develop a theory of conductance spectroscopy of the spin states of two $s = 1/2$ quantum dots coupled by the RKKY interaction. We start with a characterization of the ground state and low-energy excitation spectrum of the many-body system formed by the dots and itinerant electrons of a two-dimensional electron gas (2DEG). Next we concentrate on the case of a strong RKKY interaction, allowing us to treat the Kondo effect perturbatively and elucidate the main features of the I-V characteristic of the device sketched in Fig. 1. The measurement of differential conductance $G(V) = dI/dV$ enables one to distinguish between the singlet and triplet ground states of the two localized spins. We then consider the crossover between the singlet and triplet states on the linear conductance $G_0 \equiv G(V = 0)$. We identify signatures in the linear conductance which may allow us to distinguish the singlet-triplet crossover from a quantum phase transition between these two states. Finally, we investigate the sensitivity of the RKKY interaction to an applied magnetic flux and determine the characteristic flux needed to change the sign of the RKKY coupling.

The ground state of two localized spins interacting with each other and with a Fermi sea of itinerant electrons depends sensitively on the relations between the corresponding interaction constants. Under very special conditions,[7] requiring fine-tuning of the system parameters, a non-Fermi-liquid state of the system may be reached. Away from these special points in the parameter space, the low-energy properties of the system are that of a Fermi liquid, and we will concentrate on this generic case. The ground state of the full system, including the localized and

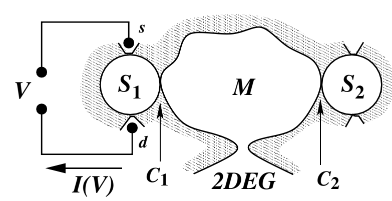

FIG. 1. The exchange interaction of $S = 1/2$ spins of dots S_1 and S_2 with the 2DEG results in the Kondo effect. The connection by weak contacts C_1 and C_2 to the bigger open dot M creates the exchange (RKKY) interaction between the two spins. The current I as a function of bias V is measured between source (s) and drain (d) contacts.

itinerant spins, is a singlet. Variation of the exchange couplings leads to a number of crossovers between different possible singlet states. The simplest Hamiltonian sufficient for describing the singlet states is

$$H = J_1 \hat{s}_1 \hat{\mathbf{S}}_1 + J_2 \hat{s}_2 \hat{\mathbf{S}}_2 + J_{12} \hat{\mathbf{S}}_1 \hat{\mathbf{S}}_2. \tag{1}$$

Here $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spins of the two dots ($S_{1,2} = 1/2$), and $\mathbf{s}_\alpha = \sum_{\sigma\sigma'} \psi^\dagger_{\alpha\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} \psi_{\alpha\sigma'}$ are the local spin densities of itinerant electrons. Electron operators $\psi_{\alpha\sigma}$ represent electron states [8] coupled to a localized spin in dot α.

First we note that at $J_{12} = 0$ the Hamiltonian Eq. (1) represents two independent $s = 1/2$ Kondo systems characterized by Kondo temperatures $T_{K\alpha} \propto \exp(-1/\nu J_\alpha)$, where ν is the band density of states at the Fermi level, $\alpha = 1, 2$. At $T = 0$, each of the two local spins is fully screened by the corresponding modes of the itinerant electrons. In these conditions, each of the dots provides a Kondo resonance for electron tunneling [8]. The independent screening, giving rise to resonances in tunneling through each of the dots, remains in effect for a sufficiently small interdot coupling. At stronger antiferromagnetic coupling, $J_{12} \gg \max_\alpha\{T_{K\alpha}\}$, the two local spins form a singlet of its own, and the Kondo resonances for the itinerant electrons vanish. In the case of ferromagnetic ($J_{12} < 0$) coupling, the two localized spins form a triplet, which is fully screened by the itinerant electrons interacting with the two dots, and the Kondo resonances for tunneling through each of the dots persist.

The above consideration shows that the zero-temperature linear conductance changes from a large value (induced by the Kondo resonance) at $(-J_{12}) \gg \max_\alpha\{T_{K\alpha}\}$, to a small value at $J_{12} \gg \max_\alpha\{T_{K\alpha}\}$. In the oversimplified representation of the device sketched in Fig. 1 by the Hamiltonian (1), these asymptotes are

$$G_U = \frac{4e^2}{\pi\hbar} \frac{G_s G_d}{(G_s + G_d)^2} \tag{2}$$

and $G = 0$, respectively (here G_s and G_d are the conductances of the junctions connecting the dot with the source and drain leads, respectively). In the special case [9] of $T_{K2} = 0$, the transition between the two asymptotes is a jump at $J_{12} = 0$ between two Fermi-liquid states. If T_{K2} is finite, the transition shifts to $J_{12} \sim T_{K1}/\ln(T_{K1}/T_{K2})$ for $T_{K2} \ll T_{K1}$, and remains at positive $J_{12} \lesssim \max_\alpha\{T_{K\alpha}\}$. This transition occurs via passing through a non-Fermi-liquid state, which belongs to the same universality class as the two-channel $s = 1/2$ Kondo problem [10]. The existence of such a non-Fermi-liquid state hinges on a special particle-hole symmetry [7] of the Hamiltonian (1). However, in the case of generic parameters of a quantum dot, the Hamiltonian of the system also includes other terms (e.g., potential scattering terms leading to the elastic cotunneling) which violate the required symmetry. In this case the quantum phase transition between the two Fermi liquids is replaced by a smooth crossover [11,12]. The zero-temperature conductance varies smoothly and monotonically with J_{12}, as shown by the solid line in Fig. 2(a) [13].

In the absence of the Zeeman splitting, the energy of exchange interaction J_{12} sets the threshold for the inelastic electron scattering, accompanied by a change of the spin state of two dots. Far above the threshold, $|eV| \gg |J_{12}|$, the processes with flip of the spin $\mathbf{S}_1$ are allowed and, in the leading order, the differential conductance $G(V)$ coincides with the conductance of a single-quantum-dot device:

$$G(V) = \frac{3}{4} \frac{G_U}{\ln^2|eV/T_{K1}|}. \tag{3}$$

Here factor $3/4$ corresponds to the square of the operator of spin $S = 1/2$, and the logarithmic term represents the Kondo renormalization of the exchange interaction in the weak coupling limit, $eV \gg T_{K1}$.

Conductance Eq. (3) arises from the scattering amplitude evaluated to the lowest order perturbation theory in the renormalized interaction constant. Within this accuracy, conductance has a sharp step at $|eV| = |J_{12}|$. The height of the step depends on the sign of J_{12}, i.e., on whether the coupling of the two localized spins is ferromagnetic or antiferromagnetic. In the case of ferromagnetic coupling, $J_{12} < 0$, conductance $G(V)$ is reduced by factor $3/2$ from the value given by Eq. (3) if the bias is lowered just below the threshold $|eV| = -J_{12}$. The renormalization group analysis of the Hamiltonian (1) shows that after the reduction at $|eV| = -J_{12}$, the differential conductance again monotonically increases as bias V decreases:

$$G(V) = \frac{2G_U}{\ln^2|eV/T_{Kt}|}. \tag{4}$$

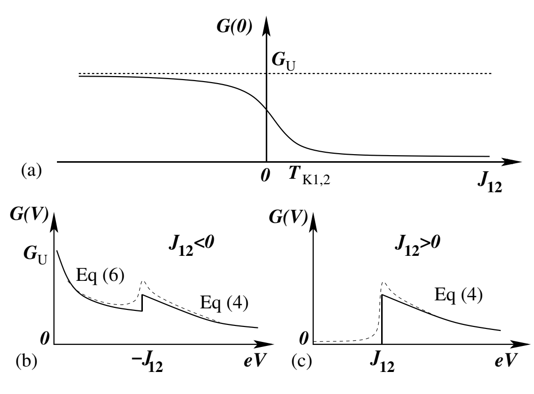

FIG. 2. (a) Linear conductance G_0 as a function of the RKKY coupling constant J_{12} at $T = 0$. The crossover from the triplet state $J_{12} < 0$ at $|J_{12}| \gg \max_\alpha\{T_{K\alpha}\}$ to a singlet state $J_{12} \gg \max_\alpha\{T_{K\alpha}\}$ takes place at $|J_{12}| \lesssim \max_\alpha\{T_{K\alpha}\}$ and results in the decrease of G_0 from a value close to the unitary limit G_U to a much smaller value controlled by the elastic cotunneling. (b), (c) Differential conductance $G(V)$ at $T = 0$ and strong RKKY coupling, $|J_{12}| \gg \max_\alpha\{T_{K\alpha}\}$. Solid lines: the second-order in exchange J_1 result; dashed lines: schematic representation of the enhancement of $G(V)$ resulting from higher order in J_1 calculation.

Here factor 2 corresponds to the square of $S = 1$ spin operator, and the logarithmic increase of $G(V)$ reflects the $S = 1$ Kondo effect with the triplet Kondo temperature T_{Kt}. In case $T_{K2} \ll T_{K1} \ll (-J_{12})$, an evaluation of the first logarithmic correction to the conductance gives $T_{Kt} = T_{K1}^2/|J_{12}|$; i.e., at large $|J_{12}|$ the triplet Kondo temperature is much lower than $\max_\alpha\{T_{K\alpha}\}$. For the antiferromagnetic coupling between the two localized spins, the particle-hole symmetric Hamiltonian (1) yields zero conductance [13].

Magnetic fields may shift and split the step in conductance $G(V)$. The proper generalization of Eq. (3) to include the Zeeman splitting $g\mu B$ by magnetic field B reads

$$G(V) = \frac{f_1(J_{12}, V, B)G_U}{\ln^2[\max\{g\mu B, eV\}/T_{K1}]}, \tag{5}$$

with a steplike function f_1, presented in Fig. 3. At $g\mu B \gg |J_{12}|, |eV|$, the localized spins are aligned along the field and $f_1 = 1/4$. If $J_{12} < 0$, Eq. (5) must be supplemented by condition $\max\{g\mu B, eV\} \gtrsim \sqrt[3]{T_{K1}^2 J_{12}}$; otherwise a $S = 1$ Kondo effect develops, see Eq. (4).

To assess the applicability of the above results obtained in the Born approximation for the electron amplitude of tunneling through the dot, one may evaluate the next-order correction to that amplitude. This correction diverges logarithmically at $|eV| = |J_{12}|$, similar to the divergence occurring in the scattering off a single localized spin in the presence of Zeeman splitting [14]. The divergence is cut off by either the temperature [14] or the relaxation rate of the localized spins in the out-of-equilibrium conditions [15]. As the result, steps in the differential conductance, obtained within the Born approximation, become asymmetric maxima of a finite width. Here we present an estimate for the correction to Eq. (4) only for the case of strongly asymmetric setup, e.g., $G_s/G_d \ll 1$, and assuming the lowest temperature, which allows for the sharpest steps in the differential conductance. In these conditions, the spin relaxation rate [16] responsible for the cutoff of the divergency is of the order of $1/\tau_s \sim |J_{12}|/[\ln^2(|J_{12}|/T_{K1})]$. The widths of the features replacing steps in the differential conductance are of the order of $\sim 1/\tau_s$, and the amplitudes of the corrections to steps described by Eq. (5) are

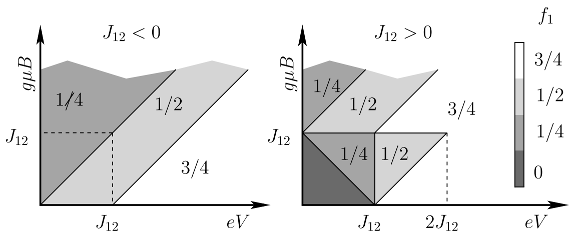

FIG. 3. The contour plot of $f_1(J_{12}, V, B)$ is shown in the bias (V)—magnetic field (B) plane. Function f_1 determines the differential conductance $G(V)$ at strong RKKY coupling $|J_{12}| \gg \max_\alpha\{T_{K\alpha}\}$, see Eq. (5).

$$\delta G \sim \frac{G_U}{\ln^3(|J_{12}|/T_{K1})} \ln\left[\ln^2 \frac{|J_{12}|}{T_{K1}}\right]. \tag{6}$$

The amplitude of the correction is small if $|J_{12}| \gg T_{K1}$, and the main effect of the higher-order terms of the perturbation theory is the smearing of the steps. In order to determine the nature of the ground state (singlet or triplet) by conductance measurements in the presence of magnetic field, the Zeeman splitting energy must exceed $1/\tau_s$ of the features in the differential conductance. The overall bias dependence of the differential conductance at $B = 0$ is sketched in Fig. 2(b) and 2(c).

Linear conductance G_0 of the dot S_1 is determined by the scattering T matrix $T_1(\varepsilon)$ for $\varepsilon \lesssim T$:

$$G_0(T, J_{12}) = G_U \int \frac{\mathrm{Im}T_1(\varepsilon)d\varepsilon}{4T\cosh^2(\varepsilon/2T)}. \tag{7}$$

In the unitary limit, $T_1(0) = i$. For $\varepsilon \sim T \gtrsim \max_\alpha\{T_{K\alpha}\}$ we may use the Born approximation [17]:

$$\mathrm{Im}T_1(\varepsilon) = \nu^2 \mathcal{J}_\varepsilon^2 \int K(\varepsilon - \varepsilon') \frac{1 + e^{\varepsilon/T}}{1 + e^{\varepsilon'/T}} d\varepsilon', \tag{8}$$

where $K(\omega) = 2\pi \sum_{\xi\xi'} \rho_\xi |\langle\xi'|\mathbf{S}_1|\xi\rangle|^2 \delta(\omega + E_\xi - E_{\xi'})$ is calculated with respect to the exact quantum states $|\xi\rangle$ of the system of two localized spins, E_ξ is the energy of state $|\xi\rangle$, and ρ_ξ is the density matrix $\rho_\xi \propto \exp(-E_\xi/T)$. The exchange constant $\mathcal{J}_\varepsilon$ is logarithmically renormalized by the Kondo effect: $\mathcal{J}_\varepsilon = 1/[\nu \ln E/T_{K1}]$ with $E = \max\{\varepsilon, T, |J_{12}|\}$.

Within Born approximation, the linear conductance depends on J_{12} only through the ratio J_{12}/T,

$$G_0 = \frac{3}{2(3 + e^{J_{12}/T})}\left[1 + \frac{J_{12}/T}{1 - e^{-J_{12}/T}}\right]\frac{G_U}{\ln^2(T/T_{K1})}. \tag{9}$$

The dependence of $G_0(T, J_{12})$ on J_{12} has a maximum in the region $|J_{12}| \lesssim T$ and conductance is higher on the triplet side of the crossover [18]. (Note, however, that the exponentially small value of conductance far in the singlet region is an artifact of our model [13].) At negative J_{12}, as temperature T decreases, the conductance $G_0(T, J_{12})$ grows and reaches G_U at $T \lesssim T_{K1}^2/|J_{12}|$. The shape of the $G_0(T, J_{12})$ vs J_{12} eventually approaches the step sketched in Fig. 2(a). In the generic case of the zero-temperature crossover between the two Fermi-liquid states, the width of the step saturates and remains finite in the limit $T \to 0$. This width is not universal and depends on the terms in the exact Hamiltonian beyond the approximation of Eq. (1). If, however, the parameters of the system are tuned properly, and the variation of J_{12} takes the system through the non-Fermi-liquid state at the certain value of J_{12}, then the step width decreases with lowering the temperature as $\sqrt{T}$; see, e.g., [19]. Finally, the limit $J_2 = 0$ within the model Eq. (1) corresponds to a sharp transition between two Fermi-liquid states at $T = 0$. A straightforward analysis, similar to Ref. [9], yields the estimate $T_{K1}/\ln(T_{K1}/T)$ for the transition width at finite temperature.

The contact interaction of the localized spins with the itinerant electrons of dot M at points C_1 and C_2 (see Fig. 1) results in the indirect exchange interaction between the localized spins. Unlike the textbook RKKY interaction facilitated by freely propagating electrons [3], here magnitude and sign of J_{12} are random, reflecting the chaotic electron motion in the dot M. Roughly, the typical value of J_{12} is $\sim\delta_1 J_{C_1} J_{C_2}$, and the typical magnetic flux Φ_c needed for changing J_{12} substantially is $\sim\Phi_0$. Here $J_{C_{1,2}}$ are the dimensionless constants of the contact exchange interaction at points $C_{1,2}$; $\delta_1 = (\nu A)^{-1}$ is the mean level spacing of one-electron energy levels in dot M, ν is the density of states of the 2DEG, A is the area of the dot M, and $\Phi_0 = hc/e$.

The RKKY coupling J_{12} may be expressed in terms of the scattering matrix $S_{1;2}$ of an electron propagating from contact C_1 to contact C_2:

$$J_{12} = -2J_{C_1}J_{C_2}\int\frac{d\varepsilon}{\pi}n(\varepsilon)\mathrm{Im}\{S_{2;1}(\varepsilon)S_{1;2}(\varepsilon)\}. \quad (10)$$

Here $n(\varepsilon)$ is the Fermi function and we assumed that electron propagation in M is spin independent: $S_{1\sigma;2\sigma'}(\varepsilon) = \delta_{\sigma\sigma'}S_{1;2}(\varepsilon)$. Within the random matrix theory, it is related in a standard way [20] to the one-electron Hamiltonian $\hat{\mathcal{H}}$ of dot M. The ensemble average $\langle J_{12}\rangle = 0$, and we calculate the correlation function [21] of the RKKY constant $J_{12}(\Phi)$ over realizations of matrix $\hat{\mathcal{H}}(\Phi)$ at two values of magnetic flux $\Phi_{1,2}$ threading dot M,

$$\langle J_{12}(\Phi_1)J_{12}(\Phi_2)\rangle = \frac{\delta_1^2}{16\pi^2}J_{C_1}^2 J_{C_2}^2\ln\left[\frac{E_{\mathrm{Th}}^2}{\mathcal{E}_+\mathcal{E}_-}\right]. \quad (11)$$

Here $\mathcal{E}_\pm = \gamma_{\mathrm{esc}} + \kappa\delta_1(\Phi_1 \pm \Phi_2)^2/\Phi_0^2$; numerical factor $\kappa \sim 1$ depends on geometry [20,22]; $\gamma_{\mathrm{esc}} = N\delta_1/(2\pi)$ is the electron escape rate from the middle dot into 2DEG through N open channels; $E_{\mathrm{Th}} = v_F/\sqrt{A}$ is the Thouless energy; and v_{F} is the Fermi velocity. At $T \gtrsim \mathcal{E}_\pm$, $\mathcal{E}_\pm$ should be replaced by T.

Equation (11) shows that the RKKY constant is symmetric with respect to the inversion of magnetic flux $\Phi \to -\Phi$, and yields the correlation flux value $\Phi_c^{J_{12}} \sim \Phi_0$. The flux $\Phi_c^{J_{12}}$ is much larger than the correlation flux $\Phi_c^G \sim \sqrt{\gamma_{\mathrm{esc}}/E_{\mathrm{Th}}}\Phi_0$ for the conductance of an open quantum dot [20]. The difference between $\Phi_c^{J_{12}}$ and Φ_c^G occurs because the contribution to J_{12} originates from energy levels within the spectrum "window" E_{Th}, whereas the conductance of a dot is usually determined by levels within much shorter energy interval γ_{esc}.

In conclusion, the presented results may help one to determine the spin states of quantum dots coupled by RKKY interaction from transport measurements; see Eq. (5) and Fig. 3. The evolution of the linear conductance with the variation of RKKY interaction constant allows one to follow the transitions between various ground states of the system. Finally, we found that the magnetic field flux needed for variation of the RKKY coupling is of the order of flux quantum Φ_0.

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*Present address: Department of Applied Physics, Yale University, New Haven, CT 06520.

[1] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, England, 1997).
[2] D. Goldhaber-Gordon *et al.*, Nature (London) **391**, 156 (1998); S. M. Cronenwett *et al.*, Science **281**, 540 (1998); J. Schmid *et al.*, Physica B (Amsterdam) **256-258**, 182 (1998).
[3] C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1987), p. 360.
[4] A. Amato, Rev. Mod. Phys. **69**, 1119 (1997).
[5] C. Jayaprakash *et al.*, Phys. Rev. Lett. **47**, 737 (1981); B. A. Jones and C. M. Varma, Phys. Rev. Lett. **58**, 843 (1987); Phys. Rev. B **40**, 324 (1989).
[6] N. J. Craig *et al.*, Science **304**, 565 (2004).
[7] I. Affleck *et al.*, Phys. Rev. B **52**, 9528 (1995); A. J. Millis *et al.*, in *Field Theories in Condensed Matter Physics*, edited by Z. Tesanovic (Addison-Wesley, Reading, MA, 1990), p. 159.
[8] M. Pustilnik and L. I. Glazman, J. Phys. Condens. Matter **16**, R513 (2004).
[9] M. Vojta *et al.*, Phys. Rev. B **65**, 140405(R) (2002).
[10] I. Affleck and A. W. W. Ludwig, Phys. Rev. B **48**, 7297 (1993).
[11] M. Pustilnik *et al.*, Phys. Rev. B **68**, 161303(R) (2003).
[12] R. M. Fye, Phys. Rev. Lett. **72**, 916 (1994).
[13] The cotunneling processes not accounted by Hamiltonian (1) result in a finite background conductance.
[14] J. Appelbaum, Phys. Rev. Lett. **17**, 91 (1966); J. A. Appelbaum, Phys. Rev. **154**, 633 (1967).
[15] J. Paaske *et al.*, Phys. Rev. B **70**, 155301 (2004).
[16] This rate can be evaluated similar to the Korringa relaxation rate; see, e.g., L. I. Glazman and M. Pustilnik, cond-mat/0501007.
[17] M. G. Vavilov *et al.*, Phys. Rev. B **68**, 075119 (2003).
[18] Linear conductance $G_0(T, J_{12})$ at fixed $|J_{12}| \gg T_{K1}$ has a maximum at $T \simeq 0.3|J_{12}|\sqrt{\ln|J_{12}|/T_{K1}}$ due to competition between the Kondo effect and the RKKY interaction; see, e.g., Ref. [17].
[19] M. Pustilnik *et al.*, Phys. Rev. B **69**, 115316 (2004).
[20] C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997); I. L. Aleiner *et al.*, Phys. Rep. **358**, 309 (2002).
[21] Statistics of the RKKY coupling in metals was studied in A. Yu. Zyuzin and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 185 (1986) [JETP Lett. **43**, 234 (1986)].
[22] K. B. Efetov, Phys. Rev. Lett. **74**, 2299 (1995).