Ruderman-Kittel-Kasuya-Yosida and Magnetic-Field Interactions in Coupled Kondo Quantum Dots

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We investigate theoretically the transport properties of two independent artificial Kondo impurities. They are coupled together via a tunable Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction. For strong enough antiferromagnetic RKKY interaction, the impurity density of states increases with the applied in-plane magnetic-field. This effect can be used to distinguish between antiferromagnetic and ferromagnetic RKKY interactions. These results may be relevant to explain some features of recent experiments by Craig *et al.* [Science **304**, 565 (2004)].

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Introduction.—Local exchange interactions between itinerant electrons and localized magnetic impurities in diluted alloys results in the Kondo effect. Its main signature is the formation of a very narrow peak at the Fermi level (E_F) in the density of states (DOS) of the localized spin at low enough temperatures $T \ll T_K$, where T_K is the Kondo temperature [1]. Magnetic impurities, even for tiny concentrations, also interact with one another via the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction, an indirect spin-spin interaction mediated by conduction electrons. The competition between both interactions leads to extremely rich and various behaviors such as spin glasses, non-Fermi-liquid fixed points, etc. [1]. Part of this complexity and richness is already embodied in the *two-impurity Kondo problem* [2–5].

Progressive advance in manufacturing nanostructures has enabled us to build up quantum dots that can act as a single spin-1/2 [6]. This has motivated the emergence of an exciting area of research based on the spin degrees of freedom, termed *spintronics*, with promising applications in quantum computing [7]. Experiments have been able to observe the Kondo effect in quantum dots [8], its most remarkable signature being the *unitary limit* of the linear conductance at zero temperature [9]. This is a consequence of the formation of a singlet state between the delocalized electrons and the localized spin in the dot. Continuous improvements of the fabrication techniques in semiconductor nanostructures have made possible the realization of more complicated structures such as double quantum dots. They can mimic the behavior of two artificial magnetic impurities coupled by a direct tunnel coupling [10]. In some recent experiments Craig *et al.* [11] have investigated a device in which two quantum dots are connected to a common open conducting region. Here RKKY and Kondo interactions compete, thus providing an experimental realization of the two-impurity Kondo problem. In Ref. [11] it was found that the strength of the nonlocal RKKY interaction can be used to control the Kondo effect in one dot by tuning gate parameters of the other dot. This constitutes the first observation of the RKKY indirect interaction in quantum dots opening the road toward the realization of multiple artificial impurity configurations with a high degree of tunability and a nonlocal spin control.

In this Letter we study the geometry depicted in Fig. 1 that corresponds to the experimental situation in [11]. Two small quantum dots are strongly connected to a central larger open area (see Fig. 1 for details). The whole system can be regarded as a triple dot. In contrast to other works dealing with the artificial realization of the two-impurity Kondo problem [12–14], we do not demand a symmetric situation where the two dots have the same Kondo temperature in order to reproduce the experimental situation. Therefore, three scales are *a priori* present in the system at zero temperature: the Kondo temperatures of each individual dots T_K^1, T_K^2 and the strength of the RKKY interaction I. When an in-plane magnetic field is applied we need to consider the Zeeman energy as well [15]. Here we analyze the transport properties in the triple dot system for various parameter ranges in relation to the experiment [11]. We also predict that an in-plane applied magnetic field competes with the antiferromagnetic (AFM) RKKY interaction and may induce a new Kondo effect in a way reminiscent

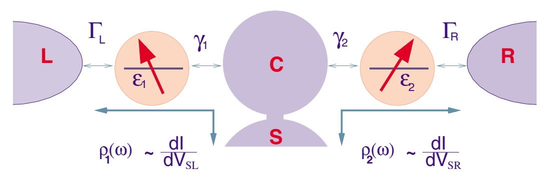

FIG. 1 (color online). Schematic picture of the setup containing two quantum dots. Each dot 1(2) is strongly connected to an open central conducting region (C) through the tunneling couplings $\gamma_{1(2)}$ and very weakly connected to its respective lead $L(R)$ by $\Gamma_{L(R)}$. Thus, the differential conductance ($dI/dV_{\rm SL(R)}$) between the S and $L(R)$ through dot 1(2) will reflect the DOS in dot 1(2).

of what happens in quantum dots with an integer number of electrons [16,17].

Model.—In the geometry depicted in Fig. 1 the two quantum dots are two magnetic impurities of spin-1/2 strongly coupled to the middle region (C) via hybridization widths γ_1, γ_2. We assume, as in the experiment, that the two dots are weakly connected to the leads through Γ_L, Γ_R such that $\Gamma_{L(R)} \ll \gamma_{1(2)}$ [11]. A simple Hamiltonian able to describe the system without the leads reads:

$$\mathcal{H} = \mathcal{H}_C + J_1 \vec{S}_1 \cdot \vec{\sigma}_1 + J_2 \vec{S}_2 \cdot \vec{\sigma}_2 + I \vec{S}_1 \cdot \vec{S}_2. \quad (1)$$

In Eq. (1) $\mathcal{H}_C$ describes a grain with a continuous DOS, $\vec{S}_{1(2)}$ is the spin operator of dot 1(2) and $\vec{\sigma}_{1(2)}$ is the local spin in region C coupled to dot 1(2). The Kondo coupling of dot 1(2) is given by $\pi\rho_c J_{1(2)} \approx \gamma_{1(2)}\{[1/(-\varepsilon_{1(2)})] + [1/(U_{1(2)} + \varepsilon_{1(2)})]\}$, where $\varepsilon_{1(2)}$ is the position of the last occupied level controlled by the dot gate voltage, $U_{1(2)}$ is the intradot Coulomb interaction, and ρ_c is the density of states in the middle region (C). The RKKY interaction is $I = J_1 J_2 \frac{\vartheta}{D}$, where D is the bandwidth of the conducting region and ϑ is a positive or negative constant depending on the interference pattern of the spin waves. The Kondo energy scales for each dot are defined by $T_K^{1(2)} \sim D \exp(-1/2\rho_c J_{1(2)})$.

In the measured nonlinear conductance [11], the zero bias anomaly (ZBA) disappears when increasing I and two small peaks at source-left lead voltage $eV_{\rm SL} = \pm I/2$ show up at equal distance from $E_F = 0$ corresponding to singlet-triplet states of the two localized spins in the dots. *A priori* the sign of I is not known. This suggest two possibilities: (i) the RKKY-like interaction is antiferromagnetic in the experiment or (ii) it is ferromagnetic with an experimental temperature $T > \max(T_K^{\rm even}, T_K^{\rm odd})$ preventing therefore the formation of the ZBA mentioned above. We shall demonstrate below how, by using an in-plane magnetic field, we can distinguish between the two situations. Our results suggest a positive sign for the RKKY-like interaction observed in the experiment.

Ferromagnetic RKKY interaction.—Let us start with $I < 0$. Depending on the relative strength of the ferromagnetic interaction and the two Kondo scales T_K^1, T_K^2 one finds different physical scenarios. For a *small* ferromagnetic coupling, either $|I| \ll T_K^1, T_K^2$ or $T_K^2 < |I| < T_K^1$, it is easy to show using renormalization group arguments that both dot spins are screened independently and each dot DOS shows the usual Kondo resonance. However, for a *large* ferromagnetic coupling ($|I| \gg T_K^1, T_K^2$) the spins of the dots add up in a $S = 1$ state. This state is screened in a two-stage procedure by conducting channels formed by the even and odd linear combination of the electrons in the central conducting region [2]. This defines two different Kondo temperatures, $T_K^{\rm even}$ and $T_K^{\rm odd}$, that are much lower than $T_K = \min(T_K^1, T_K^2)$ [2]. When T is lowered until $T \sim \min(T_K^{\rm even}, T_K^{\rm odd})$, one half of the unit of the $S = 1$ state is efficiently screened while the other half is free. By further lowering the temperature, $T \ll \min(T_K^{\rm even}, T_K^{\rm odd})$, the $S = 1$ state is completely screened leading to a Fermi liquid behavior with a Kondo resonance of width T_K in the DOS. Here, the linear conductance between the source and the left lead is $\mathcal{G}_{\rm (SL)} \sim 2e^2/h[4\Gamma_L\gamma_1/(\Gamma_L + \gamma_1)^2]$. To summarize we find that, for both cases, namely, small and large I compared with the two Kondo scales, T_K^1, T_K^2, and at very low temperatures ($T \to 0$), the DOS of each dot displays a Kondo resonance.

Antiferromagnetic RKKY interaction.—Let us now provide a quantitative analysis of the case $I > 0$. The competition between the Kondo effect and the AFM ordering results in a quantum critical point when $(I/T_K)_c \simeq 2.54$ in situations where there is a symmetry between even and odd parity channels [3] and T_K is the same Kondo temperature for both magnetic impurities. When this symmetry is broken the critical transition is replaced by a crossover [3]. When $I/T_K > (I/T_K)_c$ we have the AFM phase where the two impurities are combined into a singlet state. However, when the Kondo coupling becomes stronger, $I/T_K < (I/T_K)_c$, each impurity prefers to form its respective Kondo singlet with the delocalized electrons. This is the Kondo phase. Now we address the question of whether this competition is manifested in the transport properties when the two dots have different Kondo temperatures corresponding to the experimental geometry ($T_K^1 \neq T_K^2$ when $\varepsilon_1 \neq \varepsilon_2$ or $\gamma_1 \neq \gamma_2$).

Our results are based on a strong-coupling approximation, the so-called slave-boson mean field theory (SBMFT), which is known to capture the main physics in the Kondo regime [18]. This theory is then suitable for studying the Fermi liquid behavior of the system (i.e., $T \ll T_K$) in which the dot occupation is always 1/2 per spin. The SBMFT has been recently applied to study the Kondo effect in double quantum dots considering the same Kondo scale for each dot [12–14]. In particular, in Ref. [14] it was shown that the nonlinear conductance $\mathcal{G}(V) \equiv dI/dV$ for serial and parallel double quantum dots directly reflects the physics of the competition between the Kondo effect and the superexchange interaction.

Results.—The critical value at which the transition from the Kondo state (KS) to the AFM phase takes place can be obtained by comparing their ground state energies [13]: $E_{\rm GS}^K - E_{\rm GS}^{\rm AFM} = I/4 - (2/\pi)T_K$ (assuming $T_K = T_K^1 = T_K^2$). Thus, the transition can be estimated to occur at $(I/T_K)_c = 8/\pi \simeq 2.54$. However, this critical value changes here because both dots are not equally coupled to the central region and therefore they have different Kondo scales $T_K^1 \neq T_K^2$. We find that the critical point depends now on both Kondo temperatures as follows: $(\frac{I}{T_K^1})_c = \frac{4}{\pi}(1 + \frac{T_K^2}{T_K^1})$. For $0 \leq T_K^2 \leq T_K^1$, we have $4/\pi \leq (I/T_K^1)_c \leq 8/\pi$. Since T_K^1 and T_K^2 depend exponentially on the tunneling couplings and the level positions, a small asymmetry between these parameters induces a huge change in the ratio T_K^2/T_K^1. In order to illustrate this effect we plot in the inset of Fig. 2 the KS $\to$ AFM transition as a function of the asymmetry between the tunneling couplings γ_2/γ_1. Already with a slight asymmetry (around 10%) the transition lowers down to $(I/T_K^1)_c = (4/\pi) \approx 1.27$ in good agreement with the experimental value $(I/T_K^1)_c = 1.2$.

We solve the set of mean field equations within the SBMFT [12–14] for $\varepsilon_1 = -3.5$ and $D = 60$. This leads to a Kondo temperature for dot 1: $T_K^1 = 10^{-3}\gamma_1$. All energies are given in units of γ_1. We focus on two cases: (i) the symmetric case with $\gamma_1 = \gamma_2$ corresponding to $T_K^1 = T_K^2$ [Fig. 2(a)] and (ii) the strongly asymmetric case $\gamma_2 = \gamma_1/2$ with $T_K^2 \ll T_K^1$ [Fig. 2(b)]. Both plots are alike except that the splitting of the Kondo peak occurs at different values of I/T_K^1 since the KS $\to$ AFM transition takes place at different critical values. When $I/T_K^1 < (I/T_K^1)_c$, the RKKY interaction is not strong enough to link both spins into an AFM singlet state and the DOS displays a Kondo resonance. For the asymmetric case, the value of I/T_K^1 is increased above the critical value upon reducing ε_1 but keeping always T_K^2 very small compared to T_K^1. In both cases the DOS changes dramatically when increasing I above the critical value showing two peaks at $\omega/T_K^1 = \pm I/2$. These results clearly show that measurement of the splitting yields an estimate of the intensity of the RKKY interaction.

Effect of a magnetic field.—In what follows we focus on the symmetric situation, i.e., $T_K^1 = T_K^2$. A local magnetic field H is applied in the plane of the dots. It couples equally to the spins of both dots giving rise to an additional Zeeman term $-\vec{H}(\vec{S}_1 + \vec{S}_2)$ in Eq. (1). For $|I| < T_K$, the spins of the dots are almost independently screened. The DOS of each dot displays the usual Kondo resonance and the magnetic field ($H > T_K$) leads to a splitting of this peak at $\omega \approx \pm H$ [19]. When the RKKY interaction is ferromagnetic with $|I| > T_K$ the two spins are locked into a $S = 1$ state and at sufficiently low temperature the DOS displays a Kondo resonance. By applying a magnetic field $|I| \gg H > \max(T_K^{\rm even}, T_K^{\rm odd})$ the DOS becomes split with peaks again at $\omega \approx \pm H$. As a result the *conductance versus H always decreases for the ferromagnetic case* as in the usual $S = 1/2$ Kondo effect at low temperatures $T < |I|$.

As we anticipated, when $I > 0$ the interplay between the magnetic field and the RKKY interaction results in very different transport regimes. Next, we analyze in detail this case by using the SBMFT [14,20]. Figure 3 summarizes our results. In the left panel, we fix $I/T_K^1 < (I/T_K^1)_c$ increasing the magnetic field H [Fig. 3(a)]. Strikingly enough, we need a much higher value of the magnetic field ($H \approx 1.8T_K$) to split the DOS when the RKKY interaction is present. Clearly, both the RKKY and the Zeeman interactions compete. Whereas H tries to align both spins, the AFM RKKY interaction tries to antialign the spins of the dots. Such a behavior of the nonlinear conductance with the magnetic field has been reported experimentally in [11], suggesting that $I > 0$. In order to corroborate this picture we have considered the reversed situation in Fig. 3(b): we fix the magnetic field $H = 1.5T_K^1$ increasing I. In the absence of the RKKY interaction, the DOS displays two peaks located at $\omega \approx \pm H$ [19]. By increasing I one clearly observes that the splitting of the DOS is destroyed and a resonance at E_F emerges, restoring the Kondo effect around some critical magnetic field H_c. By further increasing I the Kondo resonance splits again. In general for $H, I \gg T_K^1$, we find four peaks in the DOS at $\pm|H \pm I|/2$ corresponding to the possible states of the two

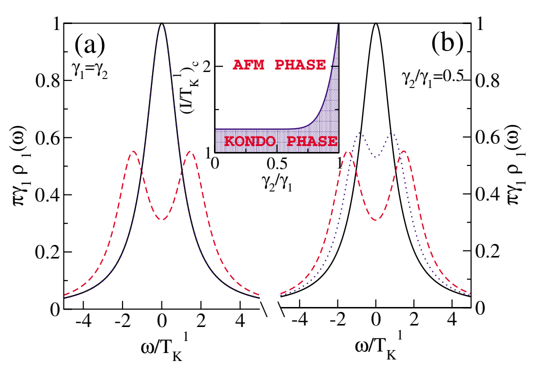

FIG. 2 (color online). (a) DOS for dot 1 in the *symmetric* case ($\gamma_1 = \gamma_2$) for $I/T_K^1 = 1$ (solid line), $I/T_K^1 = 2$ (dotted line), and $I/T_K^1 = 3$ (dashed line) [the cases for $I/T_K^1 = 1, 2, [<(I/T_K^1)_c]$ are indistinguishable since the peak width is governed by the same energy scale T_K^1 before the splitting occurs]. The critical value at which the splitting occurs is $(I/T_K^1)_c \approx 2.54$. (b) DOS for dot 1 in the *asymmetric* case ($\gamma_2 = \gamma_1/2$) for $I/T_K^1 = 1$ (solid line), $I/T_K^1 = 2$ (dotted line), and $I/T_K^1 = 3$ (dashed line). Here the critical value at which the splitting occurs is $(I/T_K^1)_c \approx 1.27$. Inset: Critical value $(I/T_K^1)_c$ vs γ_2/γ_1 at which the transition from the Kondo state to a singlet state occurs. The boundary (denoted by the solid line) separates both phases.

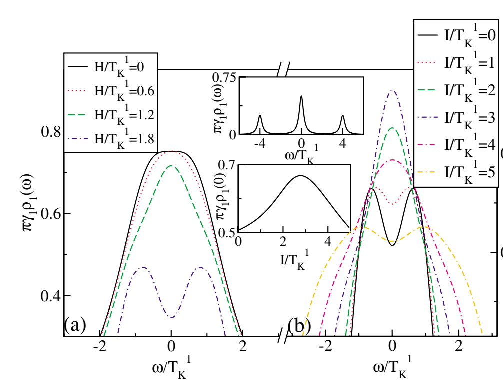

FIG. 3 (color online). (a) DOS for dot 1 for $I = 1.5T_K^1$ at different values H. (b) DOS for dot 1 for $H = 1.5T_K^1$ at different values of I/T_K^1. Upper inset: DOS for dot 1 for $H = I = 4T_K^1$. Lower inset: DOS at E_F for dot 1 as a function of I/T_K^1 for $H = 1.5T_K^1$. We set $\gamma_1 = \gamma_2$ and $T_K^1 = T_K^2$.

antiferromagnetically coupled $S = 1/2$ impurities in a magnetic field [21]. Here, the restoration of the KS occurs at the critical value $H_c \approx I$ where two peaks merge together and give a central Kondo resonance as shown in the upper inset of Fig. 3. The value H_c corresponds to the special point where the AFM singlet state and the lowest component of the triplet state become almost degenerate because of the presence of H [22]. This gives rise to a new magnetic-field induced Kondo effect rather similar to the one predicted in a quantum dot with an even number of electrons [16,17]. Nevertheless, it is worth pointing out that this occurs here as a consequence of the *nonlocal RKKY coupling*. This new magnetic-field induced Kondo effect is governed by a new Kondo scale $T_K(H_c)$ which is of order T_K in our SBMFT analysis. $T_K(H_c)$ can be evaluated using the poor man's scaling treatment developed in Ref. [17], $T_K(H_c)/T_K \sim (T_K/H)^\alpha$, where $\alpha \geq \pi/2 - 1$ depends on the ratio between the local DOS of conduction electrons in the even and odd (under parity) sectors and is minimum when this ratio is 1. Note also that the dot DOS at E_F shows a nonmonotonous behavior under the influence of an in-plane magnetic field because of the antiferromagnetic nature of the RKKY interaction (see lower inset in Fig. 3). Similar behavior of the DOS with magnetic field was reported in Ref. [15] in a different setup, a serial double dot with a direct interdot tunneling. Here we go beyond and discuss the physical origin of this effect in this geometry and give the new associated Kondo scale. Finally, the measurement of a nonmonotonous behavior of the conductance versus I/T_K^1 at low temperature would provide an unambiguous way of determining the sign of the RKKY coupling.

Conclusions.—We have shown that different transport regimes arise in the geometry studied experimentally in [11] depending on the sign of the RKKY-like interaction I. We find that these different regimes can be distinguished when a (strong) in-plane magnetic field is applied. In particular, the antiferromagnetic RKKY interaction competes with the effect of a magnetic field and from both interactions a magnetic-field induced Kondo effect emerges leading to an enhancement of the conductance with the magnetic field.

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Note added.—During the completion of this work we became aware of related work by Vavilov and Glazman [23].

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[20] A very small direct tunneling term between the dots has been added to allow a continuous transition between the AFM singlet and Kondo state in the SBMFT approximation (see Refs. [12,14]).

[21] For large H, fluctuations are important and the mean field solution gives unphysical vanishing renormalized tunneling couplings leading to divergences in the dot propagators that we regularize by hand. Going beyond mean field would regularize these divergences.

[22] In the KS → AFM transition the DOS evolves smoothly from a single peak (KS) to a double peak structure located at $\pm\kappa$ with $0 \leq \kappa \leq I/2$. Here κ characterizes the strength of the AFM order between the impurities (see Refs. [12,14]). Thus for $I \approx T_K^1$ the needed H_c to restore the Kondo effect is $H_c \approx 2\kappa \leq I$ (right panel in Fig. 3). For large $I \gg T_K^1$, the AFM state is completely established and the DOS shows the usual peaks at $\pm I/2$ (κ is maximum). In this case one finds, as expected, $H_c \sim I$ for the critical value of the magnetic field able to restore the Kondo peak (see upper inset in Fig. 3).

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