Lorentz Violation in Supersymmetric Field Theories

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We construct supersymmetric Lorentz violating operators for matter and gauge fields. We show that in the supersymmetric standard model the lowest possible dimension for such operators is five, and therefore they are suppressed by at least one power of an ultraviolet energy scale, providing a possible explanation for the smallness of Lorentz violation and its stability against radiative corrections. Supersymmetric Lorentz noninvariant operators do not lead to modifications of dispersion relations at high energies thereby escaping constraints from astrophysical searches for Lorentz violation.

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Introduction.—Recent years have seen an increase in the number of theoretical studies of Lorentz violation (LV), as well as intensified experimental efforts searching for LV signatures in terrestrial, astrophysical and cosmological settings [1,2]. For example, *effective* LV at low energies may arise in string theory due to a nonvanishing background of an antisymmetric tensor field. Alternative scenarios of quantum gravity often predict that at ultrashort distances particle dispersion relations are modified by cubic and higher terms in the energy, (see, e.g., [3]),

$$E^2 = p^2 + m^2 + b_1 \frac{E^3}{M} + b_2 \frac{E^4}{M^2} \ldots, \tag{1}$$

where b_i are some dimensionless constants. Although such conjectures are undoubtedly very speculative, if true they could provide a powerful tool of probing microscopic M^{-1} distances via LV physics.

LV operators can be classified according to their dimension. Cubic and higher order modifications of dispersion relations correspond to LV operators of at least dimension five [4]. According to naive dimension counting, the dimension D of an operator determines its scaling $\sim M^{4-D}$ with the characteristic energy scale M at which the operator is generated. Hence, dimension five operators are necessarily suppressed by one power of the ultraviolet scale M. However even Planck mass ($M_{\rm Pl}$)-suppressed operators for photons, electrons, and quarks are ruled out by a number of astrophysical constraints and precision measurements up to $\sim 10^{-5}$ level [4–7]. Even more serious problems arise with dimension three and four LV operators classified in [8], since there are no dimensional arguments as to why such operators should be small. Moreover, higher dimensional operators will in general induce lower dimensional ones through loop corrections with power-law divergent coefficients. Only additional symmetry arguments may provide genuine suppressions of such lower dimensional operators [4].

An obvious candidate for such a symmetry is supersymmetry (SUSY). Following the prevailing point of view in particle theory, we assume that at ultrashort distances (close to $1/M_{\rm Pl}$) SUSY is realized exactly. As the ultraviolet behavior of SUSY theories are free of potentially dangerous quadratic divergences, it is generally accepted as being a technical solution to the hierarchy problem. SUSY is conventionally introduced as a graded extension of the Poincaré algebra generated by translations, rotations, and Lorentz transformations, therefore, one might expect that SUSY is simply incompatible with LV physics. This is not the case because it is possible to restrict all considerations to the subalgebra generated by supercharges and translations only. In this Letter we only consider LV SUSY theories that are representations of this algebra without any further modifications. Moreover we only focus on the standard chiral and vector superfields, which are conventionally used to describe the field content of the minimal supersymmetric standard model (MSSM). Of course, the constraints of Lorentz and rotational invariance cannot be enforced anymore. However, we will see that SUSY still provides a very powerful selection rule for LV interactions. Moreover, like in conventional SUSY field theories, we expect that operators forbidden by SUSY will be suppressed by some power of $m_{\rm soft}/M$ below the soft SUSY breaking scale $m_{\rm soft}$, leading to a possible partial explanation of why the LV operators of dimension three are so tiny.

In this Letter we classify LV operators that are compatible with exact SUSY for arbitrary vector and tensor backgrounds. To this end, we describe a systematic method of constructing LV interactions in the SUSY context. We find that SUSY combined with gauge invariance severely constraints the possible form of such operators. From this analysis we conclude that the smallest dimension of LV operators within the framework of the MSSM is five. We show that these SUSY LV operators do not lead to significant modifications of dispersion relations.

Supersymmetric LV Lagrangians.—As stated earlier, LV preserves the subalgebra generated by supercharges and translations, thereby allowing the use of the superspace technique. Even though this is equivalent to a component approach, the superspace language permits the most straightforward and economical formulation of LV operators. To fix the notations we follow the textbook by Wess and Bagger [9]. The matter and gauge fields in the MSSM are described by chiral multiplets and vector multiplets. To facilitate the counting the dimensions of LV operators from their superfield expressions, we list the mass dimensions of objects appearing in this Letter in Table I. Here S denotes a chiral superfield, i.e., $\bar{D}_{\dot{\alpha}} S = 0$, and $\bar{D}_{\dot{\alpha}}$ is a super covariant derivative. The superfield strength $W_\alpha = -\frac{1}{4}\bar{D}^2(e^{-V} D_\alpha e^V)$ is obtained from the vector superfield V. With the use of Table I, it follows that the standard Lagrangian for the Wess-Zumino model,

$$L_{\rm WZ} = \int d^2\theta P(S) + {\rm H.c.} + \int d^4\theta \bar{S} S, \tag{2}$$

with a (cubic) superpotential $P(S)$ has mass dimension four. Throughout this Letter we include the superspace measures in the counting of the dimension of operators.

We construct SUSY LV operators coupled to background tensors that lead to modifications of physical observables, like a preferred direction or Lorentz frame. Our main result states that *any LV operator respecting MSSM gauge invariance and exact SUSY has dimension five or higher and therefore is suppressed by at least one power of an ultraviolet scale M.*

We show this in three steps: First we classify LV operators for chiral superfields, next we investigate consequences of gauge invariance, and finally we apply our results to the MSSM. The fundamental chiral and vector multiplets, S and V, do not carry any Lorentz indices. As only the derivatives D_α, $\bar{D}_{\dot{\alpha}}$, and ∂_m are SUSY preserving, SUSY LV interactions should be constructed by applying a number of these derivatives to superfields S and V. Consequently, any SUSY LV interaction contains two or more superfields, otherwise it is a total derivative in superspace. This rules out a LV generalization of the Fayet-Iliopoulos term $\int d^4\theta V$. The absence of external fermionic backgrounds implies that all SUSY LV operators contain an even number of fermionic derivatives D_α and $\bar{D}_{\dot{\alpha}}$. Combining these observations imply that SUSY LV starts at dimension four. In particular, we find that possible LV operators for chiral superfields (labeled by a, b, c) up to dimension five are obtained as chiral integrals ($\int d^2\theta$) of the superpotential terms

$$S_a \partial_m S_b, \qquad S_a \partial_m \partial_n S_b, \qquad S_a S_b \partial_m S_c, \tag{3}$$

and as full superspace integral ($\int d^4\theta$) of

$$\bar{S}_a \partial_m S_b, \tag{4}$$

TABLE I. Dimension of superspace objects.

obect	∂_m	θ^α	D_α	$\int d^2\theta$	$\int d^4\theta$	S	V	W_α
dim.	1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	1	0	$\frac{3}{2}$

up to total derivatives in superspace. Of all these operators only the first term in (3) has dimension four; all others have dimension five.

Next we proceed to LV in SUSY gauge theories. As D_α and ∂_m break super gauge transformations $S \to e^{-\Lambda} S$ and $e^V \to e^{\bar{\Lambda}} e^V e^{\Lambda}$, we introduce covariant derivatives $\mathcal{D}_\alpha S = e^{-V} D_\alpha (e^V S)$ and

$$\partial_m S \to \mathcal{D}_m S = -\frac{i}{4} \bar{\sigma}_m{}^{\dot{\alpha}\alpha} \mathcal{D}_{\dot{\alpha}\alpha} S = -\frac{i}{4} \bar{\sigma}_m{}^{\dot{\alpha}\alpha} \bar{D}_{\dot{\alpha}} \mathcal{D}_\alpha S. \tag{5}$$

Contrary to ∂_m, this covariant derivative does not respect chirality: $\bar{D}_{\dot{\beta}} \mathcal{D}_{\dot{\alpha}\alpha} S = 2\epsilon_{\dot{\beta}\dot{\alpha}} W_\alpha S \neq 0$, and hence LV superpotentials (3) cannot be generalized to charged chiral superfields! Consequently, the only dimension five SUSY LV operator for a charged chiral multiplet is the gauge invariant version of the Kähler LV term (4):

$$\bar{S} e^V \mathcal{D}_m S. \tag{6}$$

The constraints of gauge invariance for vector multiplets are similar to standard Lorentz preserving theories, hence possible LV terms in the SUSY gauge sector are the full superspace integral of

$$\mathrm{tr} \bar{W}_{\dot{\alpha}} e^V W_\alpha e^{-V} \tag{7}$$

of dimension five and chiral integrals of

$$\mathrm{tr} W_{(\alpha} W'_{\beta)}, \qquad \mathrm{tr} S W_{(\alpha} W'_{\beta)}, \qquad \mathrm{tr} W_\alpha \partial_m W'_\beta, \tag{8}$$

where the first expression has dimension four, while the other two have dimension five. The chiral superfield S is in an adjoint representation if V is non-Abelian, and a gauge singlet for Abelian V. Where needed, we have preformed symmetrization of α and β, denoted by (α, β), to project on LV antisymmetric tensor background, $b^{mn}(\sigma_{mn}\epsilon)^{\alpha\beta}$ (which may appear in noncommutative field theories, for example). For a single $U(1)$ or for non-Abelian gauge multiplets the first term of (8) vanishes.

Now we apply these results to the MSSM: Since all MSSM chiral superfields are charged under gauge symmetries, no LV superpotential is allowed. In particular, a LV generalization of the μ term in the Higgs sector, $H_1 \partial_m H_2$, is excluded by gauge invariance. Since MSSM contains only one $U(1)$ vector multiplet, operator $\mathrm{tr} W_{(\alpha} W'_{\beta)}$ vanishes. Therefore all dimension four SUSY LV operators in the MSSM are excluded, and the LV terms start from dimension five. Moreover, not only are dimension four LV operators forbidden in the MSSM, but also the number of dimension five operators is limited: The only three types of operators are Kähler terms (6) for MSSM chiral multiplets, interactions based on (7) and the third term in (1) for the MSSM vector multiplets.

Finally, we stress that in any SUSY theory LV is allowed only at dimension four and higher. If the spectrum of MSSM at the electroweak scale or below is extended by chiral singlets such as right-handed neutrinos, and/or by additional $U(1)$ vector multiplet(s), dimension four LV operators from [(3) and (8)] can indeed appear.

Phenomenological consequences.—As shown above there exists only three possible types of dimension five LV operators that preserve SUSY in the MSSM. We investigate phenomenological consequences of these operators and, in particular, we claim that *the dimension five SUSY LV operators do not lead to significant modifications of dispersion relations.*

The physical reason for this result can be understood from the modification of the kinetic term (6) for the scalar component z of a chiral superfield S. This modification $M^{-1}\bar{z}\partial_m\partial^2 z$ can be reduced on the equations of motion to $M^{-1}m^2\bar{z}\partial_m z$. The resulting $\sim M^{-1}m^2E$ correction of the dispersion relation is small with respect to m^2.

These arguments can be lifted to superspace. For simplicity, we focus on LV in the super quantum electrodynamics (SQED) part of the MSSM, as the extension to the full MSSM is straightforward. The theory of SQED consists of a $U(1)$ vector multiplet V and two oppositely charged chiral superfields $E_\pm$. The complete SQED Lagrangian with all dimension five SUSY LV terms is given by

$$\int d^2\theta\Big(\frac{1}{16e^2}W^2 + mE_+E_-\Big) + \text{H.c.} + \int d^4\theta \bar{E}_\pm e^{\pm V}E_\pm + \frac{1}{M}\int d^4\theta\Big(iN^m_\pm \bar{E}_\pm e^{\pm V}\mathcal{D}_m E_\pm - \frac{1}{2}N^m\bar{W}\bar{\sigma}_m W\Big) + \frac{1}{M}\int d^2\theta C^{pmn}W\sigma_{mn}\partial_p W + \text{H.c.}, \tag{9}$$

with e the electric charge and m the mass of the electron. The first line gives the standard Lagrangian for SQED, while the other two lines describe SUSY LV by external vectors $N^m_\pm$ and N^m, and a tensor $C^{pmn} = -C^{pnm}$.

To show that the dispersion relation of the electron/positron is not significantly modified, we compute the superfield equations of motion,

$$\partial^2 E_\pm - m^2\Big(1 - \frac{i}{M}(N^m_\pm - N^m_\mp)\partial_m\Big)E_\pm = 0, \tag{10}$$

up to first order in the LV and dropping all dependence on the vector superfield V. The resulting corrections to the dispersion relation,

$$E^2 = p^2 + m^2 + m^2(N^0_\pm - N^0_\mp)\frac{E}{M} + \ldots, \tag{11}$$

are drastically *smaller* than the conjectured form (1), and in fact, much smaller than m^2 as long as $E \ll M$! Further corrections with higher powers of E are suppressed by additional factors of m/M. The same holds for higher dimensional SUSY LV operators, like $\bar{E}_\pm e^{\pm V}\mathcal{D}_m\mathcal{D}_n \ldots E_\pm$. An even stronger conclusion can be reached for the photon LV operators in (9). The equation of motion in the presence of N_m,

$$\Big(1 - N^m\bar{\sigma}^{\dot{\alpha}\alpha}_m\bar{D}_{\dot{\alpha}}D_\alpha\Big)D^\beta\bar{D}^2D_\beta V = 0. \tag{12}$$

can be solved iteratively to first order in LV parameter. The zeroth order equation of motion can be applied in the second term of Eq. (12) after which it vanishes, leaving no modifications of photon propagation by N_m! Using a similar approach, we can extend this result to the C^{pmn}-proportional operator in (9).

To understand some other phenomenological consequences of SUSY LV, we present the component form of the N^m-proportional operator of (9):

$$-\frac{N^m}{2M}\int d^4\theta\bar{W}\bar{\sigma}_m W = \frac{N^p}{M}\Big[\frac{1}{2}\tilde{F}_{kp}\partial_l F^{kl} - D\partial^k F_{kp} + \eta_{pk}\lambda\sigma^k\Box\bar{\lambda} - \lambda\sigma^m\partial_m\partial_p\bar{\lambda}, \Big], \tag{13}$$

where F_{mn} is the electromagnetic field strength, λ is the photino, and D is the auxilary field. The spatial component of N^m couples to the cross product of the electric field and the electric current, $(\mathbf{E}\times\mathbf{J})\cdot\mathbf{N}$ upon the replacement of $\partial_l F^{kl}$ by the current J^k in (13). Under descrete symmetries this interaction is CPT odd, P even, C and T odd. The average of $\mathbf{E}\times\mathbf{J}$ inside a particle with charged constituents, i.e., a nucleon or a nucleus, is a vector directed along a nuclear spin $\mathbf{I}$. Following the method of [4], we estimate the size of an effective interaction between $\mathbf{N}$ and $\mathbf{I}$ to be at the level of $H_{\text{eff}} \sim (10^{-5} - 10^{-3})M^{-1}(1\text{ GeV})^2(\mathbf{N}\cdot\mathbf{I})$, where 1 GeV enters as a characteristic hadronic energy scale. This is precisely the correlation searched for by the clock comparison experiments (see, e.g., [10], and references therein) and $\mathbf{N}M^{-1}$ is limited typically at the level better than $10^{-5}M_{\text{Pl}}^{-1}$.

Lorentz violating SUSY breaking.—LV operators constructed above respect SUSY manifestly. Here we present a method to obtain LV Lagrangians that generically lead to SUSY breaking. Consider the Lagrangian

$$\int d^4\theta\tilde{V}\Psi, \quad \text{with } \tilde{V} = -n_m\theta\sigma^m\bar{\theta}. \tag{14}$$

for an arbitrary (real composite) superfield Ψ. According to Table I the superspace variables $\bar{\theta}_{\dot{\alpha}}$ and θ_α in $\tilde{V}$ effectively *reduce* the dimension of the operator by one. For example, by taking $\Psi = \bar{S}S$ we obtain LV operators of dimension three. This construction does not preserve SUSY in general: Only if

$$\int d^4x\bar{D}^2D_\alpha\Psi| = 0, \tag{15}$$

the operator (14) respects SUSY. (Here $|$ indicates that θ_α and $\bar{\theta}_{\dot{\alpha}}$ are set to zero after all superspace differentiations.)

This result can be used to show that LV by a Chern-Simons term, i.e., $n_m\epsilon^{mnkl}A_n\partial_k A_l$, does not have a SUSY extension. Using (14) we obtain a Lagrangian that contains the Chern-Simons interaction

$$M \int d^4\theta \tilde{V}\Omega = \frac{n_m}{4} M\Big(\epsilon^{mnkl} A_n \partial_k A'_l + \lambda \sigma^m \bar{\lambda}'\Big), \qquad (16)$$

where $\Omega = -\frac{1}{4}(D^\alpha V W'_\alpha + \bar{D}_{\dot\alpha} V \bar{W}'^{\dot\alpha} + V D^\alpha W'_\alpha)$ is the Chern-Simons superfield [11]. (The construction works also for a single vector multiplet $V' = V$, and (16) is super gauge invariant because $\bar{D}^2 D_\alpha \tilde{V} = 0$.) But the condition (15) implies that the Lagrangian (16) does not respect SUSY, since $\bar{D}^2 D_\alpha \Omega| = D_\alpha (W^\beta W'_\beta)|$ does not vanish. Our conclusion supports the recent result of Ref. [12] that Chern-Simons interactions require SUSY breaking. Moreover, by taking $V' = \partial^m \partial^n V$ in (16), we conclude that the only dimension five operator that leads to E^3 modification of the photon dispersion relation [4], $F^m{}_p \partial^n \tilde{F}^{kp}$ breaks SUSY explicitly!

Discussion and conclusion.—Before summarizing our main findings we comment on some recent publications [13,14] that considered the construction of dimension three and four SUSY LV interactions for a (neutral) chiral superfield, which seems to be in conflict with the main results of this Letter. As the authors observe themselves, the dimension three operators can be removed by suitable (super)-field redefinitions [13], leaving no observable effects. But they claim that the modification of the Wess-Zumino action by a symmetric tensor k^{mn} combined with modified superalgebra and SUSY transformations give rise to viable LV effects. However, their resulting dimension four LV Lagrangian (given in [13]) can be removed by the linear change of coordinates, $x'_m = x_m - k^n_m x_n$, which also brings SUSY transformations to a usual Lorentz-conserving form.

We have presented a method of obtaining manifestly supersymmetric LV interactions by allowing free space-time or an even number of super covariant derivatives to act on superfield expressions. We proved that exact SUSY requires LV to start at dimension four or higher. Gauge invariance and chirality prohibits derivatives on charged chiral superfields to appear in the superpotential. Therefore LV in the charged chiral multiplet sector begins at dimension five since extra derivatives are allowed only in kinetic terms. Applying our results to the superfield content of the MSSM we arrive at our central conclusion: All possible LV operators in MSSM have at least dimension five and therefore are suppressed by one or more powers of a large ultraviolet scale responsible for LV. Dimension five SUSY LV interactions for SQED are given in (9) with obvious generalization to full MSSM.

None of the SUSY LV operators lead to significant high-energy modifications of the dispersion relations. We find that SUSY LV operators can be reduced on the equations of motion, producing an additional suppression by m^2 and suggesting a generic form of the SUSY LV dispersion relation:

$$E^2 = p^2 + m^2\Big(1 + b_1 \frac{E}{M} + b_2 \frac{E^2}{M^2} + \ldots\Big), \qquad (17)$$

which is in sharp contrast with (1), and does not modify propagation of photons. Therefore, SUSY LV leaves no imprint on the propagation of high-energy particles and escapes constraints from astrophysical searches of LV, but can be probed with precision measurements at low energies.

As exact SUSY forbids dimension three LV operators the problem of dimensional transmutation of dimension five LV operators to dimension three with quadratically divergent loop coefficients is solved. In a more realistic theory SUSY needs to be broken, and dimension three operators may be generated but the quadratic loop divergencies are stabilized at the soft breaking scale. Details of SUSY LV phenomenology with inclusion of soft SUSY breaking and loop corrections will be investigated elsewhere [15].

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