Peres, Scudo, and Terno Reply: In his comment [1], Czachor claims that "there is something physically wrong" with the reasoning in our Letter [2] and we are reported "to point out that there are problems with relativistic quantum information theory (RQIT)" that, however, "may not be very relevant for the fundamentals of RQIT".

Since we never did point out problems with RQIT, we have nothing to say about that matter. The allegation of being "physically wrong" is based, under scrutiny, on a combination of an interesting result of Czachor and Wilczewski [3] and Czachor's possible confusion between various tests that can be performed on a quantum state and a labeling of this state [1].

In [3] it is pointed out that labeling of the discrete degree of freedom of a massive particle by a projection $l_\mu W^\mu$ of a Pauli-Lubanski vector (PLV) W^μ on a null eigenvector of a *predefined* Lorentz transformation Λ makes a transformation law for this degree of freedom under Λ independent of momentum. This is in contrast with a Wigner rotation [4] that is generally momentum dependent. Hence a reduced density matrix for this degree of freedom is invariant. It is on the strength of this result that our analysis is found deficient.

In our Letter [2] we used the Wigner spin $\mathbf{S}$ and adhered to the rest-frame labeling convention. These observables have the following property: if under a "spin operator" a linear combination of PLV components that satisfies three natural axioms is understood [5], then the triple $\mathbf{S}$ is the only such combination. For a normalized pure one-particle state

$$|\Psi\rangle = \int d\mu(\mathbf{p})\begin{pmatrix} a_1(\mathbf{p}) \\ a_2(\mathbf{p}) \end{pmatrix}|\mathbf{p}\rangle,$$

the 2×2 table ρ is obtained by tracing out the momentum degrees of freedom,

$$\rho_{ij} = \int d\mu(\mathbf{p}) a_i(\mathbf{p}) a_j^*(\mathbf{p}). \quad (1)$$

It is a density matrix because there are observables A (e.g., any operator of the form $\hat{\mathbf{n}} \cdot \mathbf{S}$, where $\hat{\mathbf{n}}$ is an arbitrary unit vector), whose expectation values can be calculated according to the trace rule $\langle A\rangle = \mathrm{tr} A\rho$. This property is the only reason why it makes sense to define a reduced or an effective density matrix [4]. We used the same definition for all observers, and one of the conclusions of [2] was that the entropy of a reduced density matrix is not Lorentz invariant, because unlike the entire state $|\Psi\rangle$, ρ is not covariant under Lorentz boosts.

We agree with Czachor that there is nothing *a priori* wrong with the yes-no observable for a massive particle that is built from a projection of PLV on a null direction. By the same token, the labeling of states that is based on the values of $l_\mu W^\mu$ is legitimate.

However, the definition of the spin density matrix [1] that follows from it and intends to cure the noncovariance of our ρ is far from satisfactory. Since there are at most two null eignevectors of a Lorentz transformation, the Λ-invariant table that is defined analogously to Eq. (1) does not have an invariant family of observables for which the trace formula applies. Moreover, given a state and an observer this table is undefined until a Lorentz transformation to some other frame is specified. In this approach a given observer has no unique spin state, but this state refers instead to a fixed pair of observers and is different for different pairs.

Consider now a pure state of *zero* spin entropy [2] that is given by $a_2 = 0$, $\int d\mu(\mathbf{p})|a_1|^2 = 1$ with $a_1(\mathbf{p})$ appreciably nonzero only for low momenta. It describes a particle (nearly) at rest with its spin up. Fix now a pair of observers that defines a null direction l^μ. Using Eq. (16) of [3] we find that in Czachor's representation both of the functions a_i^l are generically nonzero and momentum and null-direction dependent. Hence the entropies of the corresponding density matrices are nonzero and different for different pairs of observers.

To recap, the observables $l_\mu W^\mu$ may be valid and potentially useful for relativistic quantum cryptography. However, they do not lead to a viable reduced density matrix, thus voiding the conclusions of [1].

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