Surface Superconductivity of Dirty Two-Band Superconductors: Applications to MgB_2

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The minimal magnetic field H_{c2} destroying superconductivity in the *bulk* of a superconductor is smaller than the magnetic field H_{c3} needed to destroy *surface* superconductivity if the surface of a superconductor coincides with one of the crystallographic planes and is parallel to the external magnetic field. While for a dirty single-band superconductor the ratio of H_{c3} to H_{c2} is a universal temperature-independent constant 1.6946, for dirty two-band superconductors this is not the case. I show that in the latter case the interaction of the two bands leads to a novel scenario with the ratio H_{c3}/H_{c2} varying with temperature and taking values larger and smaller than 1.6946. The results are applied to MgB_2 and compared with recent experiments (A. Rydh *et al.*, cond-mat/0307445).

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Introduction.—It is well established that strong magnetic field destroys superconductivity. If an external field H applied to a type-II superconductor exceeds the second critical field H_{c2}, the *bulk* order parameter in the superconductor vanishes. However, even for $H > H_{c2}$ superconductivity might still exist in a thin layer close to the surface if H is smaller than the third critical field $H_{c3} > H_{c2}$ [1]. In this Letter, I investigate the onset of superconductivity via surface nucleation for the field H slightly below the threshold H_{c3}.

In their pioneering work [1] Saint-James and de Gennes have shown that if the external magnetic field is applied parallel to the surface of an isotropic single-band superconductor [2] with a temperature close to the transition temperature T_c the ratio H_{c3}/H_{c2} takes the universal value $\eta = 1.6946$ independently of the superconducting material. For H slightly below H_{c3} the superconducting order parameter exists within the distance $\zeta(T)$ (the coherence length of the superconductor) from the surface [1]. For distances exceeding $\zeta(T)$ the order parameter approaches zero rapidly. The dependence of the ratio H_{c3}/H_{c2} on the material properties [3–5], sample geometry and topology [6–8], and temperature [9–11] has become a subject of intensive investigations.

A novel window for investigating surface superconductivity was opened after the discovery of the two-band superconductor MgB_2 [12]. Not only has it a relatively high (≈ 40 K) T_c but also there exist two different superconducting gaps. As the consequence of this fact, various properties of MgB_2 are quite different from those of single-band superconductors. For example, the anisotropy $\gamma(T) = H_{c2}^{(ab)}(T)/H_{c2}^{(c)}(T)$ [here, $H_{c2}^{(ab)}(T)$ and $H_{c2}^{(c)}(T)$ stand for the second critical fields in the ab and c directions, respectively; note that the crystal of MgB_2 is uniaxial] of the field H_{c2} exhibits strong dependence on temperature; see, e.g., Ref. [13]. For single-band superconductors this ratio is constant.

Another puzzle is that $\gamma(T)$ varies widely in different experiments [14]. This can be attributed to the existence of surface superconductivity which might affect the observable values of H_{c2} and, hence, the anisotropy. Consequently, the determination of the third critical field H_{c3} is a important problem. In a recent experiment [15] it has been shown that H_{c3}/H_{c2} for MgB_2 is reduced.

In this Letter, I investigate the ratio H_{c3}/H_{c2} for a dirty MgB_2 crystal. The existence of two different gaps manifests itself through the remarkable dependence of H_{c3}/H_{c2} on temperature. This is in sharp contrast with the case of a dirty single-gap superconductor where $H_{c3}/H_{c2} = \eta$ in the whole temperature range. For a magnetic field lying in the ab plane of the MgB_2 crystal, I find that if one starts decreasing temperature, H_{c3}/H_{c2} first exhibits a maximum at $T \approx 0.99T_c$ and then a minimum at $T \approx 0.9T_c$. As

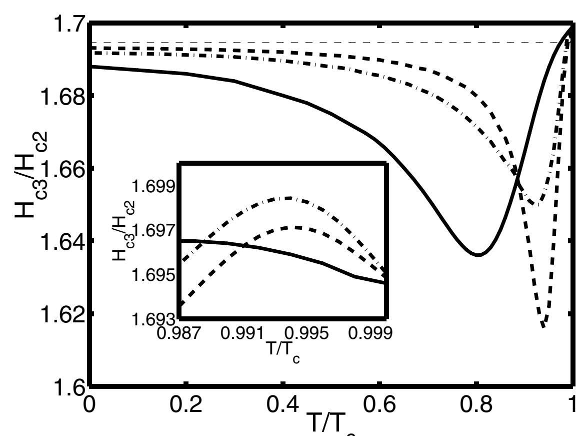

FIG. 1. H_{c3}/H_{c2} for the surface of the MgB_2 crystal coinciding with the ab plane and the external magnetic field lying in the ab plane as a function of T/T_c for different ratios of the coupling parameters and diffusivities in the two bands: coupling parameters are taken from Ref. [19] and $D_{2,c}/D_{1,c} = 100$ or 600 (dash-dotted line and dashed line, respectively); coupling parameters and $D_{2,c}/D_{1,c} \approx 126$ are taken from the comparison with experiments on the H_{c2} anisotropy [20] (solid line). For $T = T_c$, $H_{c3}/H_{c2} = 1.6946$. The horizontal dashed line is the value of 1.6946. Inset: H_{c3}/H_{c2} close to $T/T_c = 1$.

temperature decreases further, H_{c3}/H_{c2} increases and tends to a value slightly below η; see Fig. 1. Naively, one could try to use the Ginzburg-Landau theory (GLT) in order to find H_{c3}/H_{c2}. However, as will be explained below, this would lead to the ratio $H_{c3}/H_{c2} = \eta$ in the whole temperature range; i.e., one needs a more rigorous approach in order to explain the deviation of H_{c3}/H_{c2} from η.

General formalism.—An appropriate tool to investigate magnetic properties of dirty superconductors is the Usadel equations [16]. For two-band superconductors they have been derived by Koshelev and Golubov [17] and by Gurevich [18]. Since I investigate the onset of superconductivity near H_{c2} or H_{c3}, it is possible to write the Usadel equations in the linearized form

$$\omega f_\alpha + \left[-\sum_j \frac{D_{\alpha,j}}{2}\left(\nabla_j - \frac{2\pi i}{\Phi_0}A_j\right)^2 \right] f_\alpha = \Delta_\alpha, \quad (1)$$

$$\Delta_\alpha = 2\pi T \sum_{\omega>0}^{\omega_D} \Lambda_{\alpha\beta} f_\beta. \quad (2)$$

Here, $\omega = 2\pi T(n + 1/2), n = 0, 1, \ldots$ and ω_D are the Matsubara and cutoff phonon frequencies. $D_{\alpha,j}$ is the diffusion coefficient of the band $\alpha = 1, 2$ along the direction $j = x, y, z$. The indices 1 and 2 correspond to the σ and π bands, respectively. $\mathbf{A}$ is the vector potential. Δ_α and f_α are the superconducting gap and anomalous green function for the band α. The matrix $\hat{\Lambda}$ represents the strength of the coupling parameters. In the present work, I use two matrices $\hat{\Lambda}$: $\lambda_{11} \approx 0.81$, $\lambda_{22} \approx 0.285$, $\lambda_{12} \approx 0.119$, and $\lambda_{21} \approx 0.09$ (see the electronic structure calculations in Ref. [19]) and $\lambda_{11} \approx 0.695$, $\lambda_{22} \approx 0.260$, $\lambda_{12} \approx 0.177$, and $\lambda_{21} \approx 0.140$ (see the fit with the experiment in Ref. [20]). In this Letter, I will concentrate on two geometries: (i) the magnetic field is parallel to the c axis and the surface of the crystal; (ii) the surface of the superconductor coincides with the ab plane, and the field $\mathbf{H}$ lies in the ab plane. As I will show, in case (i) $H_{c3}/H_{c2} = \eta$ at any temperature. In case (ii), H_{c3}/H_{c2} is shown in Fig. 1. I assume that $\mathbf{H}$ is parallel to the z axis.

Choosing the gauge as $A_y = Hx$, I look for the solution of the form $f_\alpha \equiv f_\alpha(\omega, x)\exp(ik_y y + ik_z z)$ and $\Delta_\alpha \equiv \Delta_\alpha(x)\exp(ik_y y + ik_z z)$. In general, Eqs. (1) and (2) define a sequence of solutions corresponding to different eigenvalues $H = H_{c2}$ or $H = H_{c3}$. One should look for the maximal possible values of H_{c2} or H_{c3}. This corresponds to the case $k_z = 0$. Substituting the ansatz for f_α and Δ_α into (1) and (2), I obtain the system of equations

$$\hat{\Lambda}^{-1}\begin{pmatrix}\Delta_1(x)\\ \Delta_2(x)\end{pmatrix} = \begin{pmatrix} 2\pi T\sum_{\omega>0}^{\omega_D}\frac{1}{\omega+\hat{H}_1(x_0)} & 0 \\ 0 & 2\pi T\sum_{\omega>0}^{\omega_D}\frac{1}{\omega+\hat{H}_2(x_0)}\end{pmatrix} \times \begin{pmatrix}\Delta_1(x)\\ \Delta_2(x)\end{pmatrix}, \quad (3)$$

where

$$\hat{H}_\alpha(x_0) = -\frac{\tilde{D}_{\alpha,x}}{2}\frac{\partial^2}{\partial x^2} + \frac{\tilde{D}_{\alpha,z}}{2}\left(\frac{2\pi H}{\Phi_0}\right)^2 (x - x_0)^2, \quad (4)$$

with $\tilde{D}_{\alpha,x} = \tilde{D}_{\alpha,z} = D_{\alpha,a}$ for case (i) and $\tilde{D}_{\alpha,x} = D_{\alpha,a}$ and $\tilde{D}_{\alpha,z} = D_{\alpha,c}$ for case (ii) ($D_{\alpha,a} = D_{\alpha,b} \neq D_{\alpha,c}$ are the diffusion coefficients along the crystallographic axes). $x_0 = k_y\Phi_0/2\pi H$ is the parameter characterizing how far away the superconducting nucleus is situated from the surface. Note that x_0 is the same for the both bands. The operator (4) can be rewritten in the form $(\pi H/\Phi_0) \times \sqrt{\tilde{D}_{\alpha,x}\tilde{D}_{\alpha,z}}\hat{h}_\alpha(x'_0)$, with

$$\hat{h}_\alpha(x'_{\alpha,0}) = -\frac{\partial^2}{\partial x'^2_\alpha} + (x'_\alpha - x'_{\alpha,0})^2, \quad (5)$$

where I have made the variable substitution $x = \beta_\alpha x'_\alpha$ and $x_0 = \beta_\alpha x'_{\alpha,0}$, with $\beta_\alpha = (\tilde{D}_{\alpha,x}/\tilde{D}_{\alpha,z})^{1/4}(\Phi_0/2\pi H)^{1/2}$.

The system (3) should be solved with the boundary conditions (BC) $\partial\Delta_\alpha/\partial x|_{x=0} = 0$, and $\Delta_\alpha(x \to +\infty) \to 0$, $\alpha = 1, 2$ valid for geometries (i) and (ii); see above. For H_{c3} the BC are well established for dirty superconductors [5]. For H_{c2} the application of these BC gives the same result as the BC requiring the appearance of a superconducting nucleus in the bulk. The procedure for finding H_{c3}/H_{c2} is as follows: First, set $x_0 = 0$ in (3) and find the maximal possible field H for which the solution satisfying the BC exists. This gives H_{c2}. Next, for $x_0 \neq 0$ find the maximal field $H = H(x_0)$ for which the solution of (3) exists. Then, $H_{c3} = \max_{x_0}\{H(x_0)\}$. I would like to mention that there are complementary approaches for calculating H_{c2} based on microscopic theory [21,22] and GLT [23,24].

Here, it is instructive to study briefly the case of a single-gap superconductor. This corresponds to $\lambda_{12} = \lambda_{21} = 0$. H_{c2} and H_{c3} are then determined by those for band 1 (as $\lambda_{11} > \lambda_{22}$). The solution for $\Delta_1(x)$ is proportional to the ground-state wave function of the operator $\hat{H}_1(x_0)$ and $\Delta_2(x) = 0$. Substituting this ansatz into (3), I obtain the transcendental equation of the form

$$1 - \lambda_{11}\sum_{\omega>0}^{\omega_D}\frac{1}{\omega + (\pi H/\Phi_0)\sqrt{\tilde{D}_{1,x}\tilde{D}_{1,z}}\epsilon_0(x'_{1,0})} = 0, \quad (6)$$

with $\epsilon_0(x'_{\alpha,0})$ the lowest eigenvalue of the operator $\hat{h}_\alpha(x'_{\alpha,0})$. The field H_{c2} can be found as the solution of the above equation for $x'_{1,0} = 0$; note that $\epsilon_0(0) = 1$. Assume a certain value of the magnetic field H_{c2} is found; let us change the parameter $x'_{1,0}$. This leads to the decrease of the eigenvalue $\epsilon_0(x'_{1,0})$ [1]. In order to satisfy Eq. (6) one has to increase the field H; that is why $H_{c3} > H_{c2}$. The minimal $\epsilon_0(x'_{1,0})$ can be realized for $x'_{1,0} = 0.7618$ [1] and is equal to 0.5901 [1]. This means that $H_{c3}/H_{c2} = 1/0.5901 = \eta$ for any temperature T. Remarkably, it is not necessary to solve Eq. (6) in order to find the ratio

H_{c3}/H_{c2}, although the determination of H_{c2} or H_{c3} alone would require the complete analysis.

Case (i).—In this case, the ratio $H_{c3}/H_{c2} = \eta$ at all temperatures. This is a consequence of the fact that the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$ have identical eigenfunctions (since $\tilde{D}_{1,x}/\tilde{D}_{1,z} = \tilde{D}_{2,x}/\tilde{D}_{2,z} = 1$). The functions $\Delta_1(x)$ and $\Delta_2(x)$ are proportional to the ground-state wave function of the operator $\hat{H}_1(x_0)$ [or $\hat{H}_2(x_0)$]. The equation determining the critical fields H_{c2} and H_{c3} has the form $F[\pi D_{1,a}H\epsilon_0(x'_{1,0})/\Phi_0, \pi D_{2,a}H\epsilon_0(x'_{2,0})/\Phi_0] = 0$, with $F(y_1, y_2)$ a certain function of two arguments. Note that in the present case $\beta_1 = \beta_2$ and, consequently, $x'_{1,0} = x'_{2,0}$ and $\epsilon_0(x'_{1,0}) = \epsilon_0(x'_{2,0})$. The maximal value of H_{c3} can be realized for $x'_{1,0} = 0.7618$ and is equal to ηH_{c2} for all temperatures.

Case (ii).—If the magnetic field lies in the *ab* plane, the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$ have different eigenfunctions. This leads to a complicated transcendental equation depending on *all* eigenvalues of the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$ and not only on the ground-state ones. Equation (3) can be solved via expanding functions $\Delta_1(x)$ and $\Delta_2(x)$ over the eigenfunctions of the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$. I have truncated the basis of the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$ to subspaces consisting of 70 eigenfunctions and solved the system (3) numerically. In Fig. 1 I show the results of the numerics. Here, I take $D_{1,a}/D_{1,c} = 40.0$ and $D_{2,a}/D_{2,c} = 0.665$. These ratios can be obtained using the results for the average velocity on the MgB_2 Fermi surfaces and assuming isotropic scattering; see Refs. [25,26]. The ratio $D_{2,c}/D_{1,c}$ takes the values 100 and 600 for the matrix $\hat{\Lambda}$ calculated numerically [19]. This choice is motivated by the facts that the ratio $D_{2,c}/D_{1,c} \approx 100$ can be obtained assuming that the scattering rate of electrons is the same in the both bands. On the other hand, $R = D_{2,c}/D_{1,c} = 600$ gives a better fit with experiments on the anisotropy measurements [25]. Also, I use the matrix $\hat{\Lambda}$ and $D_{1,c}/D_{1,a} \approx 30$, $D_{1,c}/D_{1,a} \approx 0.96$, and $D_{2,c}/D_{1,c} = 126$ from the fit with experiments [20].

The results are shown in Fig. 1. For temperatures $T \lesssim 0.6T_c$ I have found that the ratio H_{c3}/H_{c2} is nearly constant and has a value slightly below η. This can be explained by the fact that at low temperatures the fields H_{c2} and H_{c3} are determined mostly by the σ band whose coherence length is much smaller than that of the π band. At low T the magnetic field H_{c2} depends on the ground-state eigenvalues of the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$ and the contribution of excited states is negligible [17,18]. The ratios $(\tilde{D}_{1,x}/\tilde{D}_{1,z})^{1/4}$ and $(\tilde{D}_{2,x}/\tilde{D}_{2,z})^{1/4}$ determining the length x_0 are equal to ≈ 2.5 and 0.9, respectively. This means that one can maximize the field H_{c3} by choosing $x'_{1,0} = 0.7618$. The length $x'_{2,0}$ then is large, and the ground-state eigenvalue of $\hat{h}_2(x'_{2,0})$ is close to 1. H_{c3}/H_{c2} then can be calculated as follows: take the zero temperature expression for H_{c2} [18,25], and make there a substitution $D_{1,j} \rightarrow D_{1,j}/\eta$. At low temperatures [18,25], $H_{c2} = \Phi_0 T_c \exp(g/2)/2\gamma(D_{1,a}D_{1,c}D_{2,a}D_{2,c})^{1/4}$, with $g = (\lambda_0^2/w^2 + \ln^2\kappa/4 + 2\lambda_- \ln\kappa/w)^{1/2} - \lambda_0/w$, $\kappa = D_{2,a}D_{2,c}/D_{1,a}D_{1,c}$, $\lambda_- = \lambda_{11} - \lambda_{22}$, $\lambda_0 = (\lambda_-^2 + 4\lambda_{12}\lambda_{21})^{1/2}$, $w = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}$, and $\ln\gamma = 0.5772$. This procedure yields $H_{c3}/H_{c2} = 1.688, 1.691$ for $D_{2,c}/D_{1,c} = 100$ and 600, respectively, and $H_{c3}/H_{c2} = 1.6728$ for $D_{2,c}/D_{1,c} = 126$. The values obtained in the numerics are slightly larger (but still smaller than η) due to a small contribution to x_0 from the π band.

If one increases temperature, the ratio H_{c3}/H_{c2} decreases and exhibits a minimum at $T \simeq 0.9T_c$. Then, the value of H_{c3}/H_{c2} goes up and takes a maximum at $T \simeq 0.99T_c$. At $T = T_c$, $H_{c3}/H_{c2} = \eta$. The nontrivial behavior of the ratio H_{c3}/H_{c2} in MgB_2 is due to the changing relative importance of the π band. While at low T it is unimportant, at high T it gives a contribution to the fields H_{c2} and H_{c3} comparable with that of the σ band. Remarkably, deviations from the GLT are maximal close to T_c, similar to the H_{c2} anisotropy [25].

Let us analyze the situation close to T_c in more detail. In particular, let us explain why at $T = T_c$, $H_{c3}/H_{c2} = \eta$. For $T_c - T \ll T_c$ the field H_{c3} is small and so are the eigenvalues of the operators $\hat{H}_1(x_0)$ and $\hat{H}_2(x_0)$; i.e., one can use the expansion $\sum_{\omega>0}^{\omega_D} 1/[\omega + \hat{H}_\alpha(x_0)] \approx \sum_{\omega>0}^{\omega_D} 1/\omega - \sum_{\omega>0}^{\omega_D} \hat{H}_\alpha(x_0)/\omega^2 + \cdots$, and Eq. (3) can be rewritten in the form

$$\hat{W}\begin{pmatrix}\Delta_1(x)\\ \Delta_2(x)\end{pmatrix} = \begin{pmatrix}\hat{R}_1(x_0) & 0\\ 0 & \hat{R}_2(x_0)\end{pmatrix}\begin{pmatrix}\Delta_1(x)\\ \Delta_2(x)\end{pmatrix}, \tag{7}$$

with $\hat{W} = \hat{\Lambda}^{-1} - \ln(2\gamma\omega_D/\pi T)\mathbb{1}_2$ and $\hat{R}_\alpha(x_0) = 2\pi T\sum_{\omega>0}^{\omega_D} 1/[\omega + \hat{H}_\alpha(x_0)] - 2\pi T\sum_{\omega>0}^{\omega_D} 1/\omega$. T_c is determined by $\det\hat{W} = 0$. Solving the system (7) for $\Delta_1(x)$, I obtain

$$(W_{11}\hat{R}_2(x_0) + W_{22}\hat{R}_1(x_0) - \hat{R}_2(x_0)\hat{R}_1(x_0))\Delta_1(x) = 0. \tag{8}$$

Equation (8) determines the fields H_{c2} and H_{c3}. To lowest order, one can neglect the term $\hat{R}_2(x_0)\hat{R}_1(x_0)$.

The equation $(W_{11}\hat{R}_2(x_0) + W_{22}\hat{R}_1(x_0))\Delta_1(x) = 0$ has the ground-state solution of the same form as Eq. (3) for a single-gap superconductor (the case $\lambda_{12} = \lambda_{21} = 0$) with $\tilde{D}_{1,x} \rightarrow D_X = W_{22}\tilde{D}_{1,x} + W_{11}\tilde{D}_{2,x}$ and $\tilde{D}_{1,z} \rightarrow D_Z = W_{22}\tilde{D}_{1,z} + W_{11}\tilde{D}_{2,z}$, and the problem of finding H_{c3} becomes equivalent to the original one considered by Saint-James and de Gennes [1]. Consequently, to lowest order in T_c the ratio H_{c3}/H_{c2} has the same value η as in the case of a single-gap superconductor. The approximation described above is equivalent to the GLT; i.e., the GLT is unable to explain deviations of H_{c3}/H_{c2} from η.

The maximum of H_{c3}/H_{c2} takes place very close to T_c and is at the boundary of the accuracy of the present numerical calculations; i.e., an analytical approach would be useful. Temperature corrections to H_{c3}/H_{c2} can be found by expanding $\hat{R}_1(x_0)$ and $\hat{R}_2(x_0)$ to order $(T_c - T)^2$. One can decompose the operator in the left-hand side of (8) as a sum $\hat{L}_1(H, x_0) + \hat{L}_2(H, x_0)$, with $\hat{L}_1(H, x_0) \propto T_c - T$ and $\hat{L}_2(H, x_0) \propto (T_c - T)^2$. Let $|\phi_0\rangle$ be the solution of the equation $\hat{L}_1(H, x_0)|\phi_0\rangle = 0$ and $H = H^{(0)}(x_0)$ the critical field to this order. The correction to the eigenvalue can be found perturbatively and are determined by the implicit relation

$$\langle\phi_0|\hat{L}_1(H, x_0)|\phi_0\rangle + \langle\phi_0|\hat{L}_2(H^{(0)}, x_0)|\phi_0\rangle = 0. \qquad (9)$$

To this order, $x_0 = 0.7618(D_X/D_Z)^{1/4}(\Phi_0/2\pi H^{(0)})^{1/2}$. Temperature corrections to H_{c3} due to change in x_0 are proportional to $(T_c - T)^3$ and can be neglected. Since $|\phi_0\rangle$, $\hat{L}_1(H, x_0)$, and $\hat{L}_2(H, x_0)$ are known, one can find H_{c3}/H_{c2} analytically. Straightforward but quite cumbersome calculations [27] show that near T_c, $H_{c3}/H_{c2} \approx \eta + b(T_c - T)/T_c$, with $b \simeq 1$ for $D_{2,c}/D_{1,c}$ in the range from 100 to 600, in accordance with the numerics; see inset in Fig. 1.

Experiment.—Recent experiments [15] show that the ratio H_{c3}/H_{c2} is reduced in the case (ii). The values $H_{c3}/H_{c2} \approx 1.5$ in the temperature range 20–30 K have been reported. For case (i), $H_{c3}/H_{c2} \approx 1.7$ [14,15]. The present theory gives that $H_{c3}/H_{c2} = \eta$ for case (i) and $H_{c3}/H_{c2} < \eta$ for case (ii), in agreement with [14,15]. There are three main sources of deviation in the exact value H_{c3}/H_{c2} between theory and experiment. First, surface quality [28] might affect H_{c3}/H_{c2}. Second, there is an error in determining the resistive transition. Third, MgB_2 is situated somewhere at the boundary of the applicability of the weak-coupling BCS theory. It would be interesting to repeat the calculation done in the present Letter starting from the Eliashberg equations.

The method described above can be generalized for an arbitrary direction of crystallographic axes with respect to the surface of a superconductor. For strongly anisotropic superconductors, surface superconductivity might disappear if the surface does not coincide with crystallographic planes [8]. Estimates [27] show that MgB_2 is sufficiently anisotropic in order to observe this kind of effects. The detailed analysis of the onset of surface superconductivity in this case is challenging for both theorists and experimentalists.

In conclusion, I have presented the calculation of the ratio H_{c3}/H_{c2} for the two-band superconductor MgB_2 in the dirty limit. Remarkably, in contrast to the case of a single-gap superconductor, the above ratio is temperature dependent. The Ginzburg-Landau theory is unable to explain deviations of H_{c3}/H_{c2} from η.

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Note added in proof.—Very recently, the effects of the interaction between electrons in different bands on vortices in MgB_2 have also been taken into account (cf. [29]).

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