Fractional Quantum Hall States in Ultracold Rapidly Rotating Dipolar Fermi Gases

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We demonstrate the experimental feasibility of incompressible fractional quantum Hall-like states in ultracold two-dimensional rapidly rotating dipolar Fermi gases. In particular, we argue that the state of the system at filling fraction $\nu = 1/3$ is well described by the Laughlin wave function and find a substantial energy gap in the quasiparticle excitation spectrum. Dipolar gases, therefore, appear as natural candidates of systems that allow us to realize these very interesting highly correlated states in future experiments.

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During the recent years, cold atom systems with strongly pronounced interparticle correlations have become a subject of intensive studies, both theoretically and experimentally. There are several ways to increase the role of interparticle interactions in gaseous trapped systems and to reach the strongly correlated regime. One of the possibilities is to employ an optical lattice where the tunneling strength between sites is smaller than the Hubbard-like onsite interaction [1]. This approach has led to a spectacular experimental observation of the Mott-Hubbard transition in atomic lattice Bose gases [2] and is nowadays a main tool to create strongly correlated systems. Another way to enhance the effects of interparticle interactions is to use a quasi-2D rotating harmonic trap [3–6]. When the rotational frequency approaches the trap frequency, i.e., in the limit of critical rotation, the single-particle energy spectrum becomes highly degenerate, and hence, the role of interparticle interactions becomes dominant. The Hamiltonian of the system in the rotating frame of reference is formally equivalent to the one of charged particles moving in a constant perpendicular magnetic field. This opens a remarkable possibility to establish a link with physics of the quantum Hall effect and to realize a large variety of strongly correlated states proposed in the context of the fractional quantum Hall effect [7] in a completely different experimental setup. Recently, the idea of composite bosons (bound states of vortices and bosonic atoms) has been used to describe the ground state of a rotating Bose-Einstein condensate in a parabolic trap in the regime of large coherence length [8–10]. The variational Laughlin-like ground states have been explored to describe the atomic [11,12] and vortex [13] states of trapped rotating Bose gases. In Ref. [14], a method of creating, manipulating, and detecting anyonic quasiparticle excitations for fractional quantum Hall bosons at filling fraction $\nu = 1/2$ in rotating Bose-Einstein condensates has been proposed. However, because of the short-range character of interparticle interactions, it was found that atomic fractional quantum Hall states are feasible only for a small number of particles. This is due to the fact that Laughlin-like states do not play any specific role in a macroscopic system, when the interaction is short ranged because they are energetically degenerate with other states. Indeed, the Jastrow prefactor in the corresponding wave functions, $\prod_{i<j}(z_i - z_j)^p$, where $z_j = x_j + iy_j$ is the coordinate of the jth particle, and p is an integer (even for bosons and odd for fermions), considerably reduces the effects of a short-range interaction and, as a consequence, gapless excitations appear. This contrasts to the case of electrons where the Coulomb interaction favors fractional quantum Hall phases by lifting the degeneracy of the ground state and provides a gap for single-particle excitations [7]. It should be noted that in some cases (when more than one Landau level is occupied in the composite particle description of the fractional quantum Hall effect [15]) the situation can be improved [16] by using the recently observed Feshbach resonance in the p-wave channel [17]. This resonance, however, is accompanied by dramatic losses, and therefore its experimental application requires a more careful analysis.

In this Letter, we demonstrate that rotating quasi-2D gaseous systems with dipole-dipole interactions could provide all necessary ingredients for the observation of fractional quantum Hall-like states. In particular, the dipole-dipole interaction favors fractional quantum Hall phases by creating a substantial gap in the single-particle excitation spectrum and makes them incompressible. We demonstrate this for the case of a quasihole excitation in the most famous Laughlin state at filling $\nu = 1/3$ in a homogeneous quasi-2D dipolar rotating Fermi gas with dipolar moments polarized perpendicular to the plane of motion. Furthermore, we discuss the possibility of providing the rotating reference frame with a quenched disorder that could ensure the robust creation and observation of fractional quantum Hall states in experiments with trapped gases.

We consider a system of N dipolar fermions rotating in an axially symmetric harmonic trapping potential with a strong confinement along the axis of rotation, the z axis. With respect to the latter, the dipoles are assumed to be aligned. Various ways of experimental realizations of ultracold dipolar gases are discussed in the review [18]. Assuming that the temperature T and the chemical potential μ are much smaller than the axial confinement, $T, \mu \ll \omega_z$, the gas is effectively two dimensional, and the Hamiltonian of the system in the rotating reference frame reads

$$\mathcal{H} = \sum_{j=1}^{N}\left(\frac{p_j^2}{2m} + \frac{m}{2}\omega_0^2 r_j^2 - \omega L_{jz}\right) + \sum_{j<k}^{N}\frac{d^2}{|\mathbf{r}_j - \mathbf{r}_k|^3}. \quad (1)$$

Here $\omega_0 \ll \omega_z$ is the radial trap frequency, ω is the rotation frequency, m is the mass of the particles, d is their dipolar moment, and L_{jz} is the projection of the angular momentum with respect to the z axis of the jth particle located at $\mathbf{r}_j = x_j\mathbf{e}_x + y_j\mathbf{e}_y$. The above Hamiltonian can be conveniently rewritten in the form

$$\mathcal{H} = \sum_{j=1}^{N}\Bigg[\underbrace{\frac{1}{2m}(\mathbf{p}_j - m\omega_0\mathbf{e}_z\times\mathbf{r}_j)^2}_{\mathcal{H}_{\text{Landau}}} + \underbrace{(\omega_0 - \omega)L_{jz}}_{\mathcal{H}_{\Delta}}\Bigg] + V_d, \quad (2)$$

where $\mathcal{H}_{\text{Landau}}$ is formally equivalent to the Landau Hamiltonian of particles with mass m and charge e moving in a constant perpendicular magnetic field with the vector potential $\mathbf{A} = (cm\omega_0/e)\mathbf{e}_z \times \mathbf{r}$, V_d is the dipole-dipole interaction [the last term in Eq. (1)], and $\mathcal{H}_\Delta$ describes the shift of single-particle energy levels as a function of their angular momentum and the difference of the frequencies $\Delta\omega = \omega_0 - \omega$.

In the limit of critical rotation $\omega \to \omega_0$, one has $\mathcal{H}_\Delta \ll \{\mathcal{H}_{\text{Landau}}, V_d\}$, and the Hamiltonian (1) describes the motion of dipolar particles in a constant perpendicular magnetic field with cyclotron frequency $\omega_c = 2\omega_0$ [8]. The spectrum of $\mathcal{H}_{\text{Landau}}$ is well-known and consists of equidistantly spaced Landau levels with energies $\varepsilon_n = \hbar\omega_c(n + 1/2)$. Each of these levels is highly degenerate and contains $N_{\text{LL}} = 1/2\pi l_0^2$ states per unit area, where $l_0 = \sqrt{\hbar/m\omega_c}$ is the magnetic length. For a given fermionic surface density n_f one can introduce the filling factor $\nu = 2\pi l_0^2 n_f$ that denotes the fraction of occupied Landau levels. Note that under the condition of critical rotation, the density of the trapped gas is uniform except at the boundary provided by an external confining potential.

For filling fractions $\nu \leq 1$, particles solely occupy the lowest Landau level and the corresponding many-body eigenfunction of $\mathcal{H}_{\text{Landau}}$ takes the form

$$\Psi(z_j) = \mathcal{N} P[z_1, \ldots, z_N]\exp\left(-\sum_{j=1}^{N}|z_j|^2/4l_0^2\right),$$

where $\mathcal{N}$ is the normalization factor and $P[\{z_j\}]$ is a totally antisymmetric polynomial in the coordinates $z_j = x_j + iy_j$ of the particles. The corresponding eigenenergy is independent of the specific choice of $P[\{z_j\}]$ and equals $N\hbar\omega_c/2$, where N is the total number of particles. This degeneracy is lifted if the dipole-dipole interaction V_d is considered. In the following, we limit ourselves to a system at filling $\nu = 1/3$, where interparticle interaction effects are most pronounced.

In this case, the trial wave functions for the ground and quasihole excited states can be taken in the form proposed by Laughlin [19],

$$\Psi_{\text{L}}(\{z_j\}) = \mathcal{N}\prod_{k<l}^{N}(z_k - z_l)^3\exp\left(-\sum_{i=1}^{N}|z_i|^2/4l_0^2\right), \quad (3)$$

$$\Psi_{\text{qh}}(\{z_j\}, \zeta_0) = \mathcal{N}_0\prod_{j=1}^{N}(z_j - \zeta_0)\Psi_{\text{L}}, \quad (4)$$

where ζ_0 is the position of the quasihole. The choice of these wave functions in the case of the considered system with dipole-dipole interactions can be justified as follows. They are exact eigenfunctions for short-range δ-like potentials and are proven to be very good trial wave functions for the Coulomb interaction problem. Actually, as it was shown by Haldane (see the corresponding contribution in [7]), the Laughlin states in the fractional quantum Hall effect are essentially unique and rigid at the corresponding filling factors ($\nu = 1/3$ in our case). They are favored by strong short-range pseudopotential components that are particularly pronounced in the case of a dipolar potential.

Another possible candidate for the ground state of our system could be a crystalline state similar to the 2D Wigner electron crystal in a magnetic field [20]. For a nonrotating dipolar Fermi gas in a 2D trap, this state has lower energy than the gaseous state for sufficiently high densities. The estimate of the stability region can be obtained from the Lindemann criterion: the ratio γ of the mean square difference of displacements in neighboring lattice sites to the square of the interparticle distance, $\gamma = \langle(\mathbf{u}_i - \mathbf{u}_{i-1})^2\rangle/a^2$, should be less than some critical value γ_c (see, e.g., Ref. [21]). The results of Ref. [22] indicate that $\gamma_c \approx 0.07$. For zero temperature, γ could be estimated as $\gamma \sim \hbar/ma^2\omega_D$, where ω_D is the characteristic (Debye) frequency of the lattice phonons, $\omega_D^2 \sim 36d^2/ma^5$. As a result, the dipolar crystal in a nonrotating gas is stable if the interparticle distance $a = (\pi n_f)^{-1/2}$ satisfies the condition $a < a_d(6\gamma_c)^2 \ll a_d$; i.e., the gas is in the strongly correlated regime, $V_d \sim d^2/a^3 \sim (a_d/a) \times (\hbar^2/ma^2) \gtrsim \varepsilon_F/(6\gamma_c)^2 \gg \varepsilon_F$.

A strong magnetic field with the cyclotron frequency larger than the Debye frequency, $\omega_c > \omega_D$, favors the crystalline state by modifying the vibrational spectrum of the crystal. In this case, $\gamma \sim \hbar/ma^2\omega_c$ [20], and the corresponding critical value is $\gamma_c \approx 0.08$ [22]. Therefore, the crystalline state is stable if $\gamma \sim \nu/2 < \gamma_c$. This limits the filling factor ν to small values $\nu < 1/6$. As a result, the ground state of the system at filling factor $\nu = 1/3$ is indeed well described by the Laughlin wave function (3). This conclusion can also be justified by a comparison of the energies per particle for these competing states. The corresponding calculations show that for $\nu = 1/3$ the Laughlin state is energetically favorable [23].

In order to prove the incompressibility of the state Ψ_L, we calculate the energy gap $\Delta\varepsilon_{qh}$ for the quasihole excitation. This gap can be expressed in terms of the pair correlation functions of the ground state $g_0(z_1, z_2)$ and the quasihole state $g_{qh}(z_1, z_2)$, respectively, as

$$\Delta\varepsilon_{qh} = \int d^2z_1 d^2z_2 V_d(z_1, z_2)[g_{qh}(z_1, z_2) - g_0(z_1, z_2)]. \quad (5)$$

For the states (3) and (4), the functions g_0 and g_{qh} have been calculated using the Monte Carlo method [24]; they can be approximated in the thermodynamic limit as [25]

$$g_0(z_1, z_2) = \frac{\nu^2}{(2\pi)^2}\left(1 - e^{-|z_1-z_2|^2/2} - 2\sum_j^{\text{odd}} \frac{C_j}{4^j j!}|z_1 - z_2|^{2j} e^{-|z_1-z_2|^2/4}\right), \quad (6a)$$

$$g_{qh}(z_1, z_2) = \frac{\nu^2}{(2\pi)^2}\left[\prod_{j=1}^{2}(1 - e^{-|z_j|^2/2}) - e^{-(|z_1|^2+|z_2|^2)/2}\left(|e^{z_1 z_2^\star/2} - 1|^2 + 2\sum_j^{\text{odd}} \frac{C_j}{4^j j!}\sum_{k=0}^{\infty}\frac{|F_{j,k}(z_1, z_2)|^2}{4^k k!}\right)\right], \quad (6b)$$

$$F_{j,k}(z_1, z_2) = \frac{z_1 z_2}{2}\sum_{r=0}^{j}\sum_{s=0}^{k}\binom{j}{r}\binom{k}{s}\frac{(-1)^r z_1^{r+s} z_2^{j+k-(r+s)}}{\sqrt{(r+s+1)(j+k+1-r-s)}}, \quad (6c)$$

where the values of the expansion coefficients C_j can be found in Ref. [25]. With respect to $\nu = 1/3$, it was argued that an accuracy better than 2% is already achieved when only the first two coefficients $C_1 = 1$ and $C_3 = -1/2$ are taken into account. In Fig. 1 we plot the difference $g_{qh} - g_0$ for the particular choice $z_1 = 3$ and $\zeta_0 = 0$. After substituting these expressions into Eq. (5) and integrating numerically, we obtain

$$\Delta\varepsilon_{qh} = (0.9271 \pm 0.019)d^2/l_0^3 \quad (7)$$

for the energy gap in the spectrum of quasiholes. Naturally, a gap of the same order of magnitude is to be expected in the spectrum of quasiparticles (quasielectrons, in the language of the fractional quantum Hall effect), although calculations in this case are much more difficult because, to the best of our knowledge, the closed or even approximate expression for the corresponding pair correlation function does not exist. The gap (7) can also be written in the form

$$\Delta\varepsilon_{qh} = (0.9271 \pm 0.019)\hbar\omega_c(a_d/l_0),$$

where $a_d = md^2/\hbar^2$ can be considered as a characteristic size of the dipole interaction. For a dipole moment of the order of 0.5 D, mass $m \sim 30$ atomic mass units, the value of a_d is of the order of 10^3 Å, and for the trap frequency $\omega_0 \sim 2\pi 10^3$ Hz, one obtains the gap $\Delta\varepsilon_{qh} \sim 30$ nK and the ratio $\Delta\varepsilon_{qh}/\hbar\omega_c < 1$. This result shows (see Fig. 2) that, on the one hand, the interparticle interaction does not mix different Landau levels, and thus the lowest Landau level approximation used in the construction of the Laughlin wave function (3) is reliable. On the other hand, it guarantees that the neglected term $\mathcal{H}_\Delta$ in the Hamiltonian (2), which is inevitably present in some experimental realizations (see below), is indeed small and does not influence the trial wave functions. We can also compare our result for the gap in the quasiparticle spectrum in the coresponding Bose system with contact interactions [12]. In that case, in order to obtain a subtantial gap (say, half of $\hbar\omega_c$), the s-wave scattering length should be comparable to the magnetic length. Typically, such large values could be achieved by using Feshbach resonance techniques. However, performing an experiment with the system close to a Feshbach resonance requires a thorough analysis of inelastic losses.

Let us now discuss possible ways of experimental realization and detection of the above states. At present,

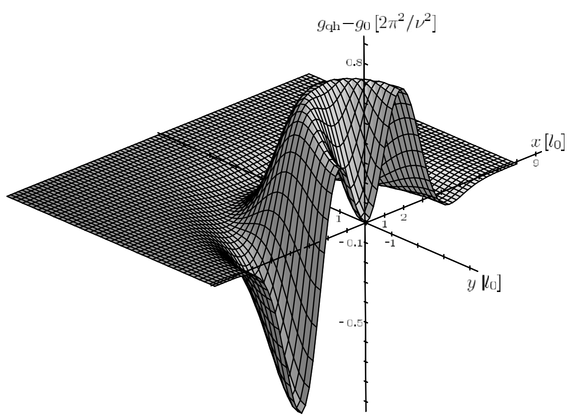

FIG. 1. The difference $g_{qh} - g_0$ as a function of $z \equiv z_2 - z_1$ for $\zeta_0 = 0$ and $z_1 = 3$. Both particles strongly avoid each other, and rotational invariance is broken by the quasihole.

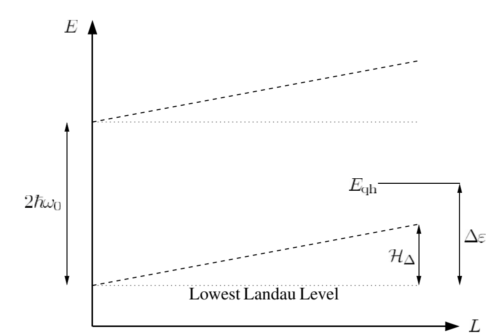

FIG. 2. Single-particle energy levels of the Hamiltonian $\mathcal{H} = \mathcal{H}_{\text{Landau}} + \mathcal{H}_\Delta$ versus their angular momentum L.

there exist two experimental methods to create rapidly rotating gas clouds. In the JILA experiments [5,6], a rotating condensate in the harmonic trap was created by evaporation of one of the spin components, and rotational rates $\omega > 99\%$ of the centrifugal limit were achieved. In this case, the term $\mathcal{H}_\Delta$ is inevitably present in the Hamiltonian and limits the total number of particles N. Namely, the condition $\mathcal{H}_\Delta < \Delta\varepsilon_{\rm qh} \lesssim \hbar\omega_c$ and the fact that single-particle states with angular momenta up to $L_z = 3N(N-1)/2$ contribute to the states (3) and (4) impose the constraint $3N(N-1)/2 < \Delta\varepsilon_{\rm qh}/\Delta\omega$. For $\Delta\omega/\omega_0 = 10^{-3}$, it gives $N < 30$. Fortunately, this bound is large enough to expect the validity of our calculations, which were performed for a homogeneous gas in the thermodynamic limit.

In the experiments of the ENS group [3,4], the bosonic gas sample was brought into rotation by stirring it with an additional laser. In addition to the harmonic potential of the optical trap, there is an extra (quartic) confining potential that allows us to reach and even exceed the critical value ω_c. In the case of critical rotation, the term $\mathcal{H}_\Delta$ can be neglected and the number of particles is limited only by the radial size of the gas cloud. We point out that in experiments of this type, it is possible to impose a quenched disorder potential in the rotating frame, generated by speckle radiation from a rotating diffractive mask [26]. The rotation of the mask should be synchronized with the stirring laser. This quenched disorder potential localizes single-particle excitations that appear in the system when the filling factor ν deviates from the value $1/3$. Therefore, it provides fractional quantum Hall states with the robustness necessary for experimental observation.

Let us finally mention possible ways of experimental detection of the fractional quantum Hall states. One of them could be the measurement of the statistics of quasiholes using the Ramsey-type interferometric method similar to that proposed in Ref. [14]. Another possibility would be to study the properties of the low energy surface (edge) modes, which are analogous to the chiral edge states of electrons in the quantum Hall effect. The corresponding analysis for a rotating bosonic cloud was recently performed in Ref. [27]. Finally, we could propose the detection of collective modes that are similar to magnetorotons and magnetoplasmons in electron quantum Hall systems (see, e.g., Girvin's contribution to Ref. [7]).

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