Pentaquarks $uudd\bar{s}$ with One Color Sextet Diquark

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The masses of pentaquarks $uudd\bar{s}$ are calculated within the framework of a semirelativistic effective QCD Hamiltonian using a diquark picture. This approximation allows a correct treatment of the confinement, assumed here to be similar to a Y junction. With only color antitriplet diquarks, the mass of the pentaquark candidate Θ with positive parity is found around 2.2 GeV. It is shown that, if a color sextet diquark is present, the lowest $uudd\bar{s}$ pentaquark is characterized by a much smaller mass with a negative parity. A mass below 1.7 GeV is computed if the masses of the color antitriplet and color sextet diquarks are taken similar.

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Introduction.—Recent experiments have reported the existence of a very narrow peak in K^+n and K^0p invariant mass distributions at 1.540 GeV [1]. This Θ resonance with isospin $I = 0$, which has been confirmed by several experimental groups in various reaction channels [2], is interpreted as a $uudd\bar{s}$ pentaquark [3]. Quantum numbers are not known, and various theories predict a $J^P = 1/2^+$ or $1/2^-$ assignment.

A popular model to describe the pentaquarks relies on the hypothesis that quarks can form diquark clusters inside the pentaquark. In this case, the confinement, which is at the origin a complicated five-body interaction with seven strings, reduces to a three-body interaction with three strings. A realistic confinement potential can then be built. The diquark picture has been proposed to explain the properties of the Θ resonance (let us note that this picture is not used in all models [4]). In Refs. [5–7], a good value is obtained for the mass, but these models do not take into account the full dynamics. In Refs. [8–10], the confinement is correctly taken into account and, as a consequence, pentaquark masses are found above 2 GeV.

Such high masses are found because the pentaquark is assumed to be composed of one antiquark and two identical color antitriplet $[ud]$ diquarks with vanishing spin and isospin. In that case, the two identical diquarks must be in a relative P wave in order to fulfill the Pauli principle, and the lowest state has a total positive parity. On the contrary, if two different diquarks are contained in a pentaquark, this P-wave penalty can be avoided, but the lowest state has a total negative parity.

In this work we show that pentaquarks can be composed with two different diquarks: one in a color antitriplet representation and the other in a color sextet representation. Provided some reasonable assumptions are made about the diquark masses, a $uudd\bar{s}$ pentaquark with a negative parity can be computed at mass near the Θ mass.

$[ud]$ *diquarks.*—All short-range interactions available between quarks, one-gluon exchange [11], Goldstone-boson exchange [12], and instanton induced [13] interactions predict that the most probable diquark which can be formed is the $[ud]$ pair in a color $\bar{3}$ representation with vanishing spin and isospin. Such a structure with those quantum numbers is allowed by the Pauli principle and is denoted D.

The instanton interaction between two quarks is a zero range interaction given in Ref. [14]. It has the structure of projectors on flavor-spin-color, and depends on two dimensioned constants g and g'. The spatial dependence of the potential is singular and is generally regularized by a Gaussian function $\rho(r)$ [14].

The instanton induced interaction is also attractive for a $[ud]$ pair in color 6 representation with spin 1 and vanishing isospin. Such a structure denoted D' is also allowed by the Pauli principle and could exist [15,16]. Despite a repulsive Coulomb contribution (see Table I), a diquark with these quantum numbers could also be formed into an exotic hadron. A very simple model developed below shows that color antitriplet and color sextet diquarks could have similar masses.

One-gluon exchange potential.—The total contribution of the one-gluon exchange process, at zero order, is

$$V_{\mathrm{OGE}} = \alpha_S \sum_{i<j=1}^{3} \frac{\tilde{\lambda}_i \tilde{\lambda}_j}{4} \frac{1}{|\vec{r}_i - \vec{r}_j|}, \tag{1}$$

where α_s is the strong coupling constant and $\tilde{\lambda}_i$ a color

TABLE I. Possible $[ud]$ diquarks, with their quantum numbers (color representation C, isospin I, and spin S). The contributions of the instanton induced force (Inst.) and of the one-gluon exchange (OGE) potential are indicated.

Notation	C	I	S	Inst.	OGE
D	$\bar{3}$	0	0	$-4g\rho(r)$	$-(2/3)\alpha_S/r$
D'	6	0	1	$-2g\rho(r)$	$+(1/3)\alpha_S/r$

operator for the ith colored object. In a $DD\bar{s}$ pentaquark, the contributions of the color operators are the same for the three possible pairs, like in a (anti)baryon: $\langle \tilde{\lambda}_i \tilde{\lambda}_j/4\rangle = -2/3$. In a $DD'\bar{s}$ pentaquark, the situation is different: $\langle \tilde{\lambda}_D \tilde{\lambda}_{\bar{s}}/4\rangle = +1/3$, $\langle \tilde{\lambda}_{D'} \tilde{\lambda}_{\bar{s}}/4\rangle = \langle \tilde{\lambda}_D \tilde{\lambda}_{D'}/4\rangle = -5/3$. The sum of the color factors for the three pairs is -2 for a $DD\bar{s}$ pentaquark and -3 for a $DD'\bar{s}$ pentaquark. So we can expect, at first approximation, a stronger attraction for the last system. This Coulomb attraction is reinforced by the possibility of a $L = 0$ total angular orbital momentum for the $DD'\bar{s}$ system.

Confinement.—The probably best possible simulation of the confinement in a three colored object system is a Y-junction potential. Each colored source generates a flux tube and the three strings connect at some point with position $\vec{r}_0$, in order to minimize the potential energy. It is expected that the energy density of the tube is proportional to the color Casimir operator of the source $\tilde{\lambda}_i^2/4$ and the length of the flux tube [17,18]. The Y-junction potential, at zero order, is then written

$$V_Y = \frac{3a}{4} \min_{\vec{r}_0} \left[\sum_{i=1}^{3} \frac{\tilde{\lambda}_i^2}{4} |\vec{r}_i - \vec{r}_0| \right], \tag{2}$$

where a is the usual string tension.

This complicated three-body interaction can be simulated by another type of junction in which the three flux tubes connect to the center of mass of the system. This one-body approximation is quite good (better than 2%) provided the string tension is slightly renormalized [19]. So, the confinement we use in this work is

$$V_{\mathrm{CM}} = f \frac{3a}{4} \sum_{i=1}^{3} \frac{\tilde{\lambda}_i^2}{4} |\vec{r}_i - \vec{r}_{\mathrm{CM}}|, \tag{3}$$

where $\vec{r}_{\mathrm{CM}}$ is center of mass coordinate. The parameter $f < 1$ rescales the interaction to simulate at best the Y junction. The possible values for the color Casimir operator are $\langle \bar{s}|\tilde{\lambda}^2/4|\bar{s}\rangle = \langle D|\tilde{\lambda}^2/4|D\rangle = 4/3$ and $\langle D'|\tilde{\lambda}^2/4|D'\rangle = 10/3$. With these numbers, an increase of the confinement interaction can be expected for a $DD'\bar{s}$ pentaquark with respect to a $DD\bar{s}$ pentaquark.

Residual interactions.—The instanton interactions, which are assumed to be responsible for the existence of the diquarks, must act also between a quark inside a diquark and the other quarks or the antiquark of the pentaquark. In Ref. [10], the residual instanton interactions are computed for the lowest $DD\bar{s}$ pentaquark. They act between the antiquark and any of the u and d quarks inside the diquarks (no such contribution is expected between the two diquarks since they are in a P wave). This decreases the mass of such a state by a quantity which is around 40 MeV. In the lowest $DD'\bar{s}$ pentaquark, the attractive instanton interactions act between all the component quarks and antiquark since the DD' system is in a S wave. A first crude estimation of this effect, which depends on the total spin of the pentaquark, gives a total potential whose strength is 2–3 times the corresponding one in a $DD\bar{s}$ pentaquark. So the mass of the lowest $DD'\bar{s}$ system could be decreased by a quantity which is around 100–150 MeV.

Supplementary residual interactions stemming from the one-gluon exchange process can act in pentaquarks. A gluon carrying both spin and color can turn a D diquark into a D' diquark. This mechanism can couple $DD\bar{s}$ and $DD'\bar{s}$ pentaquarks and can decrease the mass of the lowest system, that is to say the $DD'\bar{s}$ as we shall see below. It can also produce an oscillation $D \leftrightarrow D'$ inside a $DD'\bar{s}$ pentaquark. Such a mechanism can be generated by a color-spin interaction of type $\tilde{\lambda}_i \tilde{\lambda}_j \vec{s}_i \vec{s}_j$ between the quarks inside the diquarks. As this potential is a relativistic correction to the one-gluon exchange potential, its contribution to the mass is expected weaker than those due to the interactions considered above.

All these contributions are not easy to calculate, but they could lead to the existence of several pentaquarks with masses differing only from several tens of MeV. This must be confirmed experimentally.

Total Hamiltonian.—We use an effective QCD Hamiltonian derived in Ref. [8], but with all its auxiliary fields eliminated, as defined in Ref. [20]. The total Hamiltonian, in the center of mass frame, is a semirelativistic kinetic energy part supplemented by the Coulomb part of the one-gluon exchange potential V_{OGE} and the confinement V_{CM} described above

$$H_0 = \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2} + V_{\mathrm{OGE}} + V_{\mathrm{CM}}. \tag{4}$$

Because of their probable weak contributions, the complicated residual interactions discussed above are not taken into account here.

The particle self-energy is computed and appears as a contribution depending on the constituent particle mass

$$M = M_0 + \sum_{i=1}^{3} \frac{C(s_i, m_i, a, \delta)}{\langle \sqrt{\vec{p}_i^{\,2} + m_i^2} \rangle}, \tag{5}$$

where M_0 is an eigenvalue of H_0 and where $C(s_i, m_i, a, \delta)$ is a negative contribution for a fermion and vanishes for a boson [8]. The inverse gluonic correlation length δ is around 1 GeV (the results are not sensitive to this parameter). In pentaquarks with diquark clustering, the only contribution of the self-energy comes from the antiquark. This Hamiltonian was used in Ref. [10] for the $DD\bar{s}$ systems.

Parameters.—The values of the parameters a, α_S, δ, and m_n (n stands for u or d) are taken from a previous work about pentaquarks [8]. When the three colored sources are identical, a good value for the parameter f is around 0.94 [19]. In this Letter, we use this value for both $DD\bar{s}$ and $DD'\bar{s}$ pentaquarks.

The parameters m_D and m_s are computed in order to reproduce the masses of the baryons N and Λ considered as Dq systems. The procedure to compute these baryon

masses relies on the operators (4) and (5) adapted to a mesonlike system, as in Ref. [10]. To consider the N and Λ baryons, respectively, as pure Dn and Ds states is probably not the better approximation [21]. But our aim is just to obtain a reasonable estimation for the mass of the diquark D. A mass of 350 MeV is in good agreement with some other works [6,8,10,21]. Table II summarizes the parameters used in our calculations.

The color sextet diquark D' does not appear in any baryon. So it is not possible to fix its mass, as for the diquark D. Since a simple model yields similar masses for the D and D' diquarks (see below), $m_{D'}$ is considered here as a free parameter, with a value around m_D.

Estimation of diquark masses.—An estimation of the diquark masses can be obtained by a simple potential model, in the same spirit as in Refs. [11,16,21]. The two-quark Hamiltonian takes the following form

$$H_0 = \sum_{i=1}^{2} \sqrt{\vec{p}_i^{\,2} + m_i^2} + W_{\text{Inst.}} + W_{\text{OGE}} + \epsilon_C ar. \quad (6)$$

The mass operator takes also into account the quark self-energy [see formula (5) but for two particles]. The instanton contribution $W_{\text{Inst.}}$ and the one-gluon exchange contribution W_{OGE} are given in Table I. As diquarks are not color singlet, it is not easy to determine the better form for the "confinement." In Refs. [11,21], a potential $ar/2$ is introduced for the diquark D, based on a $\tilde{\lambda}_i\tilde{\lambda}_j$ structure for color. But such a prescription induces an anticonfining interaction for the D', and clearly is not satisfactory. Thus, the situation concerning the confinement in diquarks is quite unclear. Here we assume that the confinement is given by $\epsilon_C ar$, in which ϵ_C is a parameter in the range $[0–1]$ which could depend on the total color C of the diquark [16].

Because this Hamiltonian takes into account the instanton forces, the parameters chosen are taken from Ref. [10], except $\epsilon_{\bar{3}} = 0.65$ and $\epsilon_6 = 0.40$ which are taken from Ref. [16]. We then find $m_D = 0.531$ GeV and $m_{D'} = 0.430$ GeV. This value of m_D is larger that the one used in this work, but the important result is that $m_{D'} < m_D$. The diquarks D and D' have the same mass at 0.350 GeV with $\epsilon_{\bar{3}} = 0.38$ and $\epsilon_6 = 0.33$. Let us mention that, within this model, the diquark D (D') can be a quite small object with a size $\sqrt{\langle r^2 \rangle}$ varying from 1.0 fm (1.6 fm) to 0.5 fm (0.8 fm) when ϵ_C increases from 0.2 to 1.

Results.—The numerical algorithm to solve the two-body problem is based on the very accurate Lagrange-mesh method [22], and the technique to solve the three-body problem relies on an expansion of the wave function in terms of harmonic oscillator states with different sizes [23]. After careful checks of the convergence properties, we conclude that, with 20 quanta bases, a precision around 10 MeV can be obtained for pentaquark masses. The masses of some $uudd\bar{s}$ pentaquarks are presented in Table III.

TABLE II. Parameters of the model.

$a = 0.15$ GeV^2	$m_n = 0$
$f = 0.94$	$m_s = 0.260$ GeV
$\alpha_S = 0.39$	$m_D = 0.350$ GeV
$\delta = 1$ GeV	$m_{D'} = m_D$

The lowest $DD\bar{s}$ is characterized by $J^P = 1/2^+$ and has a mass far above the experimental value 1.540 GeV. This result is in agreement with those of Refs. [8–10].

The mass of the lowest $DD'\bar{s}$ is given with the condition $m_D = m_{D'}$. This state is characterized by $J^P = 1/2^-$. The low mass obtained is due mainly to the fact that the P-wave penalty is avoided. (The $1/2^+$ state with $L = 1$ is around 300 MeV above the $1/2^-$ state with $L = 0$.) But other effects are necessary to explain this value. The strong Coulomb potential plays an important role in the decreasing of the masses. If we put arbitrarily $\langle \tilde{\lambda}_i\tilde{\lambda}_j/4 \rangle = -2/3$, as in a $DD\bar{s}$ pentaquark, the mass of the lowest $DD'\bar{s}$ pentaquark increases by around 150 MeV. Moreover, contrary to what it is expected, the interplay between the confinement, the Coulomb forces, and the S-wave configuration makes the confining energy smaller with a $DD'\bar{s}$ internal color configuration that with a $DD\bar{s}$ one. The gain in binding energy is around 150 MeV.

Obviously, the masses of the $DD'\bar{s}$ pentaquarks depend on the D' mass. When $m_{D'}$ varies from 0.2 to 0.5 GeV, the mass of the $1/2^-$ $DD'\bar{s}$ varies from 1.550 to 1.800 GeV. The dependence is a quasilinear one, with a slope $\lesssim 1$. With reasonable values for the D' mass, the lowest $DD'\bar{s}$ state lies not too far from the experimental Θ mass. As mentioned above, some gain of binding energy could appear with the residual instanton induced interactions.

Concluding remarks.—It has been shown that dynamical models of the pentaquark $uudd\bar{s}$ within the diquark picture predict masses above 2 GeV when the two component diquarks (scalar, isospin singlet, color antitriplet) are identical [8–10].

It is shown here that an effective QCD Hamiltonian predicts that the lowest $uudd\bar{s}$ pentaquark is a $1/2^-$ state with a mass close to the Θ mass, provided this pentaquark contains two different isospin singlet diquarks: one in a color antitriplet configuration and the other in a color sextet one. This large shift in mass is obtained independently of the value chosen for the sextet diquark mass, although the precise value does obviously depend on it. The low mass obtained is due mainly to the fact that the P-wave penalty is avoided. But the particular color structures of the

TABLE III. Masses in GeV (rounded to the nearest 10 MeV) for the lowest $uudd\bar{s}$ pentaquarks, with their spin-parity J^P and quark content ($m_D = m_{D'}$).

Content	J^P	Mass	Content	J^P	Mass
$DD\bar{s}$	$1/2^+$	2.380	$DD'\bar{s}$	$1/2^-$	1.680
	$1/2^-$	2.670		$1/2^+$	1.950

Coulomb potential and of the confinement are necessary to explain this value.

Our $J^P = 1/2^-$ assignment is in agreement with the QCD sum rules which predict a negative parity for the Θ resonance [24]. The lattice QCD calculations also predict that the parity of this state is most likely negative [25].

It is possible that a lower mass, closer to the experimental values, could be computed if the contribution of the residual interactions for $DD'\bar{s}$ pentaquark are taken into account (such a work is in progress). If this is the case, it will be interesting to study the decay of this state in order to understand the reasons of the small width of the Θ resonance. This kind of calculation is complicated and out of the scope of this Letter.

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