Determining the Sign of the $b \to s\gamma$ Amplitude

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The latest Belle and *BABAR* measurements of the inclusive $\bar{B} \to X_s l^+ l^-$ branching ratio have smaller errors and lower central values than the previous ones. We point out that these results indicate that the sign of the $b \to s\gamma$ amplitude is the same as in the standard model. This underscores the importance of $\bar{B} \to X_s l^+ l^-$ in searches for new physics, and may be relevant for neutralino–dark matter analyses within the minimal supersymmetric standard model.

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The branching ratio of the inclusive radiative B-decay is one of the most important constraints for a number of new physics models because it is accurately measured and its theoretical determination is rather clean. The present world average $\mathcal{B}(\bar{B} \to X_s \gamma) = (3.52 \pm 0.30) \times 10^{-4}$ [1] agrees very well with the standard model (SM) prediction $\mathcal{B}(\bar{B} \to X_s \gamma)_{\rm SM} = (3.70 \pm 0.30) \times 10^{-4}$ [2]. A well-known way to avoid this constraint without excluding large new physics effects consists in having new physics contributions that approximately reverse the sign of the amplitude $A(b \to s\gamma)$ with respect to the SM and leave $\mathcal{B}(\bar{B} \to X_s \gamma)$ unaltered within experimental and theoretical uncertainties. Several authors pointed out that even a rather rough measurement of the inclusive $\bar{B} \to X_s l^+ l^-$ branching ratio could provide information on the sign of $A(b \to s\gamma)$ [3].

Other observables that are sensitive to the sign of $A(b \to s\gamma)$ are the forward-backward and energy asymmetries in inclusive and exclusive $b \to s l^+ l^-$ decays [3,4]. Very recently, the first measurement of the forward-backward asymmetry in $B \to K^{(*)} l^+ l^-$ was announced by the Belle Collaboration [5]. Within the limited statistical accuracy, however, the results were found to be consistent with both the SM and the "wrong-sign" $A(b \to s\gamma)$ case.

The purpose of this Letter is to point out that the present measurements of $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$ already indicate that the sign of $A(b \to s\gamma)$ is unlikely to be different from that in the SM. The experimental results that we consider are summarized in Table I.

The results in Table I are averaged over muons and electrons. The first range of the dilepton mass squared q^2 corresponds to the whole available phase space for $l = \mu$, but includes a cut for $l = e$. Moreover, the intermediate ψ and ψ' are treated as background, and a Monte Carlo simulation based on perturbative calculations is applied for the unmeasured part of the q^2 spectrum that hides under the huge ψ and ψ' peaks (see Refs. [6,7] for more details). In the second range of q^2 in Table I, theoretical uncertainties are smaller than in the first case (see below), but the experimental errors are larger due to lower statistics. As we shall see, the analyses in both regions lead to similar conclusions concerning the sign of $A(b \to s\gamma)$.

The standard model perturbative calculations are available at the next-to-next-to-leading order (NNLO) in QCD for both the considered ranges of q^2—see Refs. [8,9] for the most recent phenomenological analyses and a list of relevant references. The dominant electroweak corrections are also known [8]. In the low-q^2 domain, nonperturbative effects are taken into account in the framework of the heavy quark expansion as Λ^2/m_b^2 and Λ^2/m_c^2 corrections [10]. Analytical expressions for such corrections are also available for the full q^2 range, but they blow up in the vicinity of the intermediate ψ peak. Consequently, a cut needs to be applied, and it is no longer clear what theoretical procedure corresponds to the interpolation that is performed on the experimental side. Thus, the relative theoretical uncertainty for the full q^2 range is larger than for the low-q^2 window.

The results of the SM calculations are given in the central column of Table II. For the low-q^2 domain, they correspond to the ones of Ref. [8] with updated input values $m_{t,\rm pole} = 178.0 \pm 4.3$ GeV [11] and $\mathcal{B}(\bar{B} \to X_c l \bar{\nu}) = 10.61 \pm 0.16 \pm 0.06$ [12]. The dominant sources

TABLE I. Measurements of $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$ $[10^{-6}]$ and their weighted averages (w.a.) for two different ranges of the dilepton invariant mass squared: (a) $(2m_\mu)^2 < q^2 < (m_B - m_K)^2$ and (b) $1 < q^2 < 6$ GeV^2.

Range	Belle [6]	*BABAR* [7]	w.a.
(a)	$4.11 \pm 0.83^{+0.74}_{-0.70}$	$5.6 \pm 1.5 \pm 0.6 \pm 1.1$	4.5 ± 1.0
(b)	$1.493 \pm 0.504^{+0.382}_{-0.283}$	$1.8 \pm 0.7 \pm 0.5$	1.60 ± 0.51

TABLE II. Predictions for $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$ $[10^{-6}]$ in the standard model and with reversed sign of $\tilde{C}_7^{\rm eff}$ for the same ranges of q^2 as in Table I.

Range	SM	$\tilde{C}_7^{\rm eff} \to -\tilde{C}_7^{\rm eff}$
(a)	4.4 ± 0.7	8.8 ± 1.0
(b)	1.57 ± 0.16	3.30 ± 0.25

of uncertainty are the values of the top and bottom quark masses, as well as the residual renormalization scale dependence. For the full q^2 range, we make use of the statement in Ref. [9] that the NNLO matrix elements for $\hat{s} = q^2/m_b^2 > 0.25$ are accurately reproduced by setting the renormalization scale $\mu_b = m_b/2$ at the next-to-leading order level.

To a very good approximation, the amplitude $A(b \to s\gamma)$ is proportional to the effective Wilson coefficient $\tilde{C}_7^{\rm eff}$ $(q^2 = 0)$ that determines the strength of the $\bar{s}_L \sigma^{\alpha\beta} b_R F_{\alpha\beta}$ interaction term in the low-energy Hamiltonian. The sign of $A(b \to s\gamma)$ is therefore given by the sign of $\tilde{C}_7^{\rm eff}$ $(q^2 = 0)$. Both the value and the sign of this coefficient matter for the rare semileptonic decay. The results in the right column of Table II differ from those in the central column only by reversing the sign of $\tilde{C}_7^{\rm eff}$ in the expression for the differential $\bar{B} \to X_s l^+ l^-$ decay rate

$$\frac{d\Gamma[\bar{B} \to X_s l^+ l^-]}{d\hat{s}} = \frac{G_F^2 m_{b,\rm pole}^5 |V_{ts}^* V_{tb}|^2}{48\pi^3} \left(\frac{\alpha_{em}}{4\pi}\right)^2 (1 - \hat{s})^2 \times \left\{(1 + 2\hat{s})(|\tilde{C}_9^{\rm eff}|^2 + |\tilde{C}_{10}^{\rm eff}|^2) + \left(4 + \frac{8}{\hat{s}}\right)|\tilde{C}_7^{\rm eff}|^2 + 12\,{\rm Re}(\tilde{C}_7^{\rm eff} \tilde{C}_9^{\rm eff*})\right\}, \quad (1)$$

where $\tilde{C}_9^{\rm eff}$ and $\tilde{C}_{10}^{\rm eff}$ correspond to the low-energy interaction terms $(\bar{s}_L \gamma_\alpha b_L)(\bar{l}\gamma^\alpha l)$ and $(\bar{s}_L \gamma_\alpha b_L)(\bar{l}\gamma^\alpha \gamma_5 l)$, respectively. The definitions of all the relevant effective coefficients can be found in Sec. 5 of Ref. [13]. We stress that $\tilde{C}_i^{\rm eff}$ depend on q^2 and do *not* depend on the renormalization scale, up to residual higher-order effects. For simplicity, some of the NNLO QCD, electroweak and nonperturbative corrections are omitted in Eq. (1). However, all those corrections are taken into account in our numerical results and plots.

The sensitivity of $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$ to the sign of $\tilde{C}_7^{\rm eff}$ is quite pronounced because the last term in Eq. (1) is sizeable and it interferes destructively (in the SM) with the remaining ones. One can see that the experimental values of the $\bar{B} \to X_s l^+ l^-$ branching ratio in Table I differ from the values in the right column of Table II by 3σ in both the low-q^2 window and the full q^2 range. This fact disfavors extensions of the SM in which the sign of $\tilde{C}_7^{\rm eff}$ gets reversed while $\tilde{C}_9^{\rm eff}$ and $\tilde{C}_{10}^{\rm eff}$ receive small nonstandard corrections only.

In Fig. 1, we present constraints on additive new physics contributions to $\tilde{C}_{9,10}^{\rm eff}$ placed by the low-q^2 measurements of $\bar{B} \to X_s l^+ l^-$ (Table I), once the $\bar{B} \to X_s \gamma$ bounds on $|\tilde{C}_7^{\rm eff}|$ are taken into account. Similar plots have been previously presented in Refs. [14,15]. The two plots correspond to the two possible signs of the coefficient $\tilde{C}_7^{\rm eff}$. The regions outside the rings are excluded. Surroundings of the origin are magnified in Fig. 2 for the nonstandard case. The three lines correspond to three different values of $\mathcal{B}(\bar{B} \to X_s \gamma) \times 10^4$: 2.82, 3.52, and 4.22, which include the experimental central value as well as borders of the 90% C.L. domain. In evaluating this domain, the experimental error was enlarged by adding the SM theoretical uncertainty in quadrature. A similar procedure was applied to $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$. Its low-$q^2$ value was varied between 0.7×10^{-6} and 2.5×10^{-6}. The three lines in each plot of Figs. 1 and 2 clearly show that the exact value of $|\tilde{C}_7^{\rm eff}|$ has a minor impact on the bounds, which are therefore rather insensitive to the theoretical error estimate in $\mathcal{B}(\bar{B} \to X_s \gamma)$.

The SM point (i.e., the origin) is located barely outside the border line of the allowed region in the lower plot of Fig. 1. However, one should take into account that the overall scale in this figure is huge, and only a tiny part of the allowed region is relevant to realistic extensions of the SM. Thus, it is more instructive to look at Fig. 2, from which it is evident that a nonstandard sign of $\tilde{C}_7^{\rm eff}$ could be made compatible with experiments only by large $\mathcal{O}(1)$ new physics contributions to $\tilde{C}_{9,10}^{\rm eff}$. The SM values of $\tilde{C}_9^{\rm eff}$ and $\tilde{C}_{10}^{\rm eff}$ are around $+4.2$ and -4.4, respectively.

A case in which large nonstandard contributions to $\tilde{C}_7^{\rm eff}$ that interfere destructively with the SM ones arise naturally, while $\tilde{C}_{9,10}^{\rm eff}$ are only slightly affected, occurs in the minimal supersymmetric standard model (MSSM) with minimal flavor violation (MFV) at large $\tan\beta$, with relatively light top squark and higgsinolike chargino [3,14,16]. The maximal MFV MSSM contributions to $\tilde{C}_{9,10}^{\rm eff}$ that were found in Ref. [14] are indicated by the dashed cross in Fig. 2. As one can see, they are too small to reach the border of the allowed region. For clarity, we note that although the bounds in Ref. [14] were given for the electroweak-scale Wilson coefficients, they remain practically the same for the b-scale coefficients $\tilde{C}_{9,10}^{\rm eff}$.

Configurations of the MSSM couplings and masses for which the sign of $\tilde{C}_7^{\rm eff}$ gets reversed turn out to be relevant if no physics beyond the MSSM contributes to the intergalactic dark matter (see, e.g., Ref. [17]). In particular, configurations characterized by large mixing in the stop sector tend to be excluded by the new constraint [18]. While performing a dedicated scan over the MSSM parameters is beyond the scope of this Letter, we expect that the implementation of the $\bar{B} \to X_s l^+ l^-$ constraints will result in a significant reduction of the neutralino dark-matter-allowed region in the MSSM parameter space.

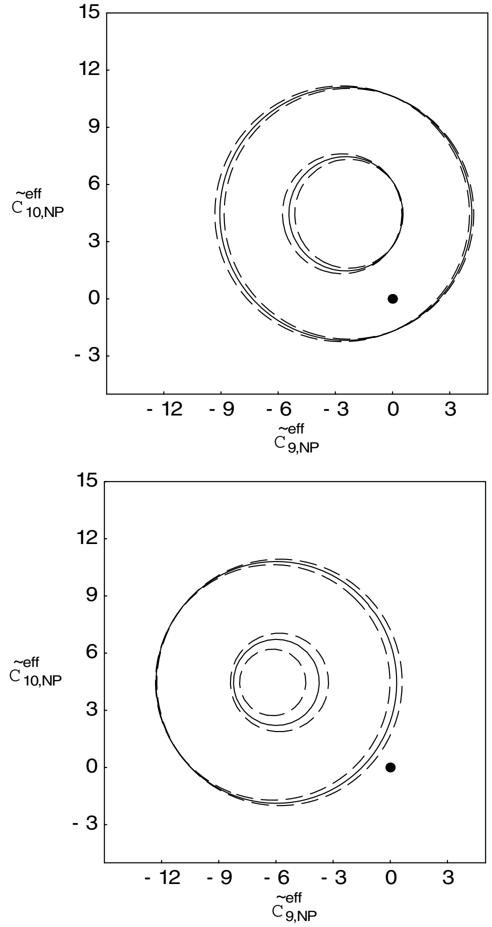

FIG. 1. Model-independent constraints on additive new physics contributions to $\tilde{C}^{\rm eff}_{9,10}$ at 90% C.L. for the SM-like (upper plot) and the opposite (lower plot) sign of $\tilde{C}^{\rm eff}_{7}$. The three lines correspond to three different values of $\mathcal{B}(\bar{B} \to X_s \gamma)$ (see the text). The regions outside the rings are excluded. The dot at the origin indicates the SM case for $\tilde{C}^{\rm eff}_{9,10}$.

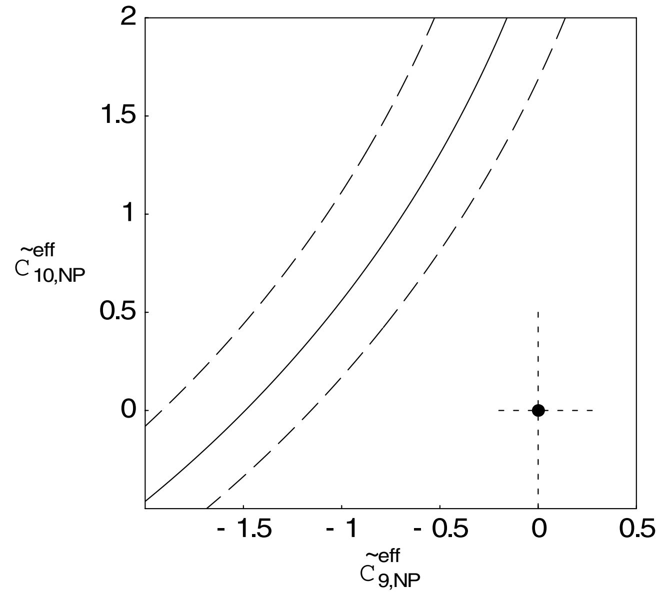

FIG. 2. Same as in the lower plot in Fig. 1. Surroundings of the origin. The maximal MFV MSSM ranges for $\tilde{C}^{\rm eff}_{9,\rm NP}$ and $\tilde{C}^{\rm eff}_{10,\rm NP}$ are indicated by the dashed cross [according to Eq. (52) of Ref. [14]].

One should be aware that in the MSSM at large $\tan\beta$, there are additional contributions suppressed by powers of the lepton masses but enhanced by $(\tan\beta)^3$. They are related to the chirality-flip operators $(\bar{s}_L b_R)(\bar{\mu}_L \mu_R)$ and $(\bar{s}_L b_R)(\bar{\mu}_R \mu_L)$ and may contribute to the muon case in a significant way. Fortunately, such contributions are bounded from above [15,19] by the experimental constraints [20] on $B^0_s \to \mu^+\mu^-$, and turn out to be irrelevant to our argument.

Another interesting example occurs in the general MSSM with R parity, where new sources of flavor and CP violation in the squark mass matrices are conveniently parametrized in terms of so-called mass insertions. The sign of the $b \to s\gamma$ amplitude can be reversed without affecting $\tilde{C}^{\rm eff}_{9,10}$ if the mass insertion $(\delta^d_{23})_{LR}$ is large and positive [21]. The new results on $\bar{B} \to X_s l^+ l^-$ exclude this possibility, and constrain significantly the case of a complex $(\delta^d_{23})_{LR}$.

To conclude, we have pointed out that the recent measurements of $\mathcal{B}(\bar{B} \to X_s l^+ l^-)$ by Belle and *BABAR* already indicate that the sign of the $b \to s\gamma$ amplitude is unlikely to be different from that in the SM. This underscores the importance of $\bar{B} \to X_s l^+ l^-$ in searches for new physics, and may be relevant for neutralino–dark matter analyses within the MSSM.

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[1] Heavy Flavor Averaging Group, J. Alexander *et al.*, hep-ex/0412073.
[2] P. Gambino and M. Misiak, Nucl. Phys. **B611**, 338 (2001); A. J. Buras, A. Czarnecki, M. Misiak, and J. Urban, Nucl. Phys. **B631**, 219 (2002).
[3] A. Ali, G. F. Giudice, and T. Mannel, Z. Phys. C **67**, 417 (1995); P. L. Cho, M. Misiak, and D. Wyler, Phys. Rev. D **54**, 3329 (1996); J. L. Hewett and J. D. Wells, Phys. Rev. D **55**, 5549 (1997); T. Goto, Y. Okada, and Y. Shimizu, Phys. Rev. D **58**, 094006 (1998).

[4] A. Ali, T. Mannel, and T. Morozumi, Phys. Lett. B **273**, 505 (1991).
[5] Belle Collaboration, K. Abe *et al.*, hep-ex/0410006.
[6] Belle Collaboration, K. Abe *et al.*, hep-ex/0408119.
[7] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **93**, 081802 (2004).
[8] C. Bobeth, P. Gambino, M. Gorbahn, and U. Haisch, J. High Energy Phys. **04** (2004) 071.
[9] A. Ghinculov, T. Hurth, G. Isidori, and Y. P. Yao, Nucl. Phys. **B685**, 351 (2004).
[10] A. Ali, G. Hiller, L. T. Handoko, and T. Morozumi, Phys. Rev. D **55**, 4105 (1997); G. Buchalla, G. Isidori, and S. J. Rey, Nucl. Phys. **B511**, 594 (1998); G. Buchalla and G. Isidori, Nucl. Phys. **B525**, 333 (1998).
[11] CDF Collaboration, P. Azzi *et al.*, hep-ex/0404010.
[12] *BABAR* Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **93**, 011803 (2004).
[13] H. H. Asatrian, H. M. Asatrian, C. Greub, and M. Walker, Phys. Lett. B **507**, 162 (2001).
[14] A. Ali, E. Lunghi, C. Greub, and G. Hiller, Phys. Rev. D **66**, 034002 (2002).
[15] G. Hiller and F. Krüger, Phys. Rev. D **69**, 074020 (2004).
[16] C. Bobeth, A. J. Buras, and T. Ewerth, hep-ph/0409293.
[17] A. Bottino, F. Donato, N. Fornengo, and S. Scopel, Phys. Rev. D **69**, 037302 (2004); **70**, 015005 (2004).
[18] S. Scopel (private communication).
[19] P. H. Chankowski and Ł. Sławianowska, Eur. Phys. J. C **33**, 123 (2004).
[20] CDF Collaboration, D. Acosta *et al.*, Phys. Rev. Lett. **93**, 032001 (2004); D0 Collaboration, V. M. Abazov, hep-ex/0410039 [Phys. Rev. Lett. (to be published)].
[21] M. Ciuchini, E. Franco, A. Masiero, and L. Silvestrini, Phys. Rev. D **67**, 075016 (2003); **68**, 079901(E) (2003).