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Hawking Radiation in Sonic Black Holes

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I present a microscopic description of Hawking radiation in sonic black holes. A one-dimensional Fermi-degenerate liquid squeezed by a smooth barrier forms a transonic flow, a sonic analog of a black hole. The quantum treatment of the noninteracting case establishes a close relationship between sonic Hawking radiation and quantum tunneling through the barrier. Quasiparticle excitations appear at the barrier and are then radiated with a thermal distribution in exact agreement with Hawking's formula. The signature of the radiation can be found in the dynamic structure factor, which can be measured in a scattering experiment. The possibility for experimental verification of this new transport phenomenon for ultracold atoms is discussed.

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Introduction.—One of the most fascinating effects in astrophysics is Hawking's prediction of black-hole radiation [1]. Hawking realized that particles such as neutrinos and photons could be created spontaneously from any black hole and emitted at just the rate that one would expect if the black hole is a body with a temperature $\hbar\kappa/2\pi k_B$ proportional to its surface gravity κ. Although Hawking's prediction combines gravitation and quantum mechanics to produce thermodynamics, it was rapidly appreciated that the radiation predicted by Hawking is more primitive and fundamental. A beautiful example of this was pointed out by Unruh [2]: The propagation of sound in a fluid or gas turning supersonic, such as in the nozzle of a rocket engine, is similar to the propagation of a scalar field close to a black hole. Using the same arguments which lead to black-hole evaporation, Unruh predicted the analog of Hawking radiation for sound waves in a moving fluid.

Although Unruh's prediction is not restricted *a priori* to a specific fluid, superfluids immediately were considered as the natural candidates for an experimental test of the sonic analog of Hawking radiation. Superfluid helium II has been the first system to be proposed for such an investigation [3]. More recently atomic Bose-Einstein condensates have been considered, as they may offer a better chance of experimental testing [4–6]. Transonic flows could be also realized in one-dimensional (1D) systems [5,7] and available 1D quantum systems, including ultracold atoms in optical potentials and waveguides [8], could be adapted to realize a sonic black hole and to investigate the sonic analog of the elusive Hawking radiation.

Sonic Hawking radiation has not been considered so far in systems of noninteracting fermions. It is the aim of this Letter to show that indeed a 1D Fermi-degenerate noninteracting gas that scatters against a very smooth potential barrier provides a clear and straightforward quantum mechanical microscopic description of the sonic analog of Hawking radiation. Sonic Hawking radiation is formed due to quantum tunneling through the top of the potential barrier. Particle-hole excitations are created at the barrier and are then radiated with a momentum distribution characterized by a temperature in exact agreement with the Hawking-Unruh formula [1,2,4]. Although the evolution of the fluid's flow is unitary, when the momentum distribution is measured locally, for instance, through the measurement of the dynamic structure factor, it is not distinguishable from a thermal (and incoherent) distribution. The general case of 1D quantum fluids as well as the possible relevance for experiments is also discussed.

Part I. Microscopic description.—Consider a Fermi-degenerate gas of particles moving in a sufficiently narrow and smooth constriction such that their motion can be considered 1D. Their stationary single-particle wave functions can be written in the adiabatic approximation [9] as $\Psi(x, y, z) = \psi(x)\xi_{0;x}(y, z)$, where $\xi_{0;x}(y, z)$ is the transverse ground-state wave function with eigenenergy $\epsilon_0(x)$. $\psi(x)$ satisfies a 1D Schrödinger equation $[-(\hbar^2/2m)\partial_x^2 + V_{\text{ext}}(x)]\psi(x) = \epsilon\psi(x)$, where V_{ext} is an effective potential that includes also the zero point energy $\epsilon_0(x)$ [10]. I assume V_{ext} very smooth and essentially nonzero only near its maximum V_{max} located at $x = 0$.

Consider a flow of particles coming from the left and colliding against the potential V_{ext}. Their particle's wave functions are given by scattering wave functions asymptotically defined by $\psi_\epsilon \propto [e^{ikx} + r(\epsilon)e^{-ikx}]\Theta(-x) + t(\epsilon)e^{ikx}\Theta(x)$, where $\epsilon = \hbar^2 k^2/2m$ is the energy eigenvalue and $r(\epsilon)$ and $t(\epsilon)$ are the reflection and transmission amplitudes, respectively. The evolution of the particle flow is unitary and the many-body wave function of the flow of particles is represented in second quantization by

$$|f\rangle = \prod_{0 \le \epsilon < \epsilon_{\text{max}}} a_\epsilon^\dagger |-\rangle, \tag{1}$$

where $a_\epsilon^\dagger$ is the creation operator of ψ_ϵ's and $|-\rangle$ is the vaccum of particles. In this state all ψ_ϵ's are occupied up to an energy ϵ_{max}, which is assumed higher than V_{max}. The noninteracting case is an ideal example of a fluid turning supersonic (see below) where the microscopic wave function of the flow (1) is given from the beginning.

Since the barrier potential is smooth the particle propagation is semiclassical and in first approximation $|t(\epsilon)|^2 \approx \Theta(\epsilon - V_{max})$. Therefore the fluid maintains in a first approximation the sharp Fermi-degenerate character of the local momentum (or velocity) distribution, which represents the "vacuum" of the black-hole sonic analog. This allows us to describe the fluid dynamic with the standard hydrodynamic equations. The velocities of the particles in the fluid are uniformly distributed within an interval (v_L, v_R). Figure 1 shows an example of velocity distribution. The right Fermi velocity $v_R = \sqrt{v_{max}^2 - 2V_{ext}(x)/m}$ is the velocity of a particle moving from the left to the right with initial velocity higher than the classical threshold velocity to go over the barrier $v_{esc} = \sqrt{2V_{max}/m}$. The left Fermi velocity v_L is the velocity of a particle starting from the top of the barrier (located at $x = 0$); i.e., $v_L = \mathrm{sgn}(x)\sqrt{v_{esc}^2 - 2V_{ext}(x)/m}$. The $\mathrm{sgn}(x)$ function accounts for the two possible different signs of the velocity that the particle can have falling either to the right or left of the barrier.

The flow velocity $v = (v_L + v_R)/2$ is the velocity of a local reference frame from where the fluid appears in equilibrium. Introducing the Fermi velocity $v_F = (-v_L + v_R)/2$ as the Fermi velocity in the comoving frame, the density is given by $n = mv_F/\pi\hbar$. With these definitions the current $\Omega = nv = mvv_F/\pi\hbar$ is conserved [11]. The Bernoulli equation $v^2/2 + h + V_{ext}/m = \mathrm{const}$, where $h(n) = \pi^2\hbar^2 n^2/2m^2$ is the enthalpy (the Fermi energy divided by the particle mass), is satisfied with $\mathrm{const} = v_{esc}^2/2 + v_{max}^2/2$. The sound velocity $[c^2 = ndh(n)/dn]$ is consistently given by the Fermi velocity $c = v_F$. It is easy to verify that the above defined particle flow is a transonic flow, namely, subsonic on the left of the barrier and super-

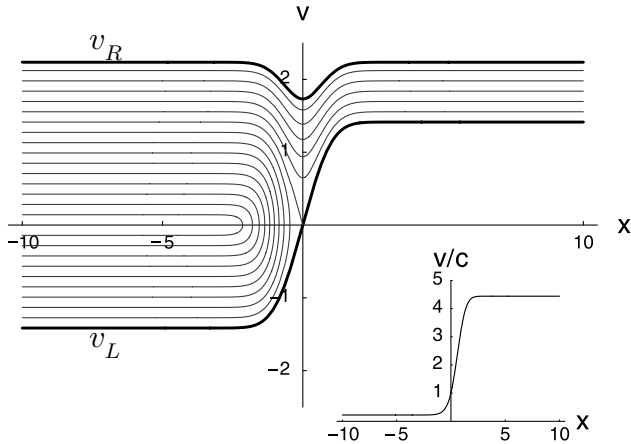FIG. 1.   Semiclassical velocity distribution as a function of the coordinate x. Velocities are uniformly distributed in the area limited by v_R (top solid line) and v_L (bottom solid line). Thinner solid lines correspond to different classical trajectories of the particles of the fluid. Here $\epsilon_{max} = 2.5$ and $V_{ext} = \exp(-x^2)$ in dimensionless units. The inset shows the Mach number v/c as a function of position x.

sonic on the right. The inset of Fig. 1 shows the behavior of the Mach number v/c in the 1D channel.

Even if the potential barrier is very smooth, its finite thickness near its top smears the semiclassical Fermi distributions at $v_L(x)$. This is due to those particles colliding against the barrier with energies comparable with V_{max}. The reflection probability is given by (see also [12])

$$|r(\epsilon)|^2 = \frac{1}{1 + \exp[2\pi(\epsilon - V_{max})/\hbar\omega_x]}, \quad (2)$$

where ω_x is the frequency of the inverted harmonic oscillator obtained expanding the barrier potential $V_{ext} = V_{max} - \frac{1}{2}m\omega_x^2 x^2 + o(x^3)$ around its maximum. Hawking's formula for a sonic black hole is given by [2,4]

$$T_H = \left(\frac{\hbar}{2\pi k_B}\right)\frac{d(v - c)}{dx}, \quad (3)$$

where the surface gravity κ in Hawking temperature $\kappa\hbar/2\pi k_B$ is replaced by $d(v - c)/dx$. For the transonic flow of fermions defined above $d(v - c)/dx = d(v_L)/dx = \omega_x$ and Hawking temperature (3) becomes

$$T_H = \frac{\hbar\omega_x}{2\pi k_B}. \quad (4)$$

Therefore the particles reflected from the barrier are distributed according to the reflection probability (2), which is identical to a thermal Fermi distribution

$$n_{T,\mu}(\epsilon) = \frac{1}{1 + \exp[(\epsilon - \mu)/k_B T]} \quad (5)$$

with a chemical potential $\mu = V_{max}$ and a temperature in exact agreement with Hawking temperature (4).

Consider a probe beam intersecting our 1D flow in a region quite far from the barrier. The measurement of the differential scattering cross sections with the probe beam allows obtaining the dynamic structure factor [13], which can be used to investigate Hawking radiation. The dynamic structure factor is defined as $S(q, \omega) = \Sigma_{l\neq f}\langle f|\rho_q|l\rangle \times \langle l|\rho_q^\dagger|f\rangle\delta(\omega - \omega_l)$, where $\omega_l = (E_l - E_f)/\hbar$ and E_l are the excitation frequencies and the energies of the eigenstates $|l\rangle$, respectively. Positive (negative) frequencies in $S(q, \omega)$ are related to absorption (release) of energy from the system. $\rho_q^\dagger = \sum_i e^{iq(x_i - R_p)}e^{-(x_i - R_p)^2/L_p^2}$ is a density operator defined locally where $L_p \gg 1/q$. Here L_p and R_p are the size and the position of the region where the probe beam and the system beam interact, respectively [14].

The dynamic structure factor of the flow is given in the subsonic region by

$$S(q, \omega) = S_{0,\mu_R}^{eq}(q, \omega)\Theta\left(\frac{\omega}{q}\right) + S_{T_H,\mu_L}^{eq}(q, \omega)\Theta\left(-\frac{\omega}{q}\right) \quad (6)$$

and in the supersonic region by

$$S(q, \omega) = [S_{0,\mu_R}^{eq}(q, \omega) + S_{T_H,\mu_L}^{eq}(q, -\omega)]\Theta\left(\frac{\omega}{q}\right), \quad (7)$$

where $\mu_L = V_{max}$ and $\mu_R = \epsilon_{max}$. $S_{T,\mu}^{eq}(q, \omega)$ is the dy-

namic structure factor of an equilibrium 1D Fermi gas at temperature T and chemical potential μ given by $S_{T,\mu}^{eq}(q,\omega) = \frac{L_{eff}m}{4\pi\hbar^2 q} e^{\hbar\omega/k_B T}/\{\cosh(\hbar\omega/k_B T) + \cosh[(\frac{m\omega^2}{2q^2} + \frac{\hbar^2 q^2}{8m} - \mu)/k_B T]\}$, where $L_{eff} = \sqrt{\pi/2}L_p$ is an effective length of the probe-system interaction region [14]. Its $T = 0$ limit is given by $S_{0,\mu}^{eq}(q,\omega) = \frac{L_{eff}m}{2\pi\hbar^2 q}\Theta(-\omega + \frac{\hbar^2 q^2}{2m} + \frac{\hbar^2 q k_F^o}{m})\Theta(\omega + \frac{\hbar^2 q^2}{2m} - \frac{\hbar^2 q k_F^o}{m})$ with $k_F^o = \sqrt{2m\mu}/\hbar$.

Because of the Pauli exclusion principle and the one dimensionality, which disconnects the Fermi "surface" into two points (represented by v_L and v_R), $S(q,\omega)$ has a typical peak structure, which contrasts the noninteracting two and three dimensional cases [13]. The peaks are related to the dispersion relation $\omega(q) = -v_L|q|\Theta(-q) + v_R|q|\Theta(q)$ valid for small q. In the supersonic region $\omega(q)$ is negative for $q < 0$ meaning that small perturbations propagate always to the right. Figure 2 shows the typical behavior of $S(q,\omega)$. The broad peaks in Fig. 2 correspond to the negative-q branch of the dispersion relation $\omega(q) = -v_L|q|$. In contrast the sharp peaks in the top of Fig. 2 correspond to the dispersion relation $\omega(q) = v_R|q|$, which is not broadened by the tunneling through the barrier. The characteristic wave vector of Hawking radiation is $q_c \sim k_B T_H/\hbar c \sim 1/2\pi^2 l_x^2 n$, where $l_x = \sqrt{m/\hbar\omega_x}$ and n is the density. Indeed at very low frequencies $\hbar\omega \ll k_B T_H$ and momentum $S(q,\omega)$ is dominated by Hawking radiation. The peak values at $|\omega| = |v_L q|$ tend to 1/4 of the corresponding zero-temperature value $L_{eff}m/2\pi\hbar^3 q$ but with a half-width $\Delta\omega = 1.76 q k_B T_H/m v_L$, which is much larger than the corresponding half-width $\Delta\omega = \hbar q^2/2m$ of the zero-temperature contribution.
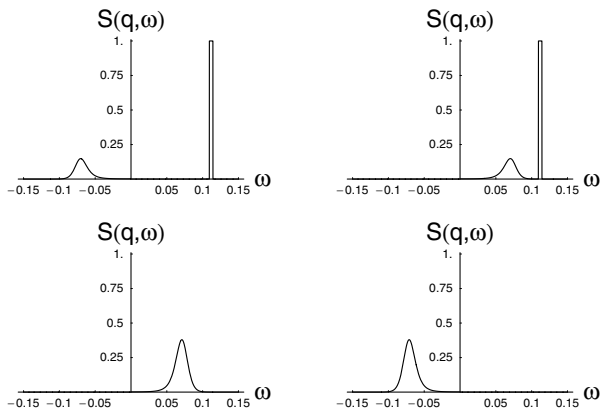

FIG. 2. Sonic Hawking radiation can be detected through the measurement of $S(q,\omega)$. Shown is the ω dependence of $S(q,\omega)$ in the subsonic (left) and supersonic (right) regions and for a positive (top) and a negative (bottom) value of q. Here $k_B T_H = 0.15 V_{max}$, $\epsilon_{max} = 2.5 V_{max}$, and $|q| = 0.05 m v_{esc}/\hbar$ is chosen in the region of small q where thermal-like transitions (broadened peaks) dominate over the zero-temperature contributions (step-like peaks). $S(q,\omega)$ is in units of $L_{eff}m/2\pi\hbar^3 q$ and ω is in units of $V_{max}/\hbar$.

The experimental observation of the widths of these peaks would provide a measure of Hawking temperature. Degenerate Fermi atoms have been only recently obtained [15]. Lithium can have a degeneracy temperature as high as 8 μK, which fixes the scale of ϵ_{max}. A reservoir of degenerate lithium could be connected to nontrapped states with a 1D channel realized by optical potentials or waveguides [16]. The barrier potential could be created by a far-blue-detuned laser sheet propagating in a direction orthogonal to that of the flow propagation and tightly focused with a waist ($1/e^2$) of twice the laser wavelength (760 nm). Hawking temperature would be of order of $T_H \sim 200$ nK and the characteristic wavelength of Hawking radiation λ_c of order of 40 μm.

The dynamic structure factor $S(q,\omega)$ in the subsonic region is equivalent to that of a 1D channel connecting two reservoirs at temperatures $T = 0$ on the left and $T = T_H$ on the right. This is the analog of Hawking radiation from a black hole into the surrounding zero-temperature space. To distinguish between this pure quantum state and a mixed thermal state would require the measurement of correlations such as those between opposite sides of the barrier. Correlated pairs of quasiparticles (particle hole), the "real" particles in an effective low-energy description, are created at the barrier and evolve in opposite directions.

The interpretation of the Hawking radiation as related to a tunnel effect is in agreement with a recent derivation of the Hawking radiation as a tunneling process of particles in a dynamical geometry due to Parikh and Wilczek [17]. A relationship with a generalized form of diffraction has been pointed out by Sanchez [18]. Moreover, electric pair production in a strong static electric field is related to a similar tunneling through an inverted harmonic oscillator [19]. As quantum tunneling is very sensitive to the details of the potential barrier, the distribution of radiated quasiparticles, related to the reflection coefficient of the barrier, can be different from (2). In particular, this suggests the possibility that some information on the supersonic part of the fluid (the analog of the classical forbidden region of a black hole) is carried by the radiation.

Part II. 1D quantum fluids.—Hawking's prediction is, in principle, valid also for strongly correlated 1D systems, such as the Tonks and Lieb-Liniger gases [20], which are currently a subject of intense interest [8] and electrons in clean short quantum wires [9,21]. Using standard hydrodynamics of 1D flow, it is possible to rewrite Hawking temperature (3) in a more general form [22] as $T_H = \eta\hbar\omega_x/2\pi k_B$, where η is defined by

$$\eta = \sqrt{\frac{3}{4} + \frac{1}{4}\left(n\frac{d^2 h}{dn^2}\Big/\frac{dh}{dn}\right)}, \qquad (8)$$

where $h(n)$ and n are the enthalpy and the density, respectively. In general η is only weakly dependent on the form of the function enthalpy $h(n)$ and Hawking temperature is basically proportional to a quantity that is related to the

propagation of a single particle of the fluid in the vicinity of the horizon. In the case of a noninteracting degenerate 1D Fermi gas considered here, η is equal to 1 since the enthalpy is proportional to the square of the density. In the case of a 1D Bose gas, η is equal to $\sqrt{3/4}$ in the mean-field regime [7] since the enthalpy is linear in the density and is equal to 1 in the Tonks-Girardeau limit [20] since the enthalpy is identical to that of a Fermi-degenerate gas in that limit. In addition to the importance of testing some aspects of black-hole evaporation, sonic Hawking radiation would be a remarkable transport phenomena in these strongly correlated quantum fluids and could be used to investigate their properties.

A quite different 1D system where one may consider to experimentally realize a sonic black hole and to test Hawking radiation would be a system of charged particles, like electrons in quantum wires, or ions in elongated traps. The long-range part of the Coulomb interaction could substantially renormalize the Hawking temperature in long wires in a similar way to how it does for the scattering from an impurity [21].

Conclusion.—A Fermi-degenerate flow of particles propagating in a 1D channel in the presence of a smooth effective barrier provides an exact solvable quantum mechanical model for the sonic analog of Hawking radiation. The agreement with Hawking's calculation is obtained in the limit of a very smooth barrier, which is the equivalent in Hawking's derivation to a slowly evolving event horizon. The thermal and incoherent characteristic of Hawking radiation is discussed in terms of a measurable quantity such as the dynamic structure factor.

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