Cutoff Anti–de Sitter Space/Conformal-Field-Theory Duality and the Quest for Braneworld Black Holes

Paul R. Anderson,[1,*] R. Balbinot,[2,†] and A. Fabbri[2,3,‡]

[1]Department of Physics, Wake Forest University, Winston-Salem, North Carolina 27109, USA

[2]Dipartimento di Fisica dell'Università di Bologna and INFN sezione di Bologna, Via Irnerio 46, 40126 Bologna, Italy

[3]Departamento de Física Teórica and IFIC, Facultad de Física, Universidad de Valencia, Burjassot-46100, Valencia, Spain

(Received 24 October 2004; published 16 February 2005)

Significant evidence is presented in favor of the holographic conjecture that "4D black holes localized on the brane found by solving the classical bulk equations in AdS_5 are quantum corrected black holes and not classical ones." The quantum correction to the Newtonian potential is computed using a numerical computation of $\langle T_{ab}\rangle$ in Schwarzschild spacetime for matter fields in the zero-temperature Boulware vacuum state. For the conformally invariant scalar field the leading order term is equivalent to that previously obtained in the weak-field approximation using Feynman diagrams and which has been shown to be equivalent, via the anti–de Sitter space/conformal-field-theory (AdS/CFT) duality, to the analogous calculation in Randall-Sundrum braneworlds. The 4D backreaction equations are used to make a prediction about the existence and the possible spacetime structure of macroscopic static braneworld black holes.

DOI: 10.1103/PhysRevLett.94.061301 PACS numbers: 04.70.Dy, 04.62.+v, 11.25.Tq, 11.25.Wx

The possibility of relating seemingly different theories via duality relations is a powerful tool which allows known results in one theory to be used to predict the outcome of difficult computations in the other. In recent years growing attention has been devoted to the so-called AdS/CFT correspondence [1], which predicts a one-to-one correspondence between a quantum gravity theory in anti–de Sitter (AdS) space and a conformal field theory (CFT) living in its boundary at infinity. A variant of this duality was proposed in [2,3] to allow for the possibility that AdS space is cut off at some finite distance L (the AdS length). This happens in the RS2 braneworld model [4] where our Universe is seen as a hypersurface, the boundary brane, which is immersed in AdS_5 space, the bulk. The presence of the brane has two primary consequences: (i) the dual CFT is cut off at the scale $1/L$ and (ii) the zero mode of 5D gravity gets trapped on the brane reproducing 4D gravity, which is then added to the dual CFT. The holographic interpretation of the Randall-Sundrum braneworlds states that the dual of the classical bulk theory is a CFT, more specifically, $\mathcal{N} = 4$ SU(N) super Yang-Mills theory in the large N planar limit, coupled to 4D gravity. In the study of quantum properties of matter-gravity systems, a widely used approach (semiclassical gravity) consists of treating gravity classically using general relativity and coupling it to quantum matter fields via the expectation value of the stress-energy tensor operator for the fields. It appears then very natural to compare 4D semiclassical results with 5D braneworld results and vice versa.

It was conjectured in [5] that for large mass black holes *4D black holes localized on the brane found by solving the classical bulk equations in* AdS_5 *are quantum corrected black holes and not classical ones*. If correct, this holographic conjecture opens a new perspective for the study of quantum effects in black hole spacetimes (for instance, the information loss problem) using 5D classical bulk physics.

The holographic conjecture explains the results of [6], where it was shown that it is not possible to match a collapsing sphere of dust on the brane with a static exterior. According to the conjecture, the deviations from staticity can be explained in terms of the Hawking radiation, which introduces time dependence into the system. It is important to mention that the conjecture is in excellent agreement with the exact solutions found for black holes localized in a $2 + 1$ brane in AdS_4 [7]. However, in AdS_5, where, despite much effort [8], black hole solutions have not been found, no actual proof of the holographic conjecture has been provided so far.

We give here a new and important check of this conjecture by using the semiclassical backreaction equations to compute the quantum corrected 4D Newtonian potential, and showing that to leading order it is equivalent to that obtained classically in the AdS_5 bulk in the weak-field limit. We then use the conjecture along with the well-known properties of the stress-energy tensor for quantized free fields in the zero-temperature Boulware state to make a prediction regarding the existence of static black hole solutions to the bulk equations.

In the weak-field limit it has been shown in [3] that the Randall-Sundrum result for the gravitational potential [9],

$$\Phi = \frac{M}{r}\left(1 + \frac{2}{3}\frac{L^2}{r^2}\right), \tag{1}$$

is equivalent to the 4D computation based on the one-loop quantum corrections to the graviton propagator [10] due to conformally invariant fields, which gives

$$\Phi = \frac{M}{r}\left(1 + \frac{\alpha}{r^2}\right). \tag{2}$$

The value of α depends on the specific numbers and types of fields considered:

$$\alpha = \frac{1}{45\pi}(12N_1 + 3N_{1/2} + N_0). \tag{3}$$

Here subscripts correspond to the spin of the field. The matching between the two expressions for the potential requires a specification of the number of degrees of freedom for each matter field species of the particular dual CFT theory; i.e., $N_1 = N^2$, $N_{1/2} = 4N^2$, $N_0 = 6N^2$, along with the relation $N^2 = \pi L^2$, which is derived from the AdS/CFT correspondence combined with the Randall-Sundrum formula involving 5D and 4D Newton's constants (see [3] for more details).

In this Letter, using semiclassical gravity, we derive the four dimensional gravitational potential by first computing the stress-energy tensor for a conformally invariant scalar field in the Boulware state [11] in Schwarzschild spacetime. The leading order behavior of this stress-energy tensor is obtained in the region far from the event horizon. Then the linearized semiclassical backreaction equations are integrated to obtain the quantum corrected Newtonian potential. The end result is then compared to Eq. (2) to check the conjecture.

The stress-energy tensor for the conformally coupled massless scalar field in the Boulware state has previously been numerically computed in [12] and in [13] analytic approximations have been computed. Moreover, in [14] an analytic approximation has been derived that can be used to obtain an approximation for the stress-energy tensor for arbitrarily coupled massless scalar fields in the Boulware state.

The analytic approximations predict that at large values of the radial coordinate r the nonzero components of the stress-energy tensor have leading order behaviors that are proportional to M^2/r^6, with M the mass of the black hole. It is clear that such a term cannot reproduce the correction in Eq. (2) (in fact, it generates a quantum correction to Φ of the order M^2/r^4). This presents a serious challenge for the holographic conjecture. One way to resolve this issue is to compute the stress-energy tensor numerically. As mentioned above, this has been done in Ref. [12]. However, it is not possible to deduce the large r behavior from the plots of the numerical results in that paper.

We have numerically computed the stress-energy tensor for massless scalar fields with arbitrary coupling to the scalar curvature in the Boulware state in Schwarzschild spacetime. The method used is the same as that given in Ref. [14], which in turn is an adaptation and generalization of the method originally used in [15–17] for the conformal scalar field in Schwarzschild spacetime. Renormalization is accomplished through the use of point splitting. In principle, one can subtract the point splitting counterterms computed in [18] from the unrenormalized stress-energy tensor and then take the limit as the points come together. In practice, it is easier to add and subtract terms using the WKB approximation for the radial modes. As shown in Ref. [14] it is possible to use the high frequency limit of the WKB approximation to write the stress-energy tensor in terms of two finite tensors $\langle T_{ab}\rangle_{\text{numerical}}$ and $\langle T_{ab}\rangle_{\text{analytic}}$ that are separately conserved. The result is

$$\begin{aligned}\langle T_{ab}\rangle_{\text{ren}} &= \langle T_{ab}\rangle_{\text{numerical}} + \langle T_{ab}\rangle_{\text{analytic}},\\ \langle T_{ab}\rangle_{\text{numerical}} &= \langle T_{ab}\rangle_{\text{unren}} - \langle T_{ab}\rangle_{\text{WKBdiv}},\\ \langle T_{ab}\rangle_{\text{analytic}} &= \langle T_{ab}\rangle_{\text{WKBdiv}} - \langle T_{ab}\rangle_{\text{ps}}.\end{aligned} \tag{4}$$

The second term can be computed analytically in any static spherically symmetric spacetime and for massless fields is the analytic approximation derived in [14] which is mentioned above.

To actually compute the stress-energy tensor numerically it is useful to add and subtract the full WKB approximation with the result that

$$\begin{aligned}\langle T_{ab}\rangle_{\text{numerical}} &= \langle T_{ab}\rangle_{\text{modes}} + \langle T_{ab}\rangle_{\text{WKBfin}},\\ \langle T_{ab}\rangle_{\text{modes}} &= \langle T_{ab}\rangle_{\text{unren}} - \langle T_{ab}\rangle_{\text{WKB}},\\ \langle T_{ab}\rangle_{\text{WKBfin}} &= \langle T_{ab}\rangle_{\text{WKB}} - \langle T_{ab}\rangle_{\text{WKBdiv}}.\end{aligned} \tag{5}$$

It turns out that $\langle T_{ab}\rangle_{\text{modes}}$ and $\langle T_{ab}\rangle_{\text{WKFfin}}$ are not separately conserved. However, for a zero-temperature massless scalar field it is possible to compute the latter analytically (except for a few integrals that must be computed numerically) for an arbitrary static spherically symmetric spacetime. Further, the mode sums converge much more rapidly than they do if one computed the quantity $\langle T_{ab}\rangle_{\text{numerical}}$ directly. The higher the order of the WKB approximation the faster the mode sums converge. In the calculations below an eighth order WKB expansion was used in $\langle T_{ab}\rangle_{\text{WKB}}$.

Because the scalar curvature is zero in Schwarzschild spacetime, it is possible to write the stress-energy tensor for arbitrary coupling in the general form [14]

$$\langle T_{ab}\rangle = C_{ab} + (\xi - \tfrac{1}{6})D_{ab}, \tag{6}$$

with ξ the coupling to the scalar curvature. The numerical results for the components of the tensors C_{ab} and D_{ab} are shown in Figs. 1 and 2. In the figures each component of the stress-energy tensor is multiplied by a factor of r^5/M. It is clear from the plots that the leading order behavior of the stress-energy tensor at large r is proportional to M/r^5 and not M^2/r^6 as predicted by the analytical approximations. We find that to leading order the stress-energy tensor is [19]

$$\begin{aligned}\langle T^t{}_t\rangle &= \frac{M}{60\pi^2 r^5} + \left(\xi - \frac{1}{6}\right)\frac{M}{4\pi^2 r^5},\\ \langle T^r{}_r\rangle &= \frac{M}{120\pi^2 r^5} - \left(\xi - \frac{1}{6}\right)\frac{M}{4\pi^2 r^5},\\ \langle T^\theta{}_\theta\rangle &= -\frac{M}{80\pi^2 r^5} + \left(\xi - \frac{1}{6}\right)\frac{3M}{8\pi^2 r^5}.\end{aligned} \tag{7}$$

This tensor is conserved to leading order in $1/r$ and for the

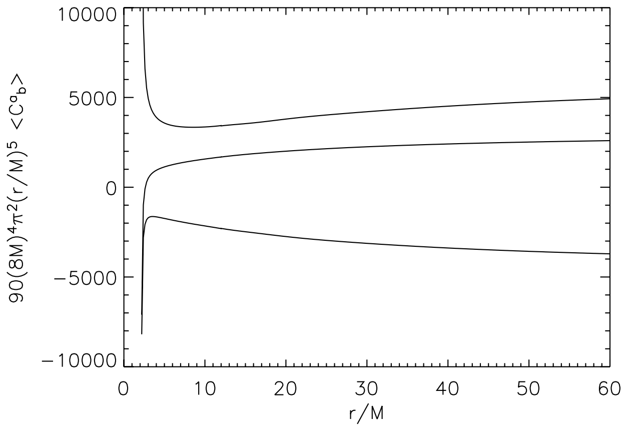

FIG. 1. The curves from top to bottom at the right of the plot correspond to $C^t{}_t$, $C^r{}_r$, and $C^\theta{}_\theta$, respectively.

conformal case, $\xi = 1/6$, is correctly traceless (the trace anomaly being of the order M^2/r^6).

The next step is to determine the quantum corrections to the Schwarzschild metric and to the Newtonian potential. These can be computed by solving the semiclassical Einstein equations (backreaction equations), which, by writing the metric as

$$ds^2 = -[1 - 2M(r)/r]e^{2\phi(r)}dt^2 + \frac{dr^2}{(1 - \frac{2M(r)}{r})} + r^2 d\Omega^2, \tag{8}$$

take the simple form

$$\partial_r M = -4\pi r^2 \langle T^t{}_t\rangle, \tag{9}$$

$$\partial_r \phi = -4\pi r \frac{\langle T^t{}_t\rangle - \langle T^r{}_r\rangle}{(1 - 2M/r)}. \tag{10}$$

At linear order, using the results (7) we find that

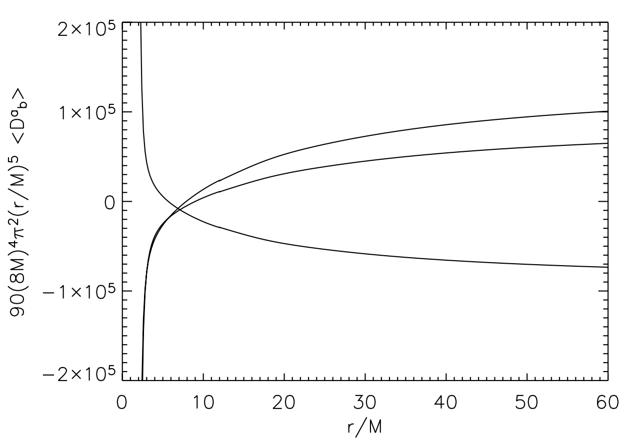

FIG. 2. The curves from top to bottom at the right of the plot correspond to $D^\theta{}_\theta$, $D^t{}_t$, and $D^r{}_r$ respectively.

$$ds^2 = -\left[1 - \frac{2M}{r}\left(1 + \frac{\alpha}{r^2}\right)\right]dt^2 + \left[1 - \frac{2M}{r}\left(1 + \frac{\beta}{r^2}\right)\right]^{-1} dr^2 + r^2 d\Omega^2, \tag{11}$$

where

$$\alpha = \frac{1}{45\pi} - \left(\xi - \frac{1}{6}\right)\frac{1}{6\pi}, \qquad \beta = \frac{1}{30\pi} + \left(\xi - \frac{1}{6}\right)\frac{1}{2\pi}. \tag{12}$$

The quantum correction to the Newtonian potential takes the form of Eq. (2) with α given in Eq. (12). For the conformal case $\xi = 1/6$ we exactly reproduce the effective potential calculation in Eq. (2). For the minimally coupled case, $\xi = 0$, Eq. (12) gives $\alpha = 1/20$ which reproduces the analogous computation with Feynman diagrams performed in [20]. To complete the proof that the classical bulk Newtonian potential is equivalent to the 4D quantum corrected potential, one should also show the matching of the results for massless spin $1/2$ and spin 1 fields. However, the agreement found for the conformal and minimally coupled scalar fields makes it reasonable to suppose that the agreement will extend to nonzero spin fields as well [21]. It would be interesting to improve the existing analytic approximations for the stress-energy tensor in the Boulware state in order to reproduce the results in Eq. (7).

The importance of our result is twofold. On one hand, it provides the first proof that calculations of the effective potential which make use of Feynman diagrams to compute corrections to the graviton propagator give the same answer as that obtained by computing the stress-energy tensor for the quantized fields and solving the linearized semiclassical backreaction equations. In addition, and this is the main focus of the present Letter, it provides a strong check of the holographic interpretation for braneworld black holes, which makes an identification between classical solutions of black holes localized on the brane and solutions to the semiclassical backreaction equations in 4D black hole spacetimes.

The holographic interpretation is quite important because, in principle, the semiclassical Einstein equations [(9) and (10)] allow one to determine not only the spacetime metric in the asymptotic region at large values of r, Eq. (11), but also the possible spacetime structure at intermediate values of r. It is well known [22] that the requirements that the stress tensor is static and vanishes asymptotically (which are equivalent to demanding a zero-temperature Boulware state for the matter fields) imply that $\langle T^a{}_b\rangle$ strongly diverges at the classical horizon of the Schwarzschild spacetime as

$$\langle T^a{}_b\rangle \sim \frac{(2N_1 + \frac{7}{4}N_{1/2} + N_0)}{30\,2^{12}\pi^2 M^4 f^2}(1, -1/3, -1/3, -1/3), \tag{13}$$

where $f = 1 - 2M/r$. A naive insertion of these quantities in the backreaction equations [(9) and (10)] gives divergent values for $M(r)$ and $\phi(r)$ in the limit $r \rightarrow 2M$. One consequence of this result is that in the linearized approximation we are considering the classical horizon gets destroyed by the quantum corrections. On the other hand, our confirmation of the holographic conjecture in the weak-field limit would seem to imply that some type of nontrivial, static vacuum solution to the classical bulk equations exists [23], although it is probably not a black hole. Both of these properties can hold only if quantum effects are large near $r = 2M$ for solutions to the semiclassical backreaction equations which have the asymptotic behavior (11). For macroscopic black holes one would not expect quantum effects to be large near the horizon. The usual interpretation of this result is that the Boulware state describes matter fields around a static star and not a black hole. In fact, the natural thing for a black hole is to evaporate via the Hawking effect. The one possible exception would be if the system has charges which allow the presence of zero-temperature solutions. Indeed, it has been shown that the stress-energy tensor of massless spin 0 and spin $1/2$ fields in the vacuum state is finite in 4D on the event horizon of the (zero-temperature) extreme Reissner-Nordström black hole [24,25].

We thank S. Fagnocchi, J. Navarro-Salas, and, in particular, R. Emparan and N. Kaloper for many useful discussions. This work has been supported in part by Grant No. PHY-0070981 from the National Science Foundation.

*Electronic address: anderson@wfu.edu
†Electronic address: balbinot@bo.infn.it
‡Electronic address: fabbria@bo.infn.it

[1] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. **323**, 183 (2000)
[2] J. Maldacena (unpublished); E. Witten (unpublished); H. Verlinde, Nucl. Phys. **B580**, 264 (2000); S. S. Gubser, Phys. Rev. D **63**, 084017 (2001); S. B. Giddings, E. Katz, and L. Randall, J. High Energy Phys. 03 (2000) 023; S. B. Giddings and E. Katz, J. Math. Phys. (N.Y.) **42**, 3082 (2001); N. Arkani-Hamed, M. Porrati, and L. Randall, J. High Energy Phys. 08 (2001) 017.
[3] M. J. Duff and J. T. Liu, Phys. Rev. Lett. **85**, 2052 (2000).
[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
[5] T. Tanaka, Prog. Theor. Phys. Suppl. **148**, 307 (2003); R. Emparan, A. Fabbri, and N. Kaloper, J. High Energy Phys. 08 (2002) 043; R. Emparan, J. Garcia-Bellido and N. Kaloper, J. High Energy Phys. 01 (2003) 079.
[6] M. Bruni, C. Germani, and R. Maartens, Phys. Rev. Lett. **87**, 231302 (2001).
[7] R. Emparan, G. T. Horowitz, and R. C. Myers, J. High Energy Phys. 01 (2000) 007; 01 (2000) 021.
[8] A. Chamblin, S. W. Hawking, and H. S. Reall, Phys. Rev. D **61**, 065007 (2000); N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezania, Phys. Lett. B **487**, 1 (2000); N. Deruelle, gr-qc/0111065; P. Kanti and K. Tamvakis, Phys. Rev. D **65**, 084010 (2002); R. Casadio, A. Fabbri, and L. Mazzacurati, Phys. Rev. D **65**, 084040 (2002); C. Charmousis and R. Gregory, Classical Quantum Gravity **21**, 527 (2004); H. Kudoh, T. Tanaka, and T. Nakamura, Phys. Rev. D **68**, 024035 (2003); H. Kudoh, Phys. Rev. D **69**, 104019 (2004); G. Kofinas, E. Papantonopoulos, and V. Zamarias, Phys. Rev. D **66**, 104028 (2002).
[9] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).
[10] M. J. Duff, Phys. Rev. D **9**, 1837 (1974); J. F. Donoghue, Phys. Rev. Lett. **72**, 2996 (1994).
[11] D. G. Boulware, Phys. Rev. D **11**, 1404 (1975).
[12] B. P. Jensen, J. G. McLaughlin, and A. C. Ottewill, Phys. Rev. D **45**, 3002 (1992).
[13] M. R. Brown and A. C. Ottewill, Phys. Rev. D **31**, 2514 (1985); V. P. Frolov and A. I. Zelnikov, Phys. Rev. D **35**, 3031 (1987).
[14] P. R. Anderson, W. A. Hiscock, and D. A. Samuel, Phys. Rev. D **51**, 4337 (1995).
[15] P. Candelas and K. W. Howard, Phys. Rev. D **29**, 1618 (1984).
[16] K. W. Howard and P. Candelas, Phys. Rev. Lett. **53**, 403 (1984).
[17] K. W. Howard, Phys. Rev. D **30**, 2532 (1984).
[18] S. M. Christensen, Phys. Rev. D **14**, 2490 (1976).
[19] The analytic values displayed in Eq. (7) have actually been obtained through a combination of computing analytically certain sums [14] that contribute to the quantity $\langle T_{ab} \rangle_{\rm WKBfin}$ and using conservation to obtain the rest. The details of this procedure will be described elsewhere. When a fitting routine from the algebraic manipulation program MATHEMATICA is used for the same sums it is found that there is agreement between the analytic and numerical values for the coefficients of the M/r^5 terms in $\langle T_{ab} \rangle$ to within approximately two significant digits.
[20] H. W. Hamber and S. Liu, Phys. Lett. B **357**, 51 (1995).
[21] In fact, it has been shown that the large r behavior of the stress energy for the massless spin $1/2$ field in the extreme Reissner-Nordström spacetime goes like M/r^5: E. D. Carlson and W. H. Hirsch (private communication).
[22] S. M. Christensen and S. A. Fulling, Phys. Rev. D **15**, 2088 (1977).
[23] R. Emparan (private communication).
[24] P. R. Anderson, W. A. Hiscock, and D. J. Loranz, Phys. Rev. Lett. **74**, 4365 (1995)
[25] E. D. Carlson, W. H. Hirsch, B. Obermayer, P. R. Anderson, and P. B. Groves, Phys. Rev. Lett. **91**, 051301 (2003).