Fast Causal Information Transmission in a Medium With a Slow Group Velocity

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It is widely believed that the velocity of information v_i encoded on an optical pulse is equal to the group velocity v_g, at least when v_g is less than the speed of light in vacuum c. On the other hand, several authors suggest that $v_i = c$, although the size of the signal traveling at this velocity may be small, thereby making it difficult to measure. Here, we measure v_i for pulses propagating through a resonant "slow-light" medium where $v_g \approx 0.006c$. We find $v_i = 1.03c^{+0.49c}_{-0.25c}$, or that $v_i \approx 168 v_g$, clearly demonstrating that the speed of information cannot be generally described by v_g, but is characterized by its own velocity.

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Currently, there is great interest in tailoring the dispersive properties of optical materials with the goal of controlling the speed of pulses of light. Several techniques for dispersion tailoring use optical fields to induce matter-field resonances, which can be designed to exhibit either large normal or anomalous dispersion near the resonances, leading to "slow" or "fast" light pulse propagation, respectively [1]. Potential applications of such media include classical and quantum information processing.

For the case when the real part of the refractive index $n(\omega)$ varies slowly over the spectral width of the pulse, it is customary to expand the field wave vector in a Taylor series centered on the pulse carrier frequency ω_0. Such an analysis leads to the concept of the group velocity $v_g = c/(n + \omega dn/d\omega)|_{\omega=\omega_0} = c/n_g$, where n_g is the frequency-dependent group index [2]. It is often assumed that information encoded on the pulse propagates at v_g [3–6].

This conventional wisdom is largely based on the work of Sommerfeld and Brillouin [7]. They used asymptotic analysis to study a step-modulated pulse propagating through a resonant absorber. Sommerfeld found that the front of the pulse (the moment when the field first becomes nonzero) propagates precisely at c, consistent with the special theory of relativity [8].

Brillouin extended Sommerfeld's analysis and found that the pulse breaks up after the front, consisting of two small wave packets (now called precursors), and a large-amplitude wave packet whose leading edge travels at the "signal" velocity v_s. The precursors arise from spectral components of the pulse above and below the resonance and are predicted to have a maximum strength of $<10^{-4}$ of the eventual signal intensity. Brillouin believed that the arrival of the signal should be associated with the arrival of the large-amplitude wave packet because of the smallness of the precursors, although Sommerfeld stressed that an extremely sensitive detector should be able to register the front of the pulse and hence measure a propagation speed of c for the signal.

Brillouin also made a surprising prediction: He found that $v_s = v_g$ when $v_g < c$. One might think that the group velocity, defined for a narrow bandwidth pulse, would not be a useful concept for a pulse containing a step discontinuity in the field amplitude. Despite this apparent contradiction, his prediction seems to hold for many cases of interest, leading to the belief that it is generally applicable [2–6].

Oughstun and Sherman [9] extended Brillouin's work and find that the precursors can be much larger. For the case when ω_0 is set to an atomic resonance, they predict that the precursors are the dominant part of the transmitted pulse, peaking immediately after the pulse front and hence one would expect that a straightforward measurement will reveal that $v_i = c$.

Recently, Chiao and collaborators [10] proposed that information is contained only in points of nonanalyticity on electromagnetic waveforms, which travel at c, and therefore that $v_i = c$ [10,11]. An example of a nonanalytic point is a discontinuity in the waveform or one of its derivatives. Recently, Parker and Walker [12] suggest that the very act of encoding information on a waveform necessarily creates points of nonanalyticity. For these proposals to be practical, the precursors generated by the point of nonanalyticity have to be large enough to be detected.

The primary purpose of this Letter is to measure v_i for pulses propagating in a resonant slow-light medium for which $v_g \ll c$. We encode one bit of information on a pulse by making a rapid unpredictable change in the pulse amplitude. We estimate the velocity of this bit by measuring the moment when it is first possible to reliably detect the bit. By comparing the propagation of pulses through the slow-light medium and vacuum, we find that $v_i \simeq c$, demonstrating that resonant pulse propagation can give rise to results that are contrary to the conventional wisdom.

We prepare the slow-light medium using an amplifying resonance, realized by inducing large atomic coherence in a laser-driven potassium vapor as shown in Fig. 1(a). The potassium vapor is contained in an uncoated Pyrex cell of length $L = 19.2$ cm and heated to obtain an atomic number density of 6.5×10^{11} atoms/cm^3. A linearly polarized coherence-preparation laser beam (frequency ω_d set to 1.91 GHz to the high-frequency side of the center of the potassium $4S_{1/2} \leftrightarrow 4P_{1/2}$ transition) is combined with the linear and orthogonally polarized information-carrying pulses. The coherence-preparation laser beam induces a Lorentzian-shaped amplifying resonance at frequency $\omega_R \simeq \omega_d + \Delta_{\rm hf}$ due to a stimulated hyperfine Raman scattering process [13], where $\Delta_{\rm hf} = 2\pi(462 \text{ MHz})$ is the hyperfine splitting of the potassium $4S_{1/2}$ state. For future reference, we denote the linewidth of the resonance (half-width at half maximum) by γ_R.

Before attempting to measure v_i, we undertake a preliminary measurement using a Gaussian-like pulse with carrier frequency $\omega_0 \simeq \omega_R$, as shown in Fig. 1(b) (filled circles). The pulse is turned on suddenly at a time $t = -1000$ ns (not shown) to a small amplitude that is well below the noise floor of our detection system. The pulse intensity then follows a Gaussian functional form ($1/e$ intensity half-width denoted by τ), attaining its peak value at 0 ns, and is switched off suddenly at $t = 1000$ ns (also not shown). For a long pulse ($\gamma_R \tau \gg 1$), we find that the pulse distortion is reduced and the pulse delay time $t_{\rm del}$ is maximized [14–17]. Unfortunately, such a long pulse also gives rise to a small value of the relative time delay $t_{\rm del}/\tau$, a quantity we need to maximize in order to distinguish between the various velocities describing the propagating pulse. Therefore, we use a pulse width of the order of $\tau \sim 1/\gamma_R$, which we find maximizes $t_{\rm del}/\tau$ but introduces a small amount of distortion [17].

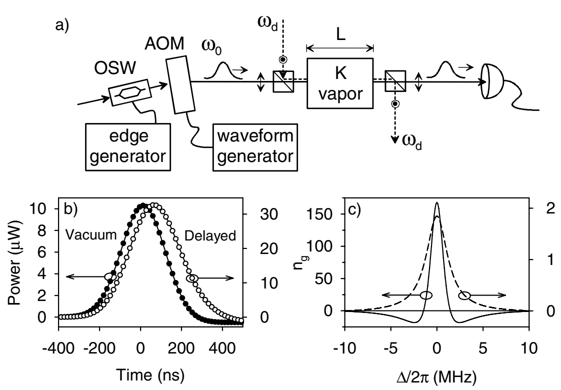

FIG. 1. Slow-light pulse propagation. (a) Experimental setup. Pulses are generated by passing a continuous-wave laser beam through an acousto-optic modulator (AOM) driven by an arbitrary waveform generator and are detected by a bandpass-coupled photoreceiver with a 3-dB response extending from 30.1 to 125 MHz. (b) Temporal evolution of Gaussian-like pulses propagating through vacuum (filled circles) and the slow-light medium (open circles). Solid lines are theoretical fits to the data. (c) Gain path length $g(\omega)L = -\omega L \text{Im}[\chi(\omega)]/c$ (dashed line) and group index (solid line) of the slow-light medium as a function of probe laser frequency.

Figure 1(b) shows the Gaussian-like pulse propagating through vacuum (filled circles) and through the slow-light medium (open circles). It is seen that the pulse is delayed by the slow-light effect and undergoes some broadening, as expected [14–17]. We do not observe any precursors related to the initial sudden turn on of the pulse at $t = -1000$ ns because their expected intensity is well below the noise floor of our detection system. A direct measurement of the waveforms reveals that the peak of the pulse propagating though the medium is delayed by 50.9 ns in comparison to the vacuum pulse. We find that the bandpass-coupled characteristic of the detector slightly shifts the peak of the pulses by an amount that is proportional to the pulse width. Correcting for this shift, we find that the peak of the pulse is delayed by $t_{\rm del}^{\rm obs} = 53.0$ ns. This experiment demonstrates that it is possible to delay a smooth pulse whose shape is chosen to suppress optical precursors; implications for information transfer and delay will be discussed below.

To obtain a quantitative understanding of our observations, we fit the waveforms to a model of the pulses and the slow-light medium. The vacuum waveform is modeled as a truncated Gaussian function passed through a bandpass filter to take into account the experimentally measured detector response. Figure 1(b) shows that the predicted vacuum waveform is in excellent agreement with the observed waveform using $\tau = 152.1$ ns. The slow-light waveform is fit by propagating the Gaussian pulse found above through a medium characterized by a linear susceptibility $\chi(\omega)$ and filtered by the detector response [18]. We find excellent agreement between the observed and predicted slow-light waveforms with a line-center intensity amplification coefficient $g_0 = 0.098$ cm^{-1} and $\gamma_R = 2\pi(1.41 \text{ MHz})$.

The narrow Raman resonance gives rise to laser beam amplification and large dispersion, as shown in Fig. 1(c). At the peak of the resonance, $n_g = 168$, resulting in $v_g/c = 5.95 \times 10^{-3}$. Such a slow group velocity will give a delay time of $t_{\rm del}^g = L/v_g - L/c = 107.6$ ns for a very long pulse, which is considerably longer than $t_{\rm del}^{\rm obs}$ because the bandwidth of the Gaussian-like pulses used in our experiment is comparable to the bandwidth of the slow-light resonances, as discussed above and in Ref. [17]. Note that the wings of the slow-light feature are surrounded by broad regions of fast light, taking on its minimum value of $n_g \simeq -18$ at $\Delta/2\pi \simeq \pm 2.4$ MHz.

We now describe our measurement of the information velocity. In the experiment, we encode one bit of information on the Gaussian-shaped waveform using a fast optical telecommunication switch (OSW) driven by a step generator (~3 ns rise time). A positive-going step is used to define one communication symbol ("1"), while a negative-going step is used to define another ("0"). The moment when the waveform goes high or low corresponds to a point of nonanalyticity [10–12]. We select these symbol waveforms because they allow us to smoothly turn on the pulse amplitude to a level above the noise floor of our detection electronics and to monitor the pulse delay.

Figure 2(a) shows the propagation of both symbols through the slow-light medium and through vacuum. Figure 2(b) shows an enlargement of the diagram in the vicinity of the transition between the symbols. From Fig. 2(a), it is seen that the slow-light medium delays the early part of the pulses during the smooth turn-on, identically to that observed for the full Gaussian-like pulses shown in Fig. 1(b). No information can yet be conveyed to a receiving party at the end of the communication channel because both symbols are the same for early times and hence they cannot be distinguished. From a simple visual inspection of the data, we see that the time when it is possible to first distinguish between the two symbols for the delayed pulses is nearly the same as the time when it is possible to first distinguish between the same two symbols propagating through vacuum. Hence, information arrives much faster than one might infer from v_g, one primary conclusion of our experiment.

The arrival time of the information is determined by observing the pulses with a receiver that attempts to identify the incoming symbols as a function of time, with a certainty characterized by the bit error rate (BER). Before the arrival of this point of nonanalyticity at the detector, we expect no detected information, corresponding to BER $= 1/2$. Once the point of nonanalyticity (and the information it carries) propagates past the detector, the BER drops as the received information grows smoothly from zero. A symbol is considered detected when the BER falls below some threshold. Hence, the detection time of information is later than the time when information is first available at the detector by an amount we call the detection latency Δt, even for pulses propagating through vacuum. Although it is possible to estimate Δt for a specific experimental apparatus, it cannot be measured directly because it requires measuring the point of nonanalyticity. In Fig. 2(b), we see that the manner in which the average symbol waveforms separate for the vacuum and delayed case are only slightly different so that the detection latency times (denoted by Δt_{vac} and Δt_{del}, respectively) should be similar. Because our analysis makes no assumption about the sources of noise in the communication system, our general experimental approach and conclusions should hold even in the limit where quantum fluctuations are dominant [20].

To quantify our results, we determine the BER for the vacuum (Fig. 3, filled circles) and delayed (open circles) pulse pairs using an integrate-and-dump matched filter technique [8]. It is seen that the BER is high for final observation times in the range between 0 and 4 ns during which the pulse amplitudes are large but not yet distinct in the presence of noise [see Fig. 2(b)]. For longer observation times, the BER drops rapidly. In the BER range between 0.1 and 0.01, chosen to keep Δt_{vac} and Δt_{del} small,

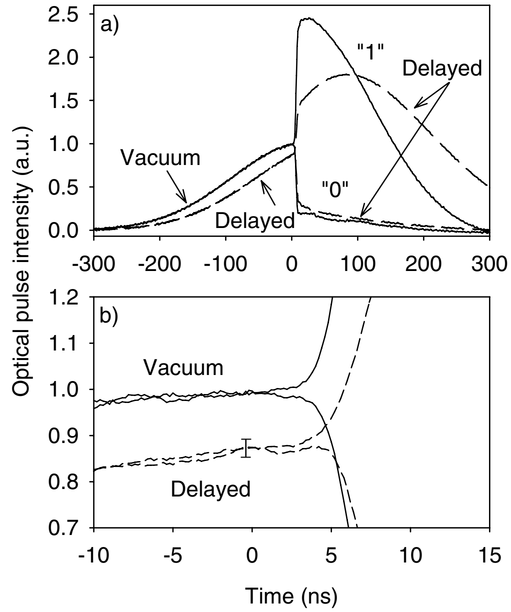

FIG. 2. Transmitting information-encoded optical pulses through a slow-light medium. (a) Transmitting 0 and 1 through vacuum (solid line) and the slow-light medium (dashed line). Each symbol is transmitted separately through the medium and vacuum, and each curve is an average of 50 pulses. (b) High-resolution plot of (a). The amplitude of the delayed and vacuum pulses have been scaled so that their heights would be the same if a Gaussian pulse propagated through the system, as in Fig. 1(b). The error bar indicates the typical standard deviation of the pulse amplitudes.

we determine the detection time for vacuum (delayed) pulse pairs Δt_{vac} (Δt_{del}) and the difference in detection times $T_i = T_{\mathrm{del}} - T_{\mathrm{vac}}$. The time difference is approximately constant for BER values over this range; its average is equal to 1.02 ± 0.1 ns. Based only on a direct measurement of T_i, one cannot determine whether an observed difference between the detection times is due to changes in the detection latencies or differences in the information velocities for vacuum ($v_{i,\mathrm{vac}}$) and the slow-light medium ($v_{i,\mathrm{del}}$) as can be seen by considering the relation among these quantities, which is given by

$$T_i = (L/v_{i,\mathrm{del}} - L/v_{i,\mathrm{vac}}) + (\Delta t_{\mathrm{del}} - \Delta t_{\mathrm{vac}}). \qquad (1)$$

To gain some insight about the importance of detection latency, we analyze a mathematical model, based on Maxwell's equations, that describes approximately our experiment. We find that the detection latency depends on the combined effects of the finite detector bandwidth and the symbol transition times, and noise in the measurement. Removing either nonideal effect would eliminate the detection latency, which, of course, is impossible in a real experiment. Consistent with previous research [10], this model predicts that $v_{i,\mathrm{del}} = v_{i,\mathrm{vac}} = c$ in the absence of noise, and hence T_i is completely deter-

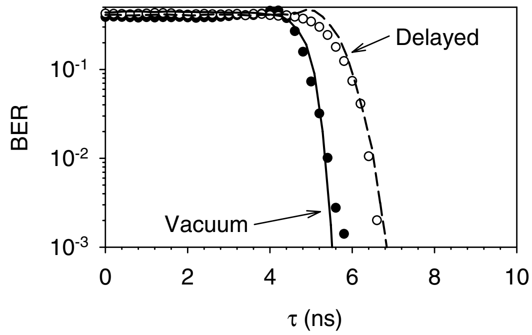

FIG. 3. Detecting the arrival of new information. Shown is the experimentally measured BER as a function of the upper limit of the integration time for the vacuum (filled circles) and delayed (open circles) pulses. Theoretical predictions based on a model of the experimental apparatus and slow-light medium are shown as lines.

mined by $(\Delta t_{\mathrm{del}} - \Delta t_{\mathrm{vac}})$. Using the same matched filtering approach, we determine the predicted BER as shown in Fig. 3. We find that $T_i = 1.04 \pm 0.1$ ns, where the error arises from uncertainty in the similarity between the model and physical experiment. The fact that $T_i \neq 0$ demonstrates that subtle changes in the shape of the symbols after information has been encoded give rise to substantial changes in the detection latency.

We arrive at our best estimate of the information velocity using the model prediction for $(\Delta t_{\mathrm{del}} - \Delta t_{\mathrm{vac}})$ in Eq. (1) and taking $v_{i,\mathrm{vac}} = c$. We find that $v_{i,\mathrm{del}} = 1.03c^{+0.49c}_{-0.25c}$, which is 168 times faster than v_g. This measurement is the primary result of this Letter and clearly demonstrates that the speed of propagating information encoded on optical waveforms is distinct from the speed of the peak of the pulse.

One possible explanation as to why we are able to measure a fast information velocity is that the large discontinuities on the symbol pulses result in large precursors, which are enhanced when the pulse frequency is near a resonance and its intensity varies rapidly compared to the response time of the slow-light medium, given approximately by $1/\gamma_R$. On the other hand, it is difficult to determine v_i accurately using the Gaussian-like pulses shown in Fig. 1(b) because the precursors are suppressed by initially switching on the pulses to a very small intensity. Thus, for the Gaussian-like pulses, there is a very large detection latency Δt between the time when information is first available at the detector and when it is detected. From this point of view, the slow-light effect does not change the speed of information; it merely adjusts the detection latency in a pulse-dependent and detection-dependent manner. Our observation may have important implications for other slow-light media because they often rely on resonant or near-resonant pulse propagation.

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