Teleportation of an Atomic Ensemble Quantum State

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We propose a protocol to achieve high fidelity quantum state teleportation of a macroscopic atomic ensemble using a pair of quantum-correlated atomic ensembles. We show how to prepare this pair of ensembles using quasiperfect quantum state transfer processes between light and atoms. Our protocol relies on optical joint measurements of the atomic ensemble states and magnetic feedback reconstruction.

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The realization of quantum networks involving optical fields and atomic ensembles is one of the most promising paths towards robust long distance quantum communication and information processing [1,2]. The efficient transfer of quantum states within that network is a key ingredient for a practical implementation [2]. Several continuous variable teleportation experiments with optical fields [3] have shown that continuously teleporting optical quantum states with a high efficiency was possible. On the other hand, the teleportation of a single atom or ion quantum state was demonstrated very recently [4]. In this Letter we present a direct scheme to teleport an atomic spin state in a way very similar to that used in the teleportation protocols for optical field states [5], which can hence be efficiently integrated within a light-atom quantum network, for instance.

Because of the long lifetime of their ground-state spins, atomic ensembles are good candidates to store and manipulate quantum states of light [6]. We show how to prepare the spin states via quantum state transfers with optical fields [7–9] and propose to achieve the teleportation of an atomic ensemble quantum state using an Einstein-Podolsky-Rosen- (EPR-)correlated pair of atomic ensembles. An optical joint measurement of the unknown ensemble (1) and one of the entangled ensemble (2) is then performed by Alice who sends the results to Bob. Using a suitable magnetic field Bob can reconstruct the input state on the other correlated ensemble (3). The quasi-ideal character of the atom-field quantum transfer processes allows high fidelity teleportation for easily accessible experimental parameters.

Another atomic teleportation protocol, relying on successive measurements alternating with optical displacements performed on two ensembles, was proposed by Kuzmich and Polzik [10]. However, this protocol requires several exchanges of information between Alice and Bob. Our scheme, being a direct adaptation of the teleportation protocols for light, needs two simultaneous measurements to achieve real-time quantum teleportation, and can easily be extended to other quantum communication and information protocols, such as entanglement swapping and quantum repeaters. This Letter successively describes the three steps of atomic teleportation: preparation, joint measurement, and reconstruction.

In this Letter we mainly consider cold atom ensembles, but our protocol also extends to atomic vapors. We consider three N-atom ensembles with an energy level structure in Λ [Fig. 1(a)]. We assume that they are placed inside optical cavities, for which the input-output theoretical treatment of the atom-field quantum fluctuations is well-adapted. They each interact with a coherent control field Ω_i and with a vacuum field A_i ($i = 1 - 3$). In reality, the Λ structure should be part of a larger $J \rightarrow J$ or $J \rightarrow J - 1$ transition, the two ground states being two Zeeman sublevels with $m_J = J, J - 1$, but after the preparation phase, the other levels play no role and can be safely ignored. To simplify the discussion we will therefore focus on the case of a $J = 1/2$ ground-state spin.

Preparation.—During the preparation stage Victor pumps the ensembles with the control fields so that their ground-state collective spins are aligned along the z axis: $\langle J_{zi} \rangle = N/2$. Ensemble 1 is assumed to be almost completely spin-polarized along z, with a small tilt corresponding to a nonzero coherence: $\langle J_{z1} \rangle \simeq N/2$ and $\langle J_{x1} \rangle$, $\langle J_{y1} \rangle \sim \mathcal{O}(\sqrt{N})$. This means that we consider small planar displacements of the spin in the vicinity of the north pole of the Bloch sphere. This approximation is all the more correct as the number of atoms is large. The quantum state of an ensemble is then determined by the ground-state coherence, the spin components J_x and J_y obeying a commutation relation $\langle [J_x, J_y] \rangle = i \langle J_z \rangle = iN/2$, similar to that of an harmonic oscillator. In the Gaussian approximation the atomic quantum state can then be represented by a noise ellipsoid in the (x, y) plane orthogonal to the mean spin [see Fig. 1(a)], in a manner very similar to the Fresnel representation of quantum optical field states. The atomic state is then completely characterized by the amplitude and phase of the coherence mean value, as well as its variances ΔJ_x^2 and ΔJ_y^2, which are equal to $N/4$ for a coherent spin state, for instance.

Let us suppose that the atomic state to be teleported is that of ensemble 1, prepared by Victor, unknown to Alice and Bob. With a suitable interaction Victor can prepare any Gaussian state (coherent state, squeezed state...) by an

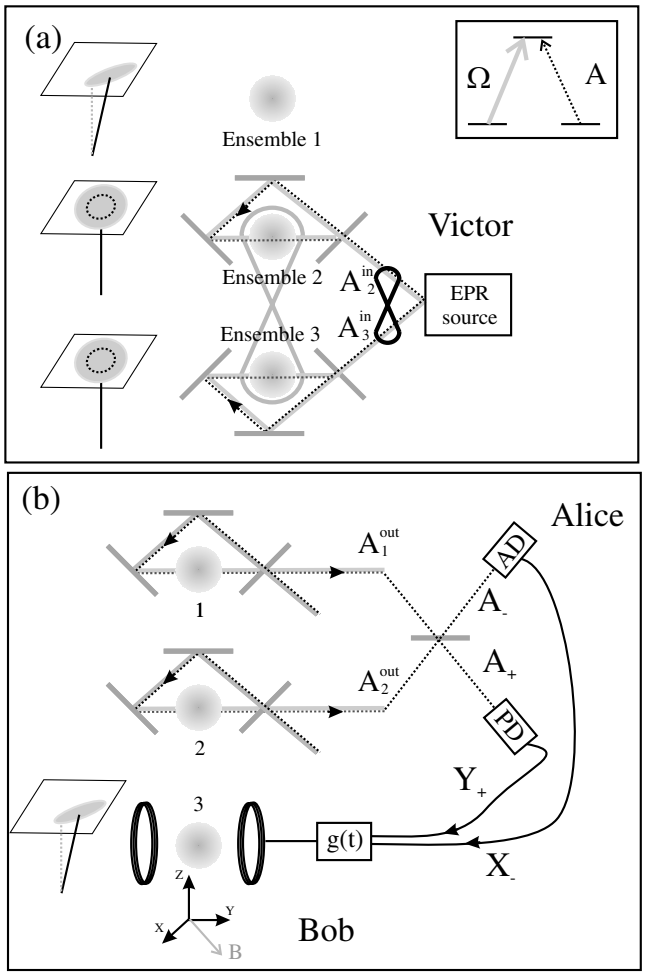

FIG. 1. Teleportation scheme: (a) *Preparation*. Insert: Λ-type level structure considered. Left: schematic atomic initial states for each ensemble (for spins 2 and 3, the dashed circle indicates the coherent spin state fluctuation distribution). (b) *Measurement and reconstruction*. Left: teleported state after reconstruction is completed. AD: amplitude detector. PD: phase detector.

adequate choice of the field state A_1^{in}, the state of which can be perfectly mapped onto the atoms. More explicitly, the coherence mean value depends on the field amplitude and phase, whereas the noise ellipsoid is given by the field quantum fluctuations. This transfer can be achieved using an electromagnetically induced transparency (EIT) (one- and two-photon resonant) or a "Raman" (large one-photon detuning, but two-photon resonant) interaction between the fields and the atoms [7,8]. In these two configurations there is little to no dissipation and the quantum fluctuations are predicted to be conserved in atom-field quantum state transfer processes. Using EPR-correlated fields Victor can also map their entanglement onto spins 2 and 3 with the same techniques [9]. Since the mean spins are parallel and equal, $J_{x2} - J_{x3}$ and $J_{y2} + J_{y3}$ are the equivalent of the usual EPR operators, satisfying $\langle [J_{x2} - J_{x3}, J_{y2} + J_{y3}] \rangle = i\langle J_{z2} - J_{z3} \rangle = 0$. We can assume without loss of generality that the fluctuations of J_{x2} and J_{x3} are correlated and those of J_{y2} and J_{y3} anticorrelated, so that the condition for their inseparability reads [11]

$$\Delta(J_{x2} - J_{x3})^2 + \Delta(J_{y2} + J_{y3})^2 < |\langle J_{z2} \rangle| + |\langle J_{z3} \rangle| = N.$$

In a symmetrical configuration the amount of atomic entanglement is given by the sum of the EPR variances (normalized to 2) [12]

$$\mathcal{I}_{2,3} = \frac{2}{N}[\Delta(J_{x2} - J_{x3})^2 + \Delta(J_{y2} + J_{y3})^2]. \qquad (1)$$

When the preparation stage is over all fields are switched off, and one disposes of an unknown atomic quantum state 1 and an EPR-correlated pair 2 and 3.

Joint measurements.—Alice then performs a simultaneous readout of ensembles 1 and 2 by rapidly switching on the control fields in cavities 1 and 2. The reverse transfer process of the preparation then takes place, and the states of spins 1 and 2 imprint in a transient manner onto the outgoing fields exiting the cavities A_1^{out} and A_2^{out}. These two fields are then mixed on a 50/50 beam splitter and Alice performs two homodyne detections of the resulting modes $A_\pm = (A_1^{\text{out}} \pm A_2^{\text{out}})/\sqrt{2}$ [Fig. 1(b)]. To obtain maximal information about the initial state, Alice measures the noise of two orthogonal quadratures—say $X_- = A_- + A_-^\dagger$ and $Y_+ = i(A_+^\dagger - A_+)$—and sends the results to Bob who disposes of ensemble 3. As we will show further, Bob can then reconstruct state 1 using a suitable magnetic field and achieve teleportation.

In more details, we assume an "EIT"-type interaction (one- and two-photon resonance), although a Raman interaction either with cold atoms or even with an atomic vapor would yield similar results. Alice rapidly switches the control field on in ensembles 1 and 2 at time $t = 0$. The outgoing modes can be expressed as a function of the initial atomic operators in ensembles 1 and 2 [8]

$$\begin{aligned} X_i^{\text{out}}(t) = {} & X_i^{\text{in}}(t) - \alpha J_{xi}(0) e^{-\tilde{\gamma}_0 t} \\ & - 2\eta^2 [X_i^{\text{in}}(t) - \tilde{\gamma}_0 \int_0^t e^{-\tilde{\gamma}_0 (t-s)} X_i^{\text{in}}(s) ds] \\ & + \beta [X_{vi}(t) - \tilde{\gamma}_0 \int_0^t e^{-\tilde{\gamma}_0 (t-s)} X_{vi}(s) ds], \end{aligned} \qquad (2)$$

$(i = 1, 2)$, with $\eta^2 = 2C/(1 + 2C)$, $\alpha = \eta\sqrt{8\tilde{\gamma}_0/N}$, $\beta = 2\eta/\sqrt{1 + 2C}$, C being the cooperativity parameter quantifying the collective strength of the atom-field coupling [7]. $\tilde{\gamma}_0$ represents the effective atomic decay rate in presence of the control field. This parameter depends on the cooperativity and the pumping rate due to the control field [8], and it is related to the duration of the transient optical pulse carrying the atomic state out of the cavity. η is actually related to the efficiency of the transfer [7], and is close to unity for large values of the cooperativity. X_{vi} is a noise atomic operator accounting for noise induced by spontaneous emission and with unity white noise spectrum. X_i^{in} is

the amplitude quadrature of the vacuum field incident on cavity i. Similar expressions hold for the phase quadratures Y_i, replacing x's, X's by y's, Y's. To derive (2) we assumed a cavity frequency bandwidth much larger than $\tilde{\gamma}_0$. In (2) the amplitude of the term proportional to $J_x(0)$ shows how the atomic state reflects on the outgoing field state. The other terms correspond to intrinsic optical field noise ($\propto X^{\text{in}}$) and added atomic noise ($\propto X_v$). The photocurrents measured by Alice can be expressed as a sum of these noise terms and the initial atomic state:

$$i_- \propto X_- \sim \text{ noise} + [J_{x1}(0) - J_{x2}(0)]e^{-\tilde{\gamma}_0 t}, \quad (3)$$

$$i_+ \propto Y_+ \sim \text{ noise} + [J_{y1}(0) + J_{y2}(0)]e^{-\tilde{\gamma}_0 t}. \quad (4)$$

By choosing the right temporal profile of her local oscillator it was shown in [8] that Alice can measure with a great efficiency the atomic states, which corresponds to the joint measurements used in the continuous variable teleportation protocols for light.

Reconstruction.—From Alice's results and his correlated ensemble 3 Bob is therefore in principle able to deduce the initial state of ensemble 1. Were we dealing with light beams, Bob could directly feed Alice's measurements to standard phase or intensity modulators to reconstruct state 1 [3,5]. The difficulty with an atomic ensemble is to physically implement the reconstruction stage. An all optical method was proposed in [10]. Another way to control the quantum fluctuations of an atomic ensemble is to use a magnetic field in order to have the spin precess in a controlled manner. Such a method was proposed to generate spin squeezing [13] and was successfully implemented recently by Geremia *et al.* to continuously monitor the atomic spin noise via feedback [14]. We propose here to use a transverse magnetic field, the components of which are proportional to Alice's homodyne detection results. Indeed, if we choose the components of the magnetic field, B_x and B_y, to be proportional to $-i_+$ and i_-, we will couple J_{x3} to i_-, and J_{y3} to i_+. Since spin 2 and 3 are initially correlated, their correlated noises will cancel leaving only spin 1 state imprinted onto that of spin 3 at the end of the reconstruction phase.

More quantitatively, the Hamiltonian corresponding to the unitary transformation that Bob performs on spin 3 is simply a $\vec{J}.\vec{B}$ coupling

$$H_B = -\lambda[B_x(t)J_{x3} + B_y(t)J_{y3}]. \quad (5)$$

The evolution equations of J_{x3} and J_{y3} are then of the form

$$\dot{J}_{x3}(t) = G(t)X_-(t), \dot{J}_{y3}(t) = G(t)Y_+(t), \quad (6)$$

in which $G(t)$ gives the electronic gain of the reconstruction process. Its temporal profile can be adjusted in order to maximize the fidelity of the reconstruction. At this point we would like to stress that choosing the right profile for this electronic gain is equivalent to choosing the right local oscillator profile in Alice's homodyne detections. We therefore choose a temporal profile in $G(t) = Ge^{-\tilde{\gamma}_0 t}$ for the gain, which we know will maximize the information that Bob gets [8]. After completion of the reconstruction, i.e., for $t \gg 1/\tilde{\gamma}_0$, the final state of J_{x3}, which we denote by J_{x3}^{out}, can be shown to be

$$J_{x3}^{\text{out}} = gJ_{x1}(0) + J_{x3}(0) - gJ_{x2}(0) + J_x^{\text{noise}} \quad (7)$$

in which $g = -G\eta/\sqrt{N\tilde{\gamma}_0}$ is the normalized gain of the teleportation protocol and J_x^{noise} is a vacuum noise operator taking into account the losses of the process. Its explicit form is not reproduced, but it is uncorrelated with the spin operators and its variance, which can be calculated from Eqs. (2) and (6), is related to the intrinsic noise added during the atom-field transfer processes: $\Delta^2 J_x^{\text{noise}} = (N/2)g^2(1-\eta^2)/\eta^2$.

We assume for simplicity initial isotropic fluctuations for the EPR-entangled ensembles, i.e., $\Delta J_{xi}^2 = \Delta J_{yi}^2 = (N/4)\cosh(2r)$ $(i = 2, 3)$, and symmetrical correlations $\langle \delta J_{x2} \delta J_{x3} \rangle = -\langle \delta J_{y2} \delta J_{y3} \rangle = (N/4)\sinh(2r)$. With these notations the inseparability criterion value (1) is then given by $\mathcal{I}_{2,3} = 2e^{-2r}$, which is 0 for perfect EPR entanglement and 2 for no entanglement. The normalized variance of J_{x3} after reconstruction is then

$$V_{x3}^{\text{out}} = g^2 V_{x1} + 2g^2 \frac{1-\eta^2}{\eta^2} + (1+g^2)\cosh(2r) - 2g\sinh(2r), \quad (8)$$

with an identical expression for the variance of J_{y3}. Note that, if the gain is set to 0, one retrieves the fact that the fluctuations of spin 3 are not modified: $V_{x3}^{\text{out}} = \cosh(2r) = V_{x3}$. Setting a *unity gain* ($g = 1$), the variances of the equivalent input noises $N_\alpha^{\text{out}} \equiv V_{\alpha 3}^{\text{out}} - g^2 V_{\alpha 1}$ $(\alpha = x, y)$ [15] are related to the EPR entanglement and the losses

$$N_{x,y}^{\text{out}} = 2e^{-2r} + 2\frac{1-\eta^2}{\eta^2}. \quad (9)$$

For high entanglement ($r \gg 1$) and negligible losses ($\eta \sim 1$) the equivalent input noises go to 0, which means that the spin 1 state has indeed been fully teleported to spin 3.

At this point we can make a few comments. First, this result is very similar to that of light beam teleportation protocols [3,5,15,16] and shows that the input noise variances go down to 0 if Alice and Bob share perfectly entangled ensembles ($r = \infty$) and in the absence of losses ($\eta = 1$). In absence of entanglement ($r = 0$), $N_x^{\text{out}} = N_y^{\text{out}} = 2$, one retrieves the fact that two units of vacuum noise are added for the measurement and the reconstruction in the protocol. A good criterion to estimate the quality of the teleportation is provided by the product of the equivalent input noise variances $V_q \equiv \sqrt{N_x^{\text{out}} N_y^{\text{out}}}$ [16]. In the absence of losses the classical limit of 2 is beaten as soon as one disposes of entanglement. The equivalent input noises being independent of the input state our teleporta-

tion protocol is unconditional, and the various measures used in light teleportation protocols [3,5,15,16] to assess the success of the teleportation are valid. Of course, for a fixed value of N, the state to be teleported has to satisfy $\langle J_{x1}\rangle, \langle J_{y1}\rangle \sim \mathcal{O}(\sqrt{N})$, else the fidelity decreases. However, teleporting larger coherences can always be achieved by increasing N.

Second, we have assumed that the measurement and the feedback times are negligible with respect to the ground-state spin lifetime, so that ensemble 3 does not evolve before the reconstruction. This approximation is fairly reasonable since the ground-state lifetime for cold atoms or paraffin-coated cells is at least of the order of several milliseconds or even up to the second [10].

Third, the intrinsic noise ($\propto 1/C$), that is, the noise which does not come from the detector quantum inefficiency or electronic noise, is expected to be rather small, thanks to the cooperative behavior of the atoms in the cavity—C can easily be made of the order of 100–1000 using low finesse cavities. This should ensure losses at the percent level and, therefore, a good teleportation. High-Q cavities are not required because the atom-field coupling is enhanced by the collective atomic behavior ($C \propto N$). Bad cavities are actually preferable since the cavity bandwidth has to be much larger than the atomic spectrum width $\tilde{\gamma}_0$.

It is also interesting to look at the physical meaning of the magnetic reconstruction. The unity gain condition $g = 1$ actually translates into the very intuitive condition that the rotation angle of spin 3 during reconstruction in a time $(2\tilde{\gamma}_0)^{-1}$ should be equal to the relative spin fluctuations: $\theta = \omega_L/(2\tilde{\gamma}_0) = 1/\sqrt{N}$, where ω_L is the Larmor frequency. This condition also gives us the order of magnitude of the magnetic field necessary to perform the reconstruction. For an interaction with $N = 10^6$ cesium atoms on the D_2 line, a gyromagnetic factor of 450 kHz/G and $\tilde{\gamma}_0 = (2\pi)225$ kHz, the amplitude of the magnetic field is about 1 mG.

Last, in order to check the quality of the teleportation, Victor can simply perform a readout of ensemble 3 with the same technique previously used by Alice and compare the output state with the input state that he had prepared. Another way to check that this teleportation scheme is successful would be for Bob not to reconstruct the atomic state, but instead, to perform an optical readout of ensemble 3 and use both his homodyne detection results and Alice's results to deduce the input state. However, in this scheme, the atomic state of 1 is never effectively teleported to ensemble 3. The spin 1 state is actually teleported to the outgoing field A_3^{out}, realizing atom-to-field teleportation.

A straightforward but nonetheless important application of our protocol for quantum communication is *atomic entanglement swapping*: if ensemble 1 in the previous scheme was initially quantum-correlated with another ensemble 0, the previous teleportation scheme ensures that ensembles 0 and 3 are entangled at the end of the process. This is of importance for the realization of quantum networks in which quantum repeaters can ensure good quality transmission of the quantum information over long distances [2].

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