Quantum Freeze of Fidelity Decay for Chaotic Dynamics

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We show that the mechanism of *quantum freeze* of fidelity decay for perturbations with a zero time average, recently discovered for a specific case of integrable dynamics [New J. Phys. **5**, 109 (2003)], can be generalized to arbitrary quantum dynamics. We work out explicitly the case of a chaotic classical counterpart, for which we find semiclassical expressions for the value and the range of the plateau of fidelity. After the plateau ends, we find explicit expressions for the asymptotic decay, which can be exponential or Gaussian depending on the ratio of the Heisenberg time to the decay time. Arbitrary initial states can be considered; e.g., we discuss coherent states and random states.

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The question of the stability of quantum time evolution with respect to small changes in the Hamiltonian has recently attracted a lot of attention [1,2]. This question is particularly important in the context of quantum information [3]. The central quantity for describing quantum stability is the fidelity $F(t) = |\langle\psi(t)|\psi_\delta(t)\rangle|^2$ where $|\psi(t)\rangle = U_0(t)|\psi\rangle$ and $|\psi_\delta(t)\rangle = U_\delta(t)|\psi\rangle$ are unperturbed and perturbed time evolutions, of perturbation strength δ, respectively, starting from the same *initial state* $|\psi\rangle$. Let the evolution operator be written as a time-ordered product $U_\delta(t) = \hat{\mathcal{T}}\exp(-i\int_0^t dt' H_\delta(t')/\hbar)$ in terms of generally time-dependent Hamiltonian $H_\delta(t) = H_0(t) + \delta H'(t)$. We assume that either $H_\delta(t)$ is time independent or, more generally, periodically forced with period τ, $H_\delta(t+\tau) = H_\delta(t)$. Then the time is measured in discrete units of τ, namely $t = n\tau$, and the former (autonomous) case is simply obtained as the limit $\tau \to 0$. The perturbed propagator for one time step can be written as $U_\delta(\tau) = U_0(\tau) \times \exp(-iV\tau\delta/\hbar)$ in terms of a Hermitian perturbation V which in the leading order perturbs the Hamiltonian, $V = H' + \mathcal{O}(\tau\delta)$. It has been shown [2] that for classically chaotic systems and for sufficiently strong perturbation and coherent initial state $|\psi\rangle$ the fidelity decay is given by classical Lyapunov exponents, and this phenomenon has been recently explained solely on the basis of classical dynamics [4]. On the other hand, for sufficiently small δ, one can express fidelity decay in terms of a power series in δ where coefficients are given as a time-correlation function of the perturbation [5,6]. The rule of thumb says that a slower decay of correlations implies a faster decay of fidelity, and vice versa. Using this approach, one can derive universal forms of fidelity decay in both cases of classically regular and chaotic dynamics and express all time scales solely in terms of classical quantities and $\hbar$.

The starting point of our analysis is the representation of fidelity $F^{(n)} = F(n\tau)$ in terms of the expectation value [5]

$$F^{(n)} = |f^{(n)}|^2, \qquad f^{(n)} = \langle\psi|M_\delta^{(n)}|\psi\rangle \tag{1}$$

of the *echo operator* $M_\delta^{(n)} := U_0(-n\tau)U_\delta(n\tau)$ which is the propagator in the interaction picture. Namely,

$$M_\delta^{(n)} = \hat{\mathcal{T}}\exp(-i\Sigma_n(V)\delta/\hbar), \tag{2}$$

where, for any operator A, $\Sigma_n(A) := \tau\sum_{n'=0}^{n-1} A_{n'}$ and $A_n := U_0(-n\tau)AU_0(n\tau)$. In case of continuous time, $M_\delta(t) = \hat{\mathcal{T}}\exp(-i\frac{\delta}{\hbar}\Sigma(V,t))$, with $\Sigma(A,t) := \int_0^t dt' A(t')$, $A(t) := U_0(-t)AU_0(t)$. The approach [5,6] using the power law expansion of (2) in δ gives to the second order

$$F^{(n)} = 1 - \frac{\delta^2}{\hbar^2}\{\langle\Sigma_n^2(V)\rangle - \langle\Sigma_n(V)\rangle^2\} + \mathcal{O}(\delta^4), \tag{3}$$

where $\langle\bullet\rangle := \langle\psi|\bullet|\psi\rangle$.

A particularly interesting special situation arises when a time averaged perturbation $\bar{V} := \lim_{n\to\infty}(n\tau)^{-1}\Sigma_n(V)$ equals zero. In general, the perturbation can be decomposed into $\bar{V}$ and the *residual* part $V = \bar{V} + V_{\mathrm{res}}$. The part $\bar{V}$ that commutes with the unperturbed evolution U_0, and is thus *diagonal* in its eigenbasis provided its spectrum is *nondegenerate,* can sometimes be put together with the unperturbed Hamiltonian H_0. This is customary in various quantum mean field approaches. Another situation of more practical relevance emerges when H_0 has an antiunitary *symmetry* T, $T^\dagger H_0 T = H_0$, changing sign of the perturbation V, $T^\dagger V T = -V$. Then the matrix of V is imaginary antisymmetric; hence, $\bar{V} = 0$. It is thus interesting to study the stability of quantum dynamics with respect to residual perturbation only (i.e., when its diagonal part exactly vanishes $\bar{V} = 0$). This problem has been addressed for the particular case of perturbed integrable dynamics , and very interesting results on extreme stability of quantum dynamics have been found.

In this Letter we show that this phenomenon of quantum freeze, namely, the saturation of fidelity to a plateau of high value, is much more general and robust as it appears in Ref. [7], and applies to arbitrary quantum evolution provided only that $\bar{V} = 0$. In particular, we work out in detail the important case of dynamics with a fully chaotic classical counterpart. We compute the plateau value (scaling as $1 - \mathrm{const} \times \delta^2$ within the second order), its range scaling as $1/\delta$, and the rate of the asymptotic decay after the plateau ends (which is either Gaussian or exponential), quantitatively in the leading order in $\hbar$ in terms of the underlying classical dynamics. The phenomenon may find a useful application in quantum computation where the fidelity error is predicted to be small and frozen in time provided only that the diagonal part of the error in each gate can be cured by some other means.

In the autonomous case ($\tau \to 0$), provided that the spectrum of H_0 is nondegenerate (which is true for a generic nonintegrable system), the perturbation is residual iff it can be written as a *time derivative* of some observable W, i.e., a commutator with H_0, $V = \frac{i}{\hbar}[H_0, W] = (d/dt)W$. Generalizing to the discrete, time-periodic case, we assume that the perturbation is of the form

$$V = \frac{1}{\tau}(W_1 - W_0) = \frac{1}{\tau}[U_0(-\tau)WU_0(\tau) - W]. \quad (4)$$

We now apply the Baker-Campbell-Hausdorff (BCH) expansion $e^A e^B = \exp[A + B + (1/2)[A, B] + \ldots]$ to the time-ordered product (2) and rewrite the echo operator

$$M_\delta^{(n)} = \exp\left\{-\frac{i}{\hbar}\left[\Sigma_n(V)\delta + \frac{1}{2}\Gamma_n\delta^2 + \ldots\right]\right\}, \quad (5)$$

where $\Gamma_n := \frac{i\tau^2}{\hbar}\sum_{n'=0}^{n-1}\sum_{n''=n'}^{n-1}[V_{n'}, V_{n''}]$. It is interesting to note that all matrix elements of Γ_n grow with n *not faster* than $\propto n$ (Sect. 2 of Ref. [7]). This becomes obvious for the special form of perturbation (4) for which it follows

$$\Sigma_n(V) = W_n - W_0, \quad (6)$$

$$\Gamma_n = \Sigma_n(R) - \frac{i}{\hbar}[W_0, W_n], \qquad R := \frac{i}{\tau\hbar}[W_0, W_1], \quad (7)$$

so the operator Γ_n is also a time sum or integral of a time-dependent operator R, minus a sort of time-correlation function which shall be neglected for systems with a strong decay of correlations studied below. In the continuous time case, $R = \frac{i}{\hbar}[W, (d/dt)W] = \hbar^{-2}[W, [W, H_0]]$ and $\Gamma(t) = \int_0^t dt' R(t') - \frac{i}{\hbar}[W(0), W(t)]$. We note that, provided W has a well defined classical limit, $\hbar \to 0$, then also V, R, and Γ_n have well defined limits since $\frac{i}{\hbar}[\bullet, \bullet]$ can be replaced by Poisson brackets. This is what we assume below, as well as that the limiting classical dynamics of U_0 is fully chaotic.

Comparing the two terms in the BCH exponential (5), we note that there should exist a time scale $t_2 \sim \delta^{-1}$, such that if $n\tau < t_2$ then the first term $\Sigma_n\delta$ dominates the second one $\frac{1}{2}\Gamma_n\delta^2$ (and higher [7]). So, let us first consider the case $n\tau < t_2$. Then we can neglect the second term and write the fidelity amplitude

$$f^{(n)} = \langle\exp(-i(W_n - W_0)\delta/\hbar)\rangle. \quad (8)$$

Expanding $f^{(n)}$ to the second order in δ, we find $F^{(n)} = 1 - \frac{\delta^2}{\hbar^2}(\kappa_0^2 + \kappa_n^2 - C_n - C_n^*)$ where $\kappa_k^2 := \langle W_k^2\rangle - \langle W_k\rangle^2$, $C_n := \langle W_n W_0\rangle - \langle W_n\rangle\langle W_0\rangle$. Using Cauchy-Schwartz inequality $|C_n| \le \kappa_0\kappa_n$ and the fact that for a bounded operator W the sequence κ_n is bounded, say, by r, we find a freeze of fidelity $1 - F^{(n)} \le 4\frac{\delta^2}{\hbar^2}r^2$, $n\tau < t_2 \propto \delta^{-1}$, for *arbitrary* quantum dynamics, irrespective of the existence and the nature of the classical limit.

Let us further assume that, due to the mixing property of classically chaotic dynamics, time correlations vanish semiclassically beyond some *mixing time scale* t_1, $C_n \to \mathcal{O}(\hbar)$ for $n\tau > t_1$, and quantum expectation values become time independent and equal, in the leading order in $\hbar$, to the classical averages over an appropriate invariant set $\langle A\rangle_{\rm cl} := \int d\mu A_{\rm cl}$. Hence, between t_1 and t_2, the fidelity freezes to a constant value [8]

$$F_{\rm plat} \approx 1 - \frac{\delta^2}{\hbar^2}(\kappa_0^2 + \kappa_{\rm cl}^2), \qquad \kappa_{\rm cl}^2 := \langle W^2\rangle_{\rm cl} - \langle W\rangle_{\rm cl}^2. \quad (9)$$

Considering two interesting extreme examples of initial states, namely, *coherent initial states* (CIS) and *random initial states* (RIS), we find the following: For CIS $\kappa_0^2 \propto \hbar$ can be neglected with respect to $\kappa_{\rm cl}^2$, whereas for RIS κ_n^2 does not depend on time; hence, $\kappa_0^2 = \kappa_{\rm cl}^2$. So within the linear-response approximation $1 - F_{\rm plat}$ is universally twice as large for RIS than for CIS. It is also worth stressing that the quantum relaxation time for CIS is $t_1 \sim t_{\rm E}$, where $t_{\rm E} = -\log\hbar/\lambda$ is the Ehrenfest time for a chaotic classical dynamics with a Lyapunov exponent λ, while for RIS $t_1 \propto \hbar^0$ is simply the classical mixing time.

One can go beyond the linear response in approximating (8) using the simple fact that in the leading order in $\hbar$ quantum observables commute and, as before, that for $n\tau > t_1$ the time correlations vanish, namely, $\langle\exp(-i\frac{\delta}{\hbar}(W_n - W_0))\rangle \approx \langle\exp(-i\frac{\delta}{\hbar}W_n)\rangle\langle\exp(i\frac{\delta}{\hbar}W_0)\rangle$:

$$F_{\rm plat} \approx |\langle\exp(-iW\delta/\hbar)\rangle_{\rm cl}\langle\exp(iW_0\delta/\hbar)\rangle|^2. \quad (10)$$

Defining a generating function in terms of the classical observable $W_{\rm cl}$, $G(z) := \langle\exp(-izW)\rangle_{\rm cl}$, one can compactly write $F_{\rm plat}^{\rm CIS} \approx |G(\delta/\hbar)|^2$ for CIS (neglecting the localized initial state average with W_0) and $F_{\rm plat}^{\rm RIS} \approx |G(\delta/\hbar)|^4$ for RIS, satisfying the *universal* relation $F_{\rm plat}^{\rm RIS} \approx (F_{\rm plat}^{\rm CIS})^2$. Curiously, the same relation is satisfied for the case of regular dynamics [7]. If the argument $z = \delta/\hbar$ is large, the analytic function $G(z)$ can be calculated generally by the method of stationary phase. In the simplest case of a single isolated stationary point $\vec{x}^*$ in N dimensions,

$$|G(z)| \approx \left|\frac{\pi}{2z}\right|^{N/2}|\det\partial_{x_j}\partial_{x_k}W(\vec{x}^*)|^{-1/2}. \quad (11)$$

This expression gives an asymptotic power law decay of the plateau height independent of the perturbation details. Note that for a finite phase space we have oscillatory *diffraction corrections* to Eq. (11) due to a finite range of integration $\int d\mu$ which in turn causes an interesting situation for specific values of z, namely, that by increasing the perturbation strength δ we can actually increase the value of the plateau.

Next we shall consider the regime of long times $n\tau > t_2$. Then the second term in the exponential of (5) dominates the first one; however, even the first term may not be negligible. Up to terms of order $\mathcal{O}(n\delta^3)$ we can factorize Eq. (5) as $M_\delta^{(n)} \approx \exp(-i\frac{\delta}{\hbar}(W_n - W_0))\exp(-i\frac{\delta^2}{2\hbar}\Gamma_n)$. When computing the expectation value $f^{(n)} = \langle M_\delta^{(n)}\rangle$ we again use the fact that in the leading semiclassical order the operator ordering is irrelevant and that, since $n\tau \gg t_1$, any time-correlation can be factorized, so also the second term of Γ_n (7) vanishes. Thus we have

$$F^{(n)} \approx F_{\text{plat}} \left| \left\langle \exp\left[-i\frac{\delta^2}{2\hbar}\Sigma_n(R)\right]\right\rangle \right|^2, \qquad n\tau > t_2. \quad (12)$$

This result is quite intriguing. It tells us that apart from a prefactor F_{plat}, the decay of fidelity with residual perturbation is formally the same as the fidelity decay with a generic nonresidual perturbation, Eqs. (1) and (2), when one substitutes the operator V with R and the perturbation strength δ with $\delta_R = \delta^2/2$. The fact that time ordering is absent in Eq. (12) as compared with (2) is semiclassically irrelevant. Thus we can directly apply the general semiclassical theory of fidelity decay [5], using a renormalized perturbation R of renormalized strength δ_R. Here we simply rewrite the key results in the "non-Lyapunov" perturbation-dependent regime, $\delta_R\tau < \hbar$. Using a classical transport rate $\sigma := \lim_{n\to\infty}\frac{1}{2n\tau}[\langle\Sigma_n^2(R)\rangle_{\text{cl}} - \langle\Sigma_n(R)\rangle_{\text{cl}}^2]$, we have either an exponential decay

$$F^{(n)} \approx F_{\text{plat}} \exp\left(-\frac{\delta^4}{2\hbar^2}\sigma n\tau\right), \qquad n\tau < t_{\text{H}}, \quad (13)$$

or a (perturbative) Gaussian decay

$$F^{(n)} \approx F_{\text{plat}} \exp\left(-\frac{\delta^4}{2\hbar^2}\sigma\frac{(n\tau)^2}{t_{\text{H}}}\right), \qquad n\tau > t_{\text{H}}. \quad (14)$$

$t_{\text{H}} = \tau\mathcal{N}/(2s)$ is the Heisenberg time, where $\mathcal{N} \sim \hbar^{-d}$ (in d degrees of freedom) is the total dimension of the Hilbert space supporting the time evolution and s is the number of different symmetry classes (of possible discrete symmetries of H_0) carrying the initial state $|\psi\rangle$. This is just the time when the integrated correlation function of R_n becomes dominated by quantum fluctuations. Comparing the linear-response expression for F_{plat} (9) with the exponential factor of Eq. (13), and similarly for Eq. (14), we obtain a semiclassical estimate of t_2:

$$t_2 \approx \min\{(t_{\text{H}}/\sigma)^{1/2}\kappa_{\text{cl}}\delta^{-1}, \kappa_{\text{cl}}^2\sigma^{-1}\delta^{-2}\}. \quad (15)$$

Interestingly, the exponential regime (13) can take place only if $t_2 < t_{\text{H}}$. If one wants to keep $F_{\text{plat}} \sim 1$ or have exponential decay in the full range until $F \sim 1/\mathcal{N}$, this implies a condition on dimensionality: $d \geq 2$. The quantum fidelity and its plateau values have been expressed (in the leading order in $\hbar$) in terms of classical quantities only. While the prefactor F_{plat} depends on the details of initial state, the exp factors of (13) and (14) *do not.* Yet, the freezing of fidelity is a purely quantum phenomenon. The corresponding *classical fidelity* (defined in [5]) does not exhibit freezing (see Fig. 1). Let us now demonstrate our theory by numerical examples.

First, we consider a quantized kicked top as an example of a one-dimensional system ($d = 1$). The system is described by quantum angular momentum $J_{x,y,z}$ with (half-)integer modulus J and the one-step propagator $U = \exp(-i\alpha J_z^2/2J)\exp(-i\pi J_y/2)$. We have chosen $\alpha = 30$ ensuring fully chaotic corresponding classical dynamics and $J = 1000$ determining the effective Planck constant $\hbar = 1/J = 10^{-3}$. The perturbation is chosen as $V = (J_x^2 - J_z^2)/2J^2$ associated with $W = J_z^2/2J^2$. The initial state is either RIS (with Gaussian random expansion coefficients) or SU(2) CIS centered at $(\varphi, \theta) = (1, 1)$. In both cases the initial state is projected on an invariant subspace of dimension $\mathcal{N} = J$ spanned by $\mathcal{H}_{\text{OE}} = \{|2m\rangle - |-2m\rangle, |2m-1\rangle + |-(2m-1)\rangle; m = 1, \ldots, J/2\}$ where $|m\rangle$ is an eigenstate of J_z. We first checked the plateau. Within the linear response (9) we have to evaluate only $\kappa_{\text{cl}}^2 = 1/45$ for the corresponding classical observable $W_{\text{cl}} = z^2/2$, giving $F_{\text{plat}}^{\text{CIS}} = 1 - (\delta J)^2\kappa_{\text{cl}}^2$, $F_{\text{plat}}^{\text{RIS}} = 1 - (\delta J)^2 2\kappa_{\text{cl}}^2$. These values give good agreement with the fidelity for weak perturbation $\delta = 10^{-3}$ shown in Fig. 1(a), whereas for strong perturbation $\delta = 10^{-2}$ shown in Fig. 1(b) the theoretical values (10) of F_{plat}, expressed in terms of the generating function $G(z)$ for CIS/RIS, have to be calculated exactly, and, indeed, the agreement is excellent. Integration over the sphere yields $G(\delta J) = \sqrt{\frac{\pi}{2\delta J}}\,\text{erfi}(e^{i\pi/4}\sqrt{\delta J/2})$. Comparing with the asymptotic general formula for $G(z)$ (11), we now also find a diffractive contribution due to the

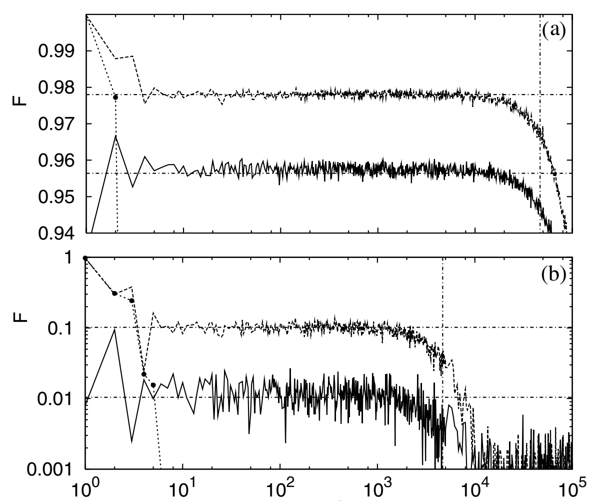

FIG. 1. $F(t)$ for the kicked top, with (a) $\delta = 10^{-3}$ and (b) $\delta = 10^{-2}$, the upper curves (dashed lines) for CIS and the lower curves (full lines) for RIS. Horizontal lines are theoretical plateau values (10), and vertical lines are theoretical values of t_2 (15). Points represent calculations of the corresponding classical fidelity for CIS which follows quantum fidelity up to the Ehrenfest barrier $t_{\text{E}} \propto \log\hbar$ (see Sect. 4 of Ref. [5]) and exhibits no freezing.

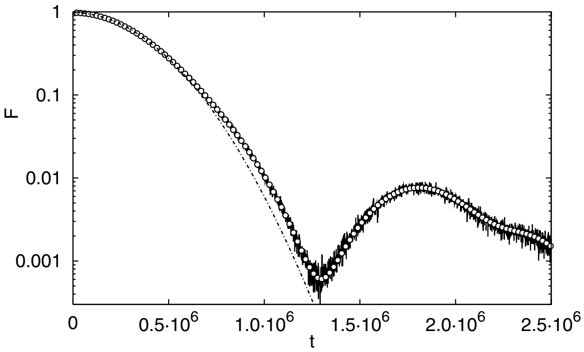

FIG. 2. Long-time Gaussian decay for CIS of a single kicked top for the same parameters as in Fig. 1(a). Full curve is a direct numerical evaluation, empty circles are numerical calculations using the renormalized strength δ_R and operator R, while the chain curve gives the theoretical decay (14).

oscillatory behavior of the complex erf function. Small (quantum) fluctuations around the theoretical plateau values in Fig. 1 lie beyond the leading order semiclassical description. In Fig. 1 we also demonstrate that the semiclassical formula (15) for t_2 works very well. The long-time Gaussian decay for the parameters of Fig. 1(a) is shown in Fig. 2. Here we compare a direct numerical calculation with the numerical calculation using a renormalized perturbation operator (7) $\delta_R R = \frac{-\delta^2}{4J^3} \times (J_x J_y J_z + J_z J_y J_x)$, and with the theoretical prediction (14) where the classical dynamics of $R_{\rm cl} = -xyz$ gives $\sigma = 5.1 \times 10^{-3}$.

To demonstrate the possibility of clean exponential long-time decay of fidelity (13) we look at a system of two ($d = 2$) coupled tops $\vec{J}_1$ and $\vec{J}_2$ given by a propagator $U = \exp(-i\varepsilon J_{z1} J_{z2}) \exp(-i\pi J_{y1}/2) \exp(-i\pi J_{y2}/2)$, with the perturbation generated by $W = A_1 \otimes \mathbb{1} + \mathbb{1} \otimes A_2$, where $A = J_z^2/2J^2$ for each top. We set $J = 1/\hbar = 100$ and $\varepsilon = 20$ in order to be in a fully chaotic regime. The initial state is always a direct product of SU(2) coherent states centered at $(\varphi, \theta) = (1, 1)$ which is subsequently projected on an invariant subspace of dimension $\mathcal{N} = J(J+1)$ spanned by a subspace $\{\mathcal{H}_{\rm OE} \otimes \mathcal{H} \setminus \mathcal{H}_{\rm OE}\}_{\rm sym}$ symmetrized with respect to the exchange of the two tops. Numerical results are shown in Fig. 3. Here we show only a long-time decay, as the situation in the plateau is qualitatively the same as for $d = 1$. For large enough perturbation one obtains an exponential decay shown in Fig. 3(a), while for smaller perturbation we have a Gaussian decay shown in Fig. 3(b). Numerical data have been successfully compared with the theory (13) and (14) using classically calculated $\sigma = 9.2 \times 10^{-3}$, and with the "renormalized" numerics using the operator R (7).

In this Letter we discussed a freeze of fidelity for arbitrary quantum evolution provided only that the diagonal part of the perturbation in the basis of the unperturbed evolution exactly vanishes. The value of the plateau can be arbitrarily close to 1 and can span over arbitrary long-

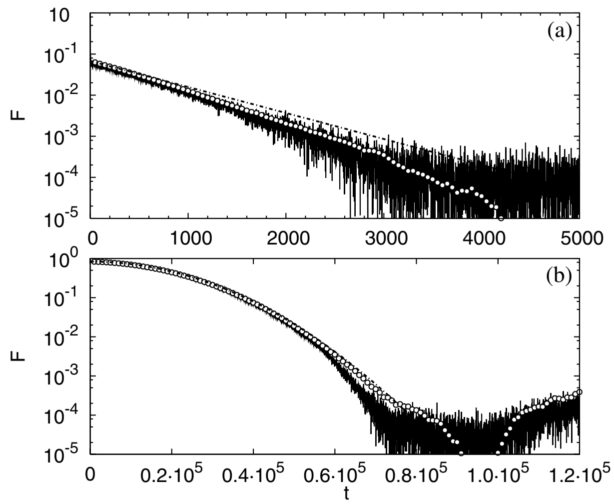

FIG. 3. Long-time fidelity decay in two coupled kicked tops. (a) For strong perturbation $\delta = 7.5 \times 10^{-2}$ we obtain an exponential decay, and (b) for smaller $\delta = 2 \times 10^{-2}$ we have a Gaussian decay. Meaning of the curves is the same as in Fig. 2.

time ranges for a sufficiently small strength of perturbation. We worked out in detail the case of systems with a fully chaotic classical limit. Our result is predicted to have an immediate application to quantum information processing. If combined with the inequality between the purity $I(t)$ of a reduced density matrix of a bipartite quantum system and the fidelity, namely $I(t) > |F(t)|^2$ [9], we predict that decoherence as characterized by $I(t)$ should also exhibit a freeze for the particular class of perturbations.

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