Spin-Wave Theory for the Dynamics Induced by Direct Currents in Magnetic Multilayers

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A spin-wave theory is presented for the magnetization dynamics in a ferromagnetic film that is traversed by spin-polarized carriers at high direct-current densities. It is shown that nonlinear effects due to four-magnon interactions arising from dipolar and surface anisotropy energies limit the growth of the driven spin wave and produce shifts in the microwave frequency oscillations. The theory explains quantitatively recent experimental results in nanometric point contacts onto magnetic multilayers showing downward frequency shifts (redshifts) with increasing current, if the external field is on the film plane, and upward shifts (blueshifts), if the field is perpendicular to the film.

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An effect that has attracted increasing attention due to its potential application in spintronic devices is the driving of the magnetization in magnetic nanostructures by high-density electric currents. Berger [1] and Slonczewski [2] first proposed that a spin-polarized current injected into a ferromagnetic thin film exerts a torque on the magnetization that opposes the effect of relaxation. This spin-transfer-induced (STI) torque can be expressed in terms of an effective magnetic field which is proportional to the current density and, as the magnetic structures shrink to nanoscale dimensions, it is expected to dominate over the classical Oersted-Ampère field created by charges in motion. Despite the controversies regarding the nature of this torque, it is generally agreed that the STI torque represents a novel mechanism for driving the magnetization [1–11].

In recent years, several authors [7,12–14] have claimed the observation of this effect in magnetotransport measurements made with point contacts onto magnetic multilayers. By varying the current intensity at a fixed value of the external magnetic field, they observed sudden changes in the resistance that were attributed to the onset of spin-wave growth at certain critical values of the current. The threshold current depends on the direction and intensity of the applied dc field. However, the evidence of the spin-wave excitation in the magnetoresistance experiments is very indirect and no information on the spatial nature of the excited modes is actually provided.

Clear-cut signatures of the presence of high-frequency spin waves have been reported by Tsoi *et al.* [15], Kiselev *et al.* [16], and Rippard *et al.* [17]. In [15], the magnetic multilayer is placed in a microwave cavity and subjected simultaneously to a direct current from a point contact and a microwave radiation field. The observed mixing of the two frequencies demonstrates that high-frequency spin waves are indeed driven by the current. On the other hand, in [16,17] microwave frequency oscillations resulting from the precession of the magnetization induced by spin-polarized currents are observed directly through contacts patterned into planar waveguides. In both cases there are intriguing features not predicted by the simple models proposed so far. In this Letter we present a spin-wave theory for this dynamical process which accounts for most of the recent experimental observations, including the blueshifts and redshifts.

We consider a ferromagnetic film traversed by a direct current with spin-polarized electrons. As shown by Berger [1] and Slonczewski [2], the interaction between the spins of the conduction electrons and the spins in the magnetic film results in a torque that can be represented by an effective field acting on the local magnetization given by [2]

$$\vec{H}_{STI} = (\beta J/\gamma M_s)\hat{z} \times \vec{M}, \tag{1}$$

where $\beta = \varepsilon\hbar\gamma/2dM_s e$, J is the current density, ε is the spin-transfer efficiency, $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio, g is the spectroscopic factor, μ_B is the Bohr magneton, e is the electron charge, d is the film thickness, and M_s is the saturation magnetization. Notice that $\hat{z}$ is the direction of the spin polarization, which is determined by the applied field that magnetizes the films. The essential feature of the STI field is that it exerts a torque on the magnetization that tends to deviate it away from equilibrium, producing an effect opposite to that of the damping. As a result, when the current exceeds a critical value J_c, the damping is overcome leading to a rapid growth of spin-wave modes supported by the film. The saturation process and other phenomena at higher currents are governed by nonlinear effects. One way to study the physics of this process is to use the Landau-Lifshitz equation, which is intrinsically nonlinear. However, this equation has only been solved analytically in the linearized approximation [4] or numerically in its full form [17]. In this Letter we employ the spin-wave formalism to describe the nonlinear dynamics of the system. First we write the magnetization as

$$\vec{M} = \hat{x}m_x e^{i\omega t} + \hat{y}m_y e^{i\omega t} + \hat{z}M_z, \tag{2}$$

where $\hat{z}$ is the equilibrium direction, $\hat{y}$ is chosen perpendicular to the film, and $m_x, m_y \ll M_z$. We will consider the cases where the external field is applied either parallel or perpendicular to the film plane. The components of the local magnetization at site j can be expanded in creation and annihilation operators of spin deviation [18], a_j^+ and a_j, as

$$m_x = \frac{g\mu_B N\sqrt{S/2}}{V}(a_j - a_j^+ a_j a_j/4NS) + \text{H.c.}, \tag{3a}$$

$$m_y = -i\frac{g\mu_B N\sqrt{S/2}}{V}(a_j - a_j^+ a_j a_j/4NS) + \text{H.c.}, \tag{3b}$$

$$M_z = M_s - \frac{g\mu_B N}{V} a_j^+ a_j, \tag{3c}$$

where N is the number of spins S in the volume V and H.c. is the Hermitean conjugate. In order to obtain the various contributions to the energy one needs to transform the spin deviation operators into the creation and annihilation operators c_k^+ and c_k for the spin-wave mode with wave vector k. This is done by means of the equation

$$a_j = \frac{1}{\sqrt{N}}\sum_k e^{ikr_j}(u_k c_k + v_k c_{-k}^+), \tag{4}$$

where u_k and v_k are the well known coefficients of the Holstein-Primakoff transformations [18] which diagonalize the Hamiltonian of the system and satisfy the relation $u_k^2 - v_k^2 = 1$. Equations (3c) and (4) can be used to express the STI field given by (1) in terms of the spin-wave operators. The expectation values of c_k^+ and c_k can be treated as classical variables, denoted by c_k^* and c_k, whose time evolution is described by the Heisenberg equation. We consider initially only the linear terms, which are determined from Zeeman, anisotropy, dipolar, interlayer exchange, intralayer exchange, and STI field contributions to the energy, that lead to

$$\frac{dc_k}{dt} = -i\omega_k c_k - (\eta_k - \beta J)c_k, \tag{5}$$

where ω_k is the frequency of the spin-wave mode with wave vector k and η_k is the corresponding relaxation rate which is phenomenologically introduced in the equation. Because of the influence of the dipolar interaction, the expression for the spin-wave frequency depends on the direction of the applied field and the direction of the wave vector with respect to the film plane. If the wave vector is perpendicular to the film plane, one has, for the case of the field perpendicular to the film plane,

$$\omega_k = \gamma(H_0 + H_{an} + H_E + Dk^2 - 4\pi M), \tag{6}$$

where H_0 is the value of the external applied field, H_{an} is the anisotropy field, H_E is the interlayer exchange, D is the exchange stiffness, and M is the effective magnetization that includes the effect of surface anisotropy. On the other hand, if the field is on the film plane, the frequency is given by

$$\omega_k = \gamma[(H_0 + H_{an} + H_E + Dk^2)(H_0 + H_{an} + H_E + Dk^2 + 4\pi M)]^{1/2}. \tag{7}$$

Equations (6) and (7) are approximately valid for a general direction of the wave vector as long as $kd \ll 1$ [19]. The solution of the linearized equation of motion (5) is straightforward: namely,

$$c_k(t) = c_k(0)e^{-i\omega_k t}e^{-(\eta_k - \beta J)t}. \tag{8}$$

Since the transverse components of the magnetization (3a) and (3b) are proportional to the amplitude of c_k, this result implies that when the current density exceeds the critical value $J_c = \eta_k/\beta$, the spin-wave mode with the lowest relaxation rate grows exponentially. This is what produces a change in the magnetic state and a corresponding step in the magnetoresistance versus current characteristics observed in experiments. Writing the relaxation rate as $\eta_k = \eta_0 + \alpha_G\omega_k$, where $\alpha_G\omega_k$ is the Gilbert contribution and η_0 is a residual value independent of the frequency, one obtains for the critical current

$$I_c = \frac{2AedM_s}{\hbar\varepsilon\gamma}(\eta_0 + \alpha_G\omega_k), \tag{9}$$

where A is the area of the current cross section, assumed to be uniform and determined by the characteristics of the point contact. Note that the second term in (9) varies with magnetic field and is responsible for the field dependence of the critical current observed in experiments [7,12–17]. For films magnetized perpendicularly to the plane, the frequency is given by (6) and the resulting critical current by

$$I_c = I_{c0} + bH_0, \tag{10a}$$

$$I_{c0} = \frac{b}{\alpha_G}[\eta_0/\gamma + \alpha_G(H_{an} + H_E + Dk^2 - 4\pi M)], \tag{10b}$$

$$b = \frac{2AedM_s\alpha_G}{\hbar\varepsilon}. \tag{10c}$$

In the magnetoresistance experiments with point contacts, the critical current is indicative of the onset of the spin-wave growth, but no direct evidence is obtained about the nature of the modes which are excited by the STI field. In the model proposed by Slonczewski [4], these modes are cylindrical waves propagating radially away from the current beam established by the point contact. The model yields an expression for the critical current identical to (10a), with the same slope b as given by (10c), but with a constant term which can be much larger than the value given by (10b). The reason for this is that the radially propagating modes have radiation losses which are much larger than the intrinsic magnon relaxation.

The field dependence of the critical current has been experimentally studied in detail by Rippard *et al.* [12] in Co/Cu multilayers, and the data were compared with the predictions of Ref. [4]. The measured critical currents exhibit a linear dependence on the field with slope b typically on the order of 0.5 mA/T and initial values I_{c0} consistently in the range 2–4 mA. Using mean values for the parameters given in [12], $4\pi M = 16.5$ kG, film thickness $d = 1.2$ nm, $D = 5$ meV nm^2, $\alpha_G = 0.02$, $\varepsilon = 0.2$, and contact diameter 35 nm, the value of b calculated with Eq. (10c), which is identical to the prediction of Ref. [4], is 0.4 mA/T, in quite good agreement with the measured value. However, the measured initial values of I_c are systematically about 1 order of magnitude smaller than the value predicted in [4]. We attribute this discrepancy to the fact that the radiation loss of the radially propagating mode with wavelength on the order of the contact diameter, assumed in [4], overwhelms the intrinsic damping of the mode. This is a strong evidence that the spin-wave mode excited by the direct current is not the cylindrical wave assumed in [4]. A further evidence of this has been recently provided by the experimental results of [16,17] which demonstrate unequivocally that the mode excited by the spin-injection current has a frequency close to the ferromagnetic resonance (FMR) value; i.e., it has $k \approx 0$.

In order to study the phenomena occurring above the onset of the spin-wave excitation, one needs to take into account the nonlinear terms in the equation of motion. The important contributions arise from three sources: the negative departure from linearity of the STI torque as the magnetization deviates from the equilibrium direction, the surface dipolar energy (demagnetizing effect), and the surface anisotropy energy. The energy density for the latter two can be expressed by the equation $E = 2\pi M_\lambda^2$, where λ is the coordinate perpendicular to the film. For an initial analysis we consider the field perpendicular to the film plane. In this case the precession of the magnetization is circular, the coefficients of the transformation (4) are $u_k = 1$ and $v_k = 0$, so that the equation of motion is considerably simplified. Using the expansions in (3) and the transformation (4) and considering that only the uniform mode $k = 0$ is present [17], one can show that

$$\frac{dc_k}{dt} = -i\omega_k c_k - (\eta_k - \beta J)c_k - \frac{\beta J}{SN} c_k^* c_k c_k - iS_k c_k^* c_k c_k. \tag{11}$$

From (11) it is straightforward to obtain the equation of motion for the number of magnons, $n_k = c_k^* c_k$. Using the normalized variables $n_k' = n_k/SN$, $t_k' = 2\eta_k t$, $r = \beta J/\eta_k = I/I_c$, where r is the driving parameter, we obtain

$$\frac{dn_k'}{dt'} = (r-1)n_k' - rn_k'^2. \tag{12}$$

This is the Bernoulli equation, which has analytical solution for an excitation r in the form of a step function applied at $t = 0$,

$$n_k'(t) = \frac{r-1}{r - v_0 e^{-(r-1)t}} \tag{13}$$

where $v_0 = r - (r-1)/n_0$, n_0 is the (normalized) initial number of magnons, assumed to be the thermal value. Equation (13) shows that with driving $r > 1$, or $I > I_c$, the number of magnons increases rapidly and saturates at times $\gg \eta_k^{-1}$ with a value

$$n_s = SN\frac{r-1}{r} = SN\frac{I - I_c}{I}. \tag{14}$$

Note that for strong driving, $r \gg 1$, the number of magnons saturates at $n_s = NS$. We can use Eqs. (3) and (4) with $u_k = 1$, $v_k = 0$, and the fact that the modulus of c_k is the square root of n_s to calculate the cone angle of the magnetization precession. For $I = 1.5I_c$, the saturation number is $n_s = NS/3$, which corresponds to a cone angle of 48.4°. Since the spin-wave theory with the expansion (3) is a perturbative approach, we expect this value to be close, but not equal, to the one obtained with the numerical solution of the Landau-Lifshitz equation [17].

The last term in Eq. (11) does not influence the number of magnons because it is pure imaginary. On the other hand, it does produce a frequency shift $\delta\omega = S_k n_k$. For $I < I_c$ this shift is negligible because the magnon number has the thermal value. However, if $I > I_c$, the magnon number is large and the shift can be appreciable. If the field is perpendicular to the film plane, the component of the magnetization entering the dipolar and surface anisotropy energy is M_z. Using (3c) and (4) it can be easily shown that the coefficient of the magnon interaction becomes $S_k = \gamma 4\pi M/NS$. This produces an upward shift in frequency with increasing driving current (blueshift), which is approximately linear in a small current range, as observed in [17]. With (14) one can obtain the frequency shift versus current. For the case the field is perpendicular to the film, the slope of this curve is

$$\frac{d\delta\omega}{dI} = \gamma\frac{4\pi M I_c}{I^2}. \tag{15}$$

Using the values given in Ref. [17], $4\pi M = 8$ kG, $g = 1.78$, $I_c = 3.95$ mA, Eq. (15) gives for $I = 8$ mA a shift of 1.2 GHz/mA. For fields applied in the plane of the film, the energy density is given by $E = 2\pi m_y^2$. In this case, with (3b) one obtains a nonlinear coefficient which is negative, so that the frequency decreases with increasing current (redshift). It turns out that with the field in the plane, the magnetization precession is elliptical, $u_k > 1$, $v_k > 0$, and the equation of motion for c_k becomes somewhat more complicated,

$$\begin{aligned}\frac{dc_k}{dt} = & -i\omega_k c_k - (\eta_k - \beta J)c_k \\ & - \frac{\beta J}{SN}[(u_k^2 + v_k^2)c_k^* c_k c_k - u_k v_k (c_k^* c_k^* c_k + c_k c_k c_k)] \\ & - iT_k(\alpha_1 c_k^* c_k c_k + \alpha_2 c_k^* c_k^* c_k + \alpha_3 c_k^* c_k^* c_k^* \\ & + \alpha_4 c_k c_k c_k),\end{aligned} \tag{16}$$

where $T_k = -\gamma\pi M/2NS$ and the other factors are (dropping the subscripts for simplicity),

$$\alpha_1 = 4u^4 - 12u^3v + 16u^2v^2 - 12uv^3 + 4v^4,$$

$$\alpha_2 = -3u^4 + 12u^3v - 18u^2v^2 + 12uv^3 - 3v^4,$$

$$\alpha_3 = -4u^3v + 8u^2v^2 - 4uv^3,$$

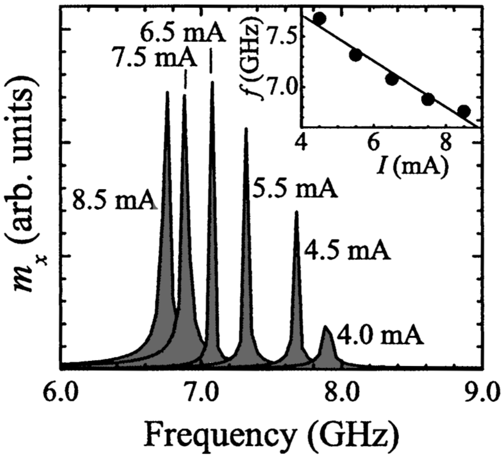

FIG. 1. Microwave frequency spectra at several values of the spin-injection current. Inset: calculated variation of f with I (symbols). The straight line represents the experimental data with slope -0.23 GHz/mA [17].

and

$$\alpha_4 = -u^4 + 4u^3v - 6u^2v^2 + 4uv^3 - v^4.$$

Note that Eq. (16) is not expected to coincide with (11) when $u_k = 1$, $v_k = 0$, because the dipolar energy arises from m_y^2 when the field is on the plane and from M_z^2 when it is perpendicular to the plane. Note also that (16) cannot be solved analytically to yield a simple expression for the frequency shift as in the case of perpendicular magnetization. This problem has been solved approximately in [20]. Numerical solutions for the spectra of the m_x component of the magnetization are shown in Fig. 1 for several values of the driving current, using the parameters of the experiments of Ref. [17], $4\pi M = 8$ kG, $\omega_k = 2\pi \times 7.9$ GHz, $g = 1.78$, $I_c = 3.95$ mA, $u_k = 1.14$, and $v_k = 0.55$. As shown in the inset of Fig. 1, the agreement with the data of Rippard *et al.* [17] is impressive, considering that there is no adjustable parameter. Note that since in the parallel configuration the precession of the magnetization is elliptical, the spectra of the M_z component of the magnetization exhibit peaks at the second harmonic, as observed experimentally [17].

In summary, we have shown that a spin-wave theory incorporating nonlinear effects due to magnon interactions explains quantitatively recent experiments of microwave oscillations produced by direct currents, applied with point contacts onto magnetic multilayers, including blue and red frequency shifts. It is expected that other nonlinear dynamic effects occur as the driving current increases and other spin-wave modes are excited. Among them we predict frequency jumps, if the new mode dominates over the previous one, and, if more than one mode coexist, mode oscillations, and several routes to chaotic behavior [21]. Whether the actual devices are able to support sufficient current densities to exhibit higher-order bifurcations remains an open question.

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