Fluctuation Conductivity of Thin Films and Nanowires Near a Parallel-Field-Tuned Superconducting Quantum Phase Transition

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We calculate the fluctuation correction to the normal state conductivity in the vicinity of a quantum phase transition from a superconducting to a normal state, induced by applying a magnetic field parallel to a dirty thin film or a nanowire with thickness smaller than the superconducting coherence length. We find that at zero temperature, where the correction comes purely from quantum fluctuations, the positive "Aslamazov-Larkin" contribution, the negative "density of states" contribution, and the "Maki-Thompson" interference contribution are all of the same order and the total correction is negative. Further, we show that, based on how the quantum critical point is approached, there are three regimes that show different temperature and field dependencies which should be experimentally accessible.

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The interest in the physics of quantum phase transitions (QPT) and of a quantum critical point (QCP) (see [1] for a review) is motivated by the explosive growth of its experimental realizations and calls for a theoretical advance. The challenge for a theory is twofold: first, it has to identify experimentally accessible systems experiencing QPT. Second, these systems should allow for a systematic and comprehensive theoretical description. In this respect, the QCP realized in dirty superconductors of reduced dimensionality [2–4] by applying a magnetic field is an exemplary phenomenon since, on the one hand, it can be explored in a controllable way in the vicinity of the critical point, and, on the other hand, it allows for a systematic quantitative theoretical study.

In this Letter we present a full systematic investigation of fluctuation corrections [5,6] to the normal state conductivity of a thin wire or a thin film in the vicinity of a QPT from a superconducting to a normal state, induced by an applied magnetic field.

We consider a thin wire (or thin film) with a diameter d (or a thickness t for the film) much smaller than the superconductor coherence length ξ, placed in a field H directed along the wire (or parallel to the film). Such a system is effectively one (two) dimensional from the point of view of pair fluctuations. In a dirty system the combined effect of the magnetic field and electron scattering gives rise to the loss of phase coherence of the Cooper pair, and, at a sufficiently strong field, to the superconductor-normal metal transition. The time τ_d of the loss of coherence caused by magnetic field can be estimated as $\tau_d D H^2 l^2/\Phi_0^2 \sim 1$, where D is the diffusion constant, $\Phi_0 \equiv \pi c/e$, and l is the characteristic sample size [7–9]. The magnetic field can be, thus, parametrized by the so-called depairing parameter $\alpha \sim 1/\tau_d$ whose exact expression for a particular geometry can be obtained from the Usadel equation [10] as

$$\alpha = \begin{cases} D(eHd/2c)^2/4 & \text{wire} \\ D(eHt/c)^2/6 & \text{film.} \end{cases} \quad (1)$$

The superconducting critical temperature T_c is related to α (see Fig. 1) via the standard relation (similar to that in the theory of paramagnetic impurities [11])

$$\ln(T_c/T_{c0}) = \psi(1/2) - \psi(1/2 + \alpha/2\pi T_c), \quad (2)$$

where ψ is the digamma function and $T_{c0} \equiv T_c(H = 0)$ is the critical temperature in the absence of a magnetic field. At $T = 0$ the superconductivity breaks down at $\alpha_{c0} \equiv \alpha_c(T = 0) = \pi T_{c0}/2\gamma$, where $\ln\gamma \approx 0.577$ is the Euler constant. The realization of an analogous QCP in a superconductor with paramagnetic impurities was first suggested in Ref. [12] (see also Ref.[13]). Since in our case the depairing parameter α depends on H, it allows for a well controlled exploration of the QPT and its vicinity by varying the applied magnetic field.

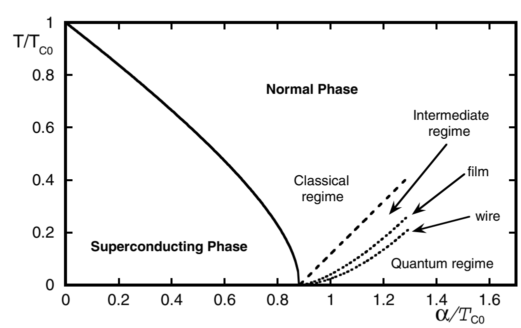

FIG. 1. Phase diagram of a superconducting nanowire (thin film) in a parallel magnetic field (parametrized by α). The boundary between the quantum and the intermediate regime, shown by a dotted line, is different for a wire and film as marked in the figure while all other boundaries coincide.

Our approach is based on the diagrammatic perturbation theory. Note that the technique using the time-dependent Ginzburg-Landau formalism that was adapted in Refs. [12,13] accounts only for the direct "Aslamazov-Larkin" (AL) type of contribution [14] to the fluctuation conductivity that comes from the charge transfer via fluctuating Cooper pairs, but misses the zero-temperature contribution to the correction. On the contrary, our approach takes care of all the contributions including the "density of states" (DOS) part resulting from the reduction of the normal single-electron density of states at the Fermi level, and the more indirect "Maki-Thompson" (MT) interference contribution [15,16]. At zero temperature, where the correction comes purely from quantum fluctuations, these turn out to be of the same order as the AL contribution.

Our result for the fluctuational correction to the normal state conductivity can be conveniently presented as a sum of the zero-temperature and finite-temperature contributions:

$$\delta\sigma(\alpha, T) = \delta\sigma_0(\alpha) + \delta\sigma_T(\alpha, T). \qquad (3)$$

The zero-temperature correction is given by

$$\delta\sigma_0(\alpha) = -\frac{2De^2}{\pi d(2+d)}\int\frac{d^dq}{(2\pi)^d}\frac{(Dq^2)^2}{\alpha_q^3\ln(\alpha_q/\alpha_c)}, \qquad (4)$$

where $\alpha_q \equiv \alpha + Dq^2/2$ and $d = 1, 2$ correspond to a wire and film, respectively. Its magnitude decreases monotonically with increasing field; this leads to a *negative* magnetoresistance. Note that a negative magnetoresistance was found also in granular superconductors [17] and in thin films in *perpendicular* magnetic field [18]. Shown in Fig. 2 are plots of the dimensionless correction

$$\delta\bar{\sigma}_0(\alpha) = e^{-2}\times\begin{cases}(D/\alpha_{c0})^{-1/2}\delta\sigma_0(\alpha) & \text{wire}\\ \delta\sigma_0(\alpha) & \text{film.}\end{cases} \qquad (5)$$

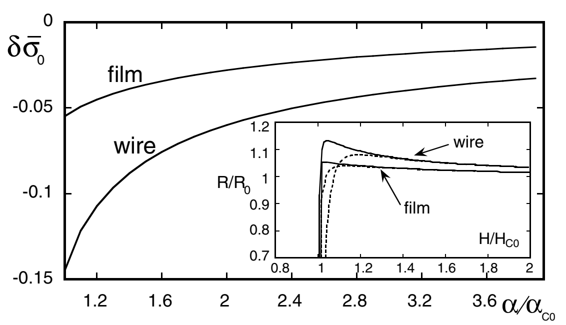

FIG. 2. Dimensionless zero-temperature fluctuation conductivity correction (5) as a function of depairing parameter α. Inset shows dependence of the resistivity on the magnetic field for temperatures $T/\alpha_{c0} = 0.01$ (upper curves) and $T/\alpha_{c0} = 0.1$ (lower curves). Resistivity is normalized to the high field resistivity R_0 while the magnetic field is normalized to the critical field H_{c0} at $T = 0$. The values for the high field resistivity are taken as $R_0^{-1} = e^2/c$ for a film and $R_0^{-1} = e^2(D/\alpha_{c0})^{1/2}$ for a wire.

When expanded around the QCP, the dimensionless conductivity close to the QCP is given by

$$\delta\bar{\sigma}_0(\alpha) = \delta\bar{\sigma}_0(\alpha = \alpha_{c0}) + b(\alpha - \alpha_{c0})/\alpha_{c0}, \qquad (6)$$

with the numerical coefficient $b = 0.386$ and $b = 0.070$ for a wire and film, respectively. Note that since the upper critical dimension for our model at zero temperature is 2, the result (6) is expected to hold only outside the quantum Ginzburg region for both wire and film.

In the vicinity of the QCP ($T, \alpha - \alpha_c \ll \alpha_{c0}$), the field dependence of $\delta\sigma_T(\alpha, T)$ turns out to be more singular than that of $\delta\sigma_0(\alpha)$ and for $T > \alpha - \alpha_c(T)$ its leading term is given by

$$\delta\sigma_T(\alpha, T) = e^2\times\begin{cases}\frac{\sqrt{D}T}{4\sqrt{2}[\alpha-\alpha_c(T)]^{3/2}} & \text{wire}\\ \frac{T}{4\pi[\alpha-\alpha_c(T)]} & \text{film}\end{cases} \qquad (7)$$

while for $T < \alpha - \alpha_c$ it is

$$\delta\sigma_T(\alpha, T) = e^2\times\begin{cases}\frac{\pi\sqrt{D}T^2}{12\sqrt{2}[\alpha-\alpha_c(T)]^{5/2}} & \text{wire}\\ \frac{T^2}{18[\alpha-\alpha_c(T)]^2} & \text{film.}\end{cases} \qquad (8)$$

The contributions $\delta\sigma_T$ and $\delta\sigma_0$ become comparable at

$$T \equiv T_0(\alpha) \sim \begin{cases}(\alpha-\alpha_{c0})^{7/4}/\alpha_{c0}^{3/4} & \text{wire}\\ (\alpha-\alpha_{c0})^{3/2}/\alpha_{c0}^{1/2} & \text{film.}\end{cases} \qquad (9)$$

The key point is that the behavior of the fluctuation corrections to the conductivity depends on the way one approaches the QCP and we can identify three regimes in the vicinity of the QCP that show qualitatively different behaviors as illustrated in Fig. 1. There is a "classical" regime for $T > \alpha - \alpha_c$, where the correction is given by Eq. (7); an "intermediate" regime for $\alpha - \alpha_c > T > T_0(\alpha)$, where the correction behaves according to Eq. (8); and a "quantum" regime for $T_0(\alpha) > T$, where the behavior crosses over to an essentially zero-temperaturelike behavior which is not singular and almost temperature independent with the fluctuation correction dominated by $\delta\sigma_0$ as given by Eqs. (4)–(6).

Since in the quantum region the correction to conductivity is negative whereas in the classical and intermediate region it is positive, we predict a nonmonotonic behavior of the resistivity as a function of the magnetic field at finite temperature. The corresponding plots are shown in the inset of Fig. 2 for nanowires and thin films. Such a behavior was indeed reported in experiments on amorphous Nb thin films in Ref. [2], and recently on bismuth ultrathin films in Ref. [3]; while for nanowires, to the best of our knowledge, it was not reported yet.

To derive our main results, we carry out a microscopic calculation within the standard framework of the temperature diagrammatic technique in a disordered electron system [19–21] in the diffusive limit (inverse mean free time $\tau^{-1} \gg T, \alpha$). The main building block of the diagrammatic technique in the presence of the BCS interaction is the so-called "Cooperon," the ladder diagram that describes coherent scattering by impurities in the particle-particle channel. In the presence of a parallel magnetic field, it is given by

$$C(\Omega, q) = \frac{1}{|\Omega| + Dq^2 + 2\alpha}, \tag{10}$$

where Ω is the bosonic Matsubara frequency and $\mathbf{q}$ is the momentum in the effective dimension. Using Eq. (10) in a standard way, one obtains the "fluctuation propagator," i.e., the impurity-averaged sum over the ladder diagrams corresponding to the electron-electron interaction in the Cooper channel,

$$K^{-1}(\Omega, q) = \ln\left(\frac{T}{T_c^0}\right) - \psi\left(\frac{1}{2}\right) + \psi\left(\frac{1}{2} + \frac{\alpha_q + |\Omega|/2}{2\pi T}\right), \tag{11}$$

where Ω is the bosonic Matsubara frequency. The pole of Eq. (11) defines the boundary between the superconducting and normal phases given by Eq. (2). At low temperatures, $T \ll \alpha_{c0}$, the fluctuation propagator reduces to $K^{-1}(\Omega, q) = \ln[(\alpha_q + |\Omega)|/2)/\alpha_c(T)]$.

The fluctuation correction to the conductivity is obtained as usual from the Kubo formalism with the appropriate analytic continuation. The standard set of diagrams constituting the AL, DOS, and MT contributions is shown in Fig. 3.

The AL contribution [Fig. 3(a)] can be expressed as a sum of two terms: $\delta\sigma^{\rm AL} = \delta\sigma_1^{\rm AL} + \delta\sigma_2^{\rm AL}$ with

$$\frac{\delta\sigma_1^{\rm AL}}{e^2} = \frac{D^2}{2\pi T d}\int \frac{d^d q}{(2\pi)^d}\frac{d\Omega}{\mathrm{sh}^2\frac{\Omega}{2T}}[\mathrm{Im}\{K(-i\Omega, q)\Gamma_{\Omega\mathbf{q}}^2\} \times \mathrm{Im}\{K(-i\Omega, q)\} + (\mathrm{Im}\{K(-i\Omega, q)\Gamma_{\Omega\mathbf{q}}\})^2], \tag{12}$$

$$\frac{\delta\sigma_2^{\rm AL}}{e^2} = \frac{D^2 i}{8\pi^4 T^3}\int \frac{d^d q d\Omega}{(2\pi)^d} q_x^2 \psi'\left(\frac{1}{2} + \frac{\alpha_q - i\Omega/2}{2\pi T}\right) \times \psi''\left(\frac{1}{2} + \frac{\alpha_q - i\Omega/2}{2\pi T}\right)K^2(-i\Omega, q), \tag{13}$$

where $\Gamma_{\Omega\mathbf{q}} \equiv (\mathbf{q}/2\pi T)\psi'[1/2 + (\alpha_q - i\Omega/2)/2\pi T]$. The contribution $\delta\sigma_1^{\rm AL}$ appears only at finite temperature while $\delta\sigma_2^{\rm AL}$ contains the contribution that survives even at $T = 0$. Note that $\delta\sigma_2^{\rm AL}$ that describes the quantum regime results from differentiating $\Gamma_{\mathbf{q}}(-i\Omega)$ with respect to Ω and is missed within the usual static approximation ($\Omega = 0$).

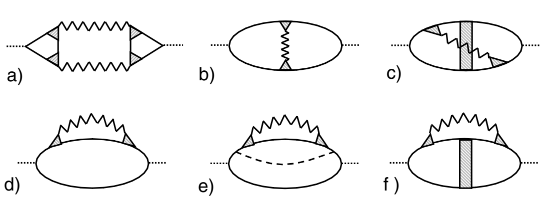

FIG. 3. Diagrams for the fluctuation conductivity: (a) Aslamazov-Larkin diagram, (b)–(c) Maki-Thompson diagrams, and (d)–(f) density of state diagrams. The full line stands for the disorder averaged normal state Green's function, the wavy line for the fluctuation propagator K, the shaded rectangle for the Cooperon C, and the shaded triangle for the vertex $C/2\pi\nu\tau$.

The MT correction is given by $\delta\sigma^{\rm MT} = \delta\sigma_1^{\rm MT} + \delta\sigma_2^{\rm MT}$, where

$$\frac{\delta\sigma_1^{\rm MT}}{e^2} = \frac{D}{2\pi}\int \frac{d^d q d\omega d\Omega}{(2\pi)^d}\mathrm{th}\frac{\omega}{2T}K(-i\Omega, q) \times \left[\frac{\mathrm{cth}\frac{\Omega}{2T}}{(\alpha_q - i\omega_+)^3} + \frac{i}{2T\mathrm{sh}^2\frac{\Omega}{2\mathrm{T}}}\frac{1}{\alpha_q^2 + \omega_+^2}\right], \tag{14}$$

$$\frac{\delta\sigma_2^{\rm MT}}{e^2} = -\frac{3D^2}{2\pi}\int \frac{d^d q d\omega d\Omega}{(2\pi)^d}\frac{\mathrm{th}\frac{\omega}{2T}}{\mathrm{th}\frac{\Omega}{2T}}\frac{q_x^2 K(-i\Omega, q)}{(\alpha_q - i\omega_+)^4}, \tag{15}$$

with $\omega_+ \equiv \omega + \Omega/2$. For low temperatures, the contributions $\delta\sigma_1^{\rm MT}$ [Fig. 3(b)] and $\delta\sigma_2^{\rm MT}$ [Fig. 3(c)] are of the same order and both need to be taken into account. On the contrary, at high temperatures, the diagram [Fig. 3(c)] having an extra Cooperon propagator is of a lower order.

The DOS fluctuation correction is given by the expression $\delta\sigma^{\rm DOS} = \delta\sigma_1^{\rm DOS} + \delta\sigma_2^{\rm DOS}$ where

$$\frac{\delta\sigma_1^{\rm DOS}}{e^2} = \frac{D}{2\pi}\int \frac{d^d q d\omega d\Omega}{(2\pi)^d}\mathrm{th}\frac{\omega}{2T}K(-i\Omega, q) \times \left[\frac{\mathrm{cth}\frac{\Omega}{2T}}{(\alpha_q - i\omega_+)^3} - \frac{i}{2T\mathrm{sh}^2\frac{\Omega}{2T}}\mathrm{Re}\frac{1}{(\alpha_q - i\omega_+)^2}\right], \tag{16}$$

$$\frac{\delta\sigma_2^{\rm DOS}}{e^2} = \frac{D^2}{4\pi}\int \frac{d^d q d\omega d\Omega}{(2\pi)^d} q_x^2 \mathrm{th}\frac{\omega}{2T}K(-i\Omega, q) \times \left[\frac{-3\mathrm{cth}\frac{\Omega}{2T}}{(\alpha_q - i\omega_+)^4} + \frac{i}{T\mathrm{sh}^2\frac{\Omega}{2T}}\mathrm{Re}\frac{1}{(\alpha_q - i\omega_+)^3}\right]. \tag{17}$$

The correction $\delta\sigma_1^{\rm DOS}$ corresponds to diagrams shown in Figs. 3(d) and 3(e) while $\delta\sigma_2^{\rm DOS}$ is given by diagrams from Fig. 3(f) having an extra Cooperon propagator.

In the zero-temperature limit, one can see that, apart from $\delta\sigma_2^{\rm AL}$, only the terms in the DOS and MT contributions that have a $\mathrm{cth}(\Omega)$ survive. Integrating these terms over frequency Ω by parts, we come to the result of Eq. (4). It is interesting to note that at $T = 0$, we find that the AL correction is positive and the DOS correction is negative, as expected, while the MT correction which does not have a prescribed sign is negative and related to $\delta\sigma^{\rm AL}$ via $\delta\sigma^{\rm MT} = -\delta\sigma^{\rm AL}/2$.

At nonzero temperatures, the terms having a factor $1/\mathrm{sh}(\Omega/2T)$ need to be included. One finds that the leading contribution close to the QCP comes from $\delta\sigma_1^{\rm AL}$ and the evaluation of this term in different regimes leads to the results Eqs. (7) and (8) discussed earlier.

Finally we would like to point out an interesting experimental realization of QCP in a quantum wire, that of a hollow cylinder with thin walls [4]. In this case the pair-breaking parameter reads

$$\alpha = D\left[\frac{eH}{4c}\left(-4n + \frac{eH}{c}(r_1^2 + r_2^2)\right) + n^2\frac{\ln(r_2/r_1)}{r_2^2 - r_1^2}\right], \quad (18)$$

where r_1 and r_2 are the inner and outer radii, respectively, and n is an arbitrary integer. For a thin cylinder ($r_1 \approx r_2 \approx r$) it reduces to $\alpha = (D/2r^2)(\Phi/\Phi_0 - n)^2$, where Φ is the flux enclosed by the cylinder, thereby rendering the classic Little-Park oscillations [7] of T_c as can be seen from Eq. (2). Interestingly, for a cylinder with small enough radius, $r < r_c = \sqrt{D\gamma/4\pi T_{c0}}$, it is possible to push the T_c down to zero at magnetic fields corresponding to half-integer fluxes $\Phi = \Phi_0(1/2 + n)$, as was experimentally observed in Ref. [4]. While the positive fluctuation contribution to conductivity that we would associate with the classical regime was clearly observed, the behavior expected for the quantum and intermediate regimes was not reported so far.

In conclusion, we have investigated the fluctuation correction to the normal state conductivity in the vicinity of a parallel-field-induced QCP in dirty samples of reduced dimensions, taking into account both quantum and thermal fluctuations within the diagrammatic perturbation theory. Our key finding is that there are three regimes that show a qualitatively different behavior ranging from quantum to classical. The particular temperature and field behavior of the conductivity is dictated by the choice of path in approaching the QCP while making the measurement. We have found that, for a nanowire (or for a hollow cylinder) as well as for a thin film, the zero-temperature conductivity correction that also governs the quantum regime is *negative*, which means that the quantum pairing fluctuations increase the resistance to the charge flow. Our findings imply that experiments should detect a *negative* magnetoresistance in the quantum regime.

To make a detailed comparison with our theory, the weak localization [22] and Altshuler-Aronov [21] corrections must be subtracted from the experimental conductivity. However, inclusion of these corrections will not affect the predicted negative sign of the magnetoresistance at low temperatures. Indeed, the Altshuler-Aronov correction does not depend on the magnetic field, while the weak localization correction is determined by the Cooperon propagator (10) via $\delta\sigma = -(2e^2D/\pi)\int d^dq/(2\pi)^dC(0, q)$ and thus gives also a negative contribution to the magnetoresistance. We hope that our findings will stimulate further experimental measurements of fluctuating conductivity in quantum wires where the negative magnetoresistance at low temperatures to the best of our knowledge was not yet found and further in-depth analysis for films in which case the negative magnetorsesistance was indeed observed.

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