

## Determination of the Stiffness of the Nuclear Symmetry Energy from Isospin Diffusion

Lie-Wen Chen,<sup>1,\*</sup> Che Ming Ko,<sup>1</sup> and Bao-An Li<sup>2</sup>

<sup>1</sup>*Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843-3366, USA*

<sup>2</sup>*Department of Chemistry and Physics, P.O. Box 419, Arkansas State University, State University, Arkansas 72467-0419, USA*

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With an isospin- and momentum-dependent transport model, we find that the degree of isospin diffusion in heavy-ion collisions at intermediate energies is affected by both the stiffness of the nuclear symmetry energy and the momentum dependence of the nucleon potential. Using a momentum dependence derived from the Gogny effective interaction, recent experimental data from NSCL-MSU on isospin diffusion are shown to be consistent with a nuclear symmetry energy given by  $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{1.05}$  at subnormal densities. This leads to a significantly constrained value of about  $-550$  MeV for the isospin-dependent part of the isobaric incompressibility of isospin asymmetric nuclear matter.

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Knowledge of the density dependence of nuclear symmetry energy is important for understanding not only the structure of radioactive nuclei [1–4] but also many important issues in nuclear astrophysics [5,6], such as nucleosynthesis during presupernova evolution of massive stars and the cooling of protoneutron stars. Although the nuclear symmetry energy at normal nuclear matter density is known to be around 30 MeV from the empirical liquid-drop mass formula [7,8], its values at other densities are poorly known [9,10]. Studies based on either microscopic many-body theories or phenomenological approaches have thus far given widely divergent predictions on the density dependence of nuclear symmetry energy [11]. Empirically, the incompressibility of asymmetric nuclear matter is essentially undetermined [12], even though the incompressibility of symmetric nuclear matter at its saturation density  $\rho_0 = 0.16 \text{ fm}^{-3}$ , has been determined to be  $231 \pm 5$  MeV from nuclear giant monopole resonances [13], and the equation of state at densities of  $2\rho_0 < \rho < 5\rho_0$  has been constrained by measurements of collective flows in nucleus-nucleus collisions [14].

In light of the new opportunities provided by radioactive beams, a lot of interest and activity has recently been devoted to extracting information on the density dependence of nuclear symmetry energy from reactions induced by such nuclei [15–26]. In particular, isospin diffusion in heavy-ion collisions is found to depend sensitively on the density dependence of nuclear symmetry energy [27–29]. Within a momentum-independent transport model, in which the nuclear potential depends only on local nuclear density, the isospin diffusion data from recent experiments at the NSCL-MSU (National Superconducting Cyclotron Laboratory at Michigan State University) was found to favor a quadratic density dependence for the interaction part of nuclear symmetry energy [30]. This conclusion has stimulated much interest because of its implications to nuclear many-body theories and nuclear astrophysics. However, the nuclear potential acting on a nucleon is known to depend also on its momentum. For nuclear iso-

scalar potential, its momentum dependence is well known and is important in extracting information about the equation of state of symmetric nuclear matter [14,31–38]. Very recently, the momentum dependence of the isovector (symmetry) potential [11,39–41] was also shown to be important for understanding a number of isospin related phenomena in heavy-ion reactions [42–44]. It is thus necessary to include momentum dependence in both the isoscalar and isovector potentials for studying the effect of nuclear symmetry energy on isospin diffusion. In this Letter, we shall show that the isospin diffusion data are consistent instead with a softer symmetry energy that is nearly linear in density within the momentum-dependent effective interaction used in the present study.

Our study is based on the isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU04) transport model that uses experimental nucleon-nucleon cross sections in free space and includes the momentum dependence in both the isoscalar and isovector potentials [42]. Although the momentum dependence of the isoscalar potential is known empirically, that of the isovector potential is not as well determined [45]. In the IBUU04 model, it is based on the Gogny effective interactions. Specifically, the potential  $U(\rho, \delta, \mathbf{p}, \tau)$  for a nucleon with isospin  $\tau$  ( $1/2$  for neutrons and  $-1/2$  for protons) and momentum  $\mathbf{p}$  in asymmetric nuclear matter at total density  $\rho$  is given by [40]

$$\begin{aligned}
 U_{\text{MDI}}(\rho, \delta, \mathbf{p}, \tau) = & A_u \frac{\rho_{\tau'}}{\rho_0} + A_l \frac{\rho_{\tau}}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^{\sigma} (1 - x \delta^2) \\
 & - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^{\sigma}} \delta \rho_{\tau'} + \frac{2C_{\tau,\tau}}{\rho_0} \\
 & \times \int d^3\mathbf{p}' \frac{f_{\tau}(\mathbf{r}, \mathbf{p}')}{1 + (\mathbf{p} - \mathbf{p}')^2/\Lambda^2} + \frac{2C_{\tau,\tau'}}{\rho_0} \\
 & \times \int d^3\mathbf{p}' \frac{f_{\tau'}(\mathbf{r}, \mathbf{p}')}{1 + (\mathbf{p} - \mathbf{p}')^2/\Lambda^2}, \quad (1)
 \end{aligned}$$

where  $\rho_{\tau}$  and  $\rho_{\tau'}$  denote proton or neutron density with  $\tau \neq \tau'$ ; and  $\delta \equiv (\rho_n - \rho_p)/\rho$  is the isospin asymmetry. The

$f_\tau(\mathbf{r}, \mathbf{p})$  denotes the phase-space distribution function at coordinate  $\mathbf{r}$  and momentum  $\mathbf{p}$ . The corresponding momentum-dependent interaction (MDI) leads to an incompressibility of  $K_0 = 211$  MeV for the symmetric nuclear matter at saturation density. On the right-hand side of Eq. (1), the first four terms with  $\sigma = 3/4$  and  $B = 106.35$  MeV describe the momentum-independent interaction. The terms with parameters  $C_{\tau,\tau} = -11.7$  MeV and  $C_{\tau,\tau'} = -103.4$  MeV describe the momentum-dependent interaction of a nucleon of isospin  $\tau$  and momentum  $\mathbf{p}$  with like and unlike nucleons in the background fields, respectively. With the parameter  $\Lambda = 1.0p_F^0$ , where  $p_F^0$  denotes nucleon Fermi momentum at  $\rho_0$ , the isoscalar potential  $[U_n(\rho, p) + U_p(\rho, p)]/2$  coincides with predictions from the variational many-body theory using inputs constrained by nucleon-nucleon scattering data [42], and the isovector potential  $[U_n(\rho, p) - U_p(\rho, p)]/2$  also agrees with the momentum dependence of the Lane potential extracted from low-energy nucleon-nucleus scattering experiments [42] and recent neutron-nucleus data at 96 MeV [46].

The parameter  $x$  in Eq. (1) is introduced to mimic the various theoretical predictions for the density dependence of the nuclear symmetry energy  $E_{\text{sym}}(\rho)$ , which is defined via the parabolic approximation to the nucleon specific energy in an isospin asymmetric nuclear matter [9,10], i.e.,

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + \mathcal{O}(\delta^4). \quad (2)$$

With  $x = 1$ , for instance, the  $E_{\text{sym}}(\rho)$  is that predicted by a Hartree-Fock calculation using the Gogny effective interaction [40]. The parameters  $A_l(x)$  and  $A_u(x)$  are  $A_l(x) = -120.57 + 2Bx/(\sigma + 1)$  and  $A_u(x) = -95.98 - 2Bx/(\sigma + 1)$ , respectively. Including also the well-known contribution from nucleon kinetic energies, i.e.,  $E_{\text{sym}}^{\text{kin}}(\rho) = (2^{2/3} - 1)\frac{3}{5}E_F^0(\rho/\rho_0)^{2/3} \approx 13.0(\rho/\rho_0)^{2/3}$ , the density dependence of nuclear symmetry energy is shown in Fig. 1 for  $x = 1, 0, -1$ , and  $-2$ .

The interaction part of nuclear symmetry energy can be parametrized by  $E_{\text{sym}}^{\text{pot}}(\rho) = F(x)\rho/\rho_0 + [18.6 -$

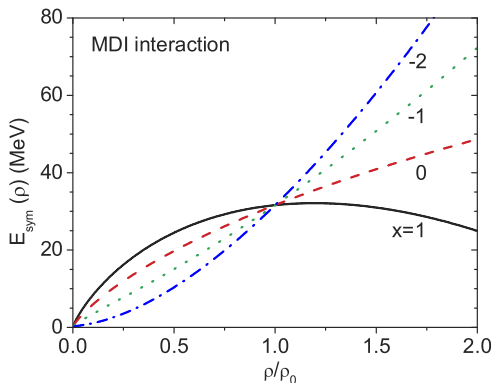


FIG. 1 (color online). Density dependence of nuclear symmetry energy for different values of the  $x$  parameter in Eq. (1).

$F(x)](\rho/\rho_0)^{G(x)}$ , with  $F(x)$  and  $G(x)$  given in Table I for  $x = 1, 0, -1$ , and  $-2$ . Also shown are other characteristics of the symmetry energy, including its slope  $L \equiv 3\rho_0(dE_{\text{sym}}/d\rho)_{\rho=\rho_0}$  and curvature  $K_{\text{sym}} \equiv 9\rho_0^2(d^2E_{\text{sym}}/d\rho^2)_{\rho=\rho_0}$  at  $\rho_0$ , as well as the isospin-dependent part  $K_{\text{asy}} \approx K_{\text{sym}} - 6L$  [20] of the isobaric incompressibility of asymmetric nuclear matter  $K(\delta) = K_0 + K_{\text{asy}}\delta^2$  [47,48].

For comparison, we have also constructed the following momentum-independent soft Bertsch–Kruse–Das Gupta (SBKD) potential that has  $K_0 = 200$  MeV [49] and exactly the same  $E_{\text{sym}}(\rho)$  as the MDI interaction:

$$U_{\text{SBKD}}(\rho, \delta, \tau) \equiv -356\rho/\rho_0 + 303(\rho/\rho_0)^{7/6} + 4\tau E_{\text{sym}}^{\text{pot}}(\rho) + [18.6 - F(x)][G(x) - 1](\rho/\rho_0)^{G(x)}\delta^2. \quad (3)$$

Isospin diffusion in heavy-ion collisions can in principle be studied by examining the average isospin asymmetry of the projectilelike residue in the final state. Since reactions at intermediate energies are complicated by preequilibrium particle emission and production of neutron-rich fragments at midrapidity, differences of isospin diffusions in mixed and symmetric systems are usually used to minimize these effects [30]. To study isospin diffusion in  $^{124}\text{Sn} + ^{112}\text{Sn}$  reactions at  $E = 50$  MeV/nucleon and an impact parameter of  $b = 6$  fm, we thus also consider the reaction systems  $^{124}\text{Sn} + ^{124}\text{S}$  and  $^{112}\text{Sn} + ^{112}\text{Sn}$  and  $^{124}\text{Sn} + ^{112}\text{Sn}$  at same energy and impact parameter as in Ref. [30]. The degree of isospin diffusion in the reaction  $^{124}\text{Sn} + ^{112}\text{Sn}$  is then measured by [50]

$$R_i = \frac{2X_{^{124}\text{Sn}+^{112}\text{Sn}} - X_{^{124}\text{Sn}+^{124}\text{Sn}} - X_{^{112}\text{Sn}+^{112}\text{Sn}}}{X_{^{124}\text{Sn}+^{124}\text{Sn}} - X_{^{112}\text{Sn}+^{112}\text{Sn}}}, \quad (4)$$

where  $X$  is the average isospin asymmetry  $\langle\delta\rangle$  of the  $^{124}\text{Sn}$ -like residue defined as the composition of nucleons with local densities higher than  $\rho_0/20$  and velocities larger than  $1/2$  the beam velocity in the c.m. frame. A density cut of  $\rho_0/8$  is found to give almost the same results. In the ideal case, the value of  $R_i$  ranges between 0.05 and 1 from complete mixing to full transparency.

As an example of present more realistic calculations, we show in Fig. 2 the time evolutions of  $R_i$  and average central density calculated with  $x = -1$  using both MDI and SBKD interactions. It is seen that the isospin diffusion

TABLE I. The parameters  $F$  (MeV),  $G$ ,  $K_{\text{sym}}$  (MeV),  $L$  (MeV), and  $K_{\text{asy}}$  (MeV) for different values of  $x$ .

$x$	$F$	$G$	$K_{\text{sym}}$	$L$	$K_{\text{asy}}$
1	107.232	1.246	-270.4	16.4	-368.8
0	129.981	1.059	-88.6	62.1	-461.2
-1	3.673	1.569	94.1	107.4	-550.3
-2	-38.395	1.416	276.3	153.0	-641.7

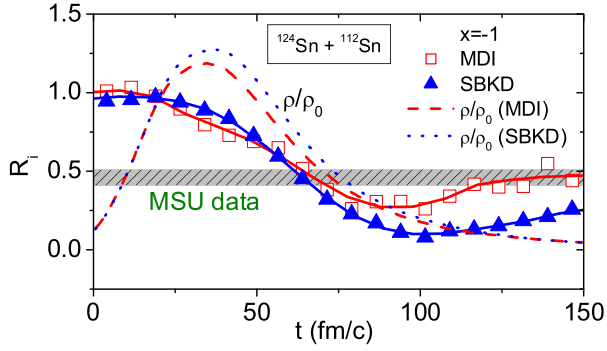


FIG. 2 (color online). Time evolutions of  $R_i$  and average central density for MDI and SBKD interactions with  $x = -1$ .

process occurs mainly from about 30 to 80 fm/c corresponding to average central density from about  $1.2\rho_0$  to  $0.3\rho_0$ . However, the value of  $R_i$  still changes slightly with time until after about 120 fm/c when projectilelike and targetlike residues are well separated. This is partly due to the fact that the isovector potential remains appreciable at low density as shown in Fig. 3, where the symmetry potential  $(U_n - U_p)/2\delta$  is shown as a function of momentum [panel (a)] or density [panel (b)] for the MDI interaction and as a function of density for the SBKD interaction [panel (c)]. Also, evaluating isospin diffusion  $R_i$  based on three reaction systems, which have different time evolutions for the projectile residue as a result of different total energies and numbers of nucleons, further contributes to the change of  $R_i$  at low density. For the two interactions considered here, the main difference between the values for  $R_i$  appears in the expansion phase when densities in the participant region are well below  $\rho_0$ . The experimental data from MSU are seen to be reproduced nicely by the MDI interaction with  $x = -1$ , while the SBKD interaction with  $x = -1$  leads to a significantly lower value for  $R_i$  as the strength of the momentum-independent symmetry potential is stronger (see Fig. 3), which has been shown to enhance the isospin diffusion [27,29,30].

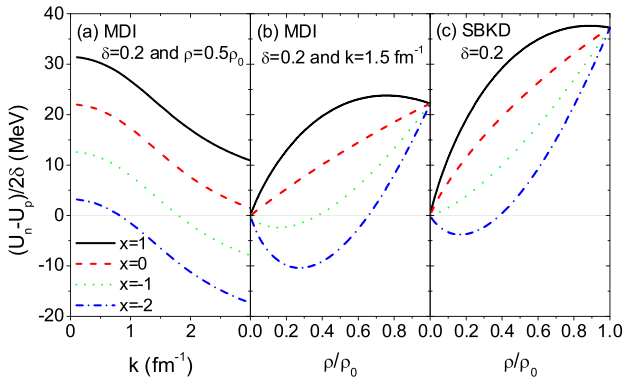


FIG. 3 (color online). Symmetry potential as a function of momentum (a) or density (b) with the MDI interaction and SBKD interaction (c).

To see how isospin diffusion depends on the density dependence of nuclear symmetry energy, we show in Fig. 4 the final saturated value for  $1 - R_i$ , which measures the degree of isospin diffusion, as a function of  $K_{\text{asy}}$  for both MDI and SBKD interactions. It is obtained by averaging the value of  $1 - R_i$  after 120 fm/c with error bars corresponding to its dispersion, whose magnitude is similar to the error band shown in Ref. [30] for the theoretical results from the BUU model. For the SBKD interaction without momentum dependence, the isospin diffusion decreases monotonically (i.e., increasing value for  $R_i$ ) with increasing strength of  $K_{\text{asy}}$  as the corresponding isovector potential is mostly positive and decreases with increasing stiffness of  $E_{\text{sym}}(\rho)$  in the whole range of considered  $x$  parameters. The isospin diffusion is reduced when the momentum-dependent interaction MDI is used as the momentum dependence weakens the strength of symmetry potential except for  $x = -2$ . As seen in Fig. 3, the symmetry potential in the MDI interaction has the smallest strength for  $x = -1$  as it is close to zero at  $k \approx 1.5 \text{ fm}^{-1}$  and  $\rho/\rho_0 \approx 0.5$ , and increases again with further hardening of the symmetry energy, e.g.,  $x = -2$ , when it becomes largely negative at all momenta and densities. The MDI interaction with  $x = -1$  thus gives the smallest degree of isospin diffusion among the interactions considered in the present study and reproduces the MSU data as already shown in Fig. 2.

The symmetry energy in the MDI interaction with  $x = -1$  is  $E_{\text{sym}}(\rho) = 13.0(\rho/\rho_0)^{2/3} + 3.7\rho/\rho_0 + 14.9(\rho/\rho_0)^{1.57} \approx 31.6(\rho/\rho_0)^{1.05}$ . It leads to a value of  $K_{\text{asy}} \approx -550 \text{ MeV}$  for the isospin-dependent part of the isobaric incompressibility of asymmetric nuclear matter, which should be compared to the published constraint of  $-566 \pm 1350 < K_{\text{asy}} < 139 \pm 1617 \text{ MeV}$  extracted earlier from studying giant monopole resonances [12].

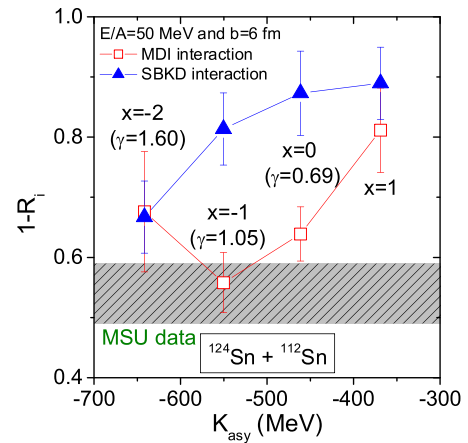


FIG. 4 (color online). The degree of isospin diffusion as a function of  $K_{\text{asy}}$  with the MDI and SBKD interactions.  $\gamma$  is the parameter for fitting the corresponding symmetry energy with  $E_{\text{sym}}(\rho) = 31.6(\rho/\rho_0)^\gamma$ .

It is worthwhile to mention that, if the isoscalar part of the SBKD potential in Eq. (3) is replaced with the momentum-dependent Yukawa interaction (MDYI) potential of Gale *et al.* [33], which has a similar  $K_0$  to those for the MDI and SBKD potentials, the resulting  $R_i = 0.37 \pm 0.07$  is much closer to that obtained with the MDI ( $R_i = 0.44 \pm 0.05$ ) than with the SBKD ( $R_i = 0.19 \pm 0.06$ ) interaction for  $x = -1$ . The strongly repulsive momentum-dependent isoscalar potential thus reduces the effect of isovector potential on the reaction dynamics. Results from the present study are therefore not much affected by the uncertainty in the momentum dependence of the isovector potential.

In summary, we have used an isospin- and momentum-dependent transport model to study isospin diffusion in heavy-ion collisions at intermediate energies. We find that the diffusion of isospins happens mostly during the expansion stage when the density is below normal nuclear matter density. The momentum dependence in the nuclear potential plays an important role and affects the sensitivity of the degree of isospin diffusion to the density dependence of nuclear symmetry energy. Aside from the uncertainty due to residual effects of preequilibrium particle emission and cluster formation, the present study shows that within the context of the mean-field interactions considered here recent experimental data from NSCL-MSU on isospin diffusion is consistent with a nuclear symmetry energy that is nearly linear in density at subnormal densities. This leads to a significantly constrained value of about  $-550$  MeV for the isospin-dependent part of the isobaric incompressibility of isospin asymmetric nuclear matter.

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\*Current address: Institute of Theoretical Physics, Shanghai Jiao Tong University, Shanghai 200240, China.

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