Experimental Demonstration of a Nondestructive Controlled-NOT Quantum Gate for Two Independent Photon Qubits

Zhi Zhao,[1,2] An-Ning Zhang,[1] Yu-Ao Chen,[1] Han Zhang,[1] Jiang-Feng Du,[1] Tao Yang,[1] and Jian-Wei Pan[1,2]
[1]*Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230027, People's Republic of China*
[2]*Physikaliches Institut, Universität Heidelberg, Philisophenweg 12, 69120 Heidelberg, Germany*
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Universal logic gates for two quantum bits (qubits) form an essential ingredient of quantum information processing. However, photons, one of the best candidates for qubits, suffer from a lack of strong nonlinear coupling, which is required for quantum logic operations. Here we show how this drawback can be overcome by reporting a proof-of-principle experimental demonstration of a nondestructive controlled-NOT (CNOT) gate for two independent photons using only linear optical elements in conjunction with single-photon sources and conditional dynamics. Moreover, we exploit the CNOT gate to discriminate all four Bell states in a teleportation experiment.

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The controlled-NOT (CNOT) or similar logic operations between two individual quantum bits (qubits) are essential for various quantum information protocols such as quantum communication [1–3] and quantum computation [4]. In recent years, certain quantum logic gates have been experimentally demonstrated, for example, in ion traps [5,6] and high-finesse microwave cavities [7]. These achievements open many possibilities for future quantum information processing (QIP) with single atoms. Another promising system for QIP is to use single photons. This is due to the photonic robustness against decoherence and the availability of single-qubit operation. However, it has been very difficult to achieve the necessary logic operations for two individual photon qubits since the physical interaction between photons is much too small.

Surprisingly, Knill, Laflamme, and Milburn (KLM) have shown that nondeterministic quantum logic operations can be performed using linear optical elements, additional photons (ancilla) and postselection based on the output of single-photon detectors [8]. The original proposal by KLM, though elegant, is not economical in its use of optical components. Various schemes have been proposed to reduce the complexity of the KLM scheme while improving its theoretical efficiency [9–11]. Remarkably, a recent scheme proposed by Nielsen [12] suggests that, without using the elaborate teleportation and Z-measurement error correction in the KLM scheme, any nontrivial linear optical gate that succeeds with finite probability is sufficient to obtain efficient quantum computation. Hence, this scheme significantly simplifies the experimental implementation of linear optical quantum computation (LOQC)

A crucial requirement in the schemes of LOQC is the so-called *classical feed-forwardability*, that is, it must be, in principle, possible to detect when the gate has succeeded by performing some appropriate measurement on ancilla photons [8,12]. This information can then be feed-forwarded for conditional future operations on the photonic qubits to achieve efficient LOQC.

Recently destructive CNOT operations have been realized using linear optical elements [13–16]. However, as they necessarily destroy the output state such logic operations are not classically feed-forwardable and have little practical significance. Fortunately, it has been suggested [10] that a destructive CNOT gate together with the quantum parity check can be combined with a pair of entangled photons to implement a nondestructive (conventional) CNOT gate that satisfies the feed-forwardability criterion.

In this paper, we present for the first time a proof-of-principle demonstration of a nondestructive CNOT gate for two independent photons, realizing the proposal of Pittman, Jacobs, and Franson [10]. The quality of such a CNOT gate is further demonstrated by discriminating all four Bell states in an experiment on quantum teleportation.

In our experiment, we consider qubits implemented as the polarization states of photons. We define the horizontal polarization state $|H\rangle$ as logic 1, and the vertical one $|V\rangle$ as logic 0. As shown in Fig. 1(a), one can achieve the desired nondestructive CNOT gate for photons 2 and 5 by performing a quantum parity check on photons 2 and 3 and a destructive CNOT operation on photons 4 and 5, where photons 3 and 4 are in the state $|\Psi^-\rangle_{34}$, which is one of the four Bell states:

$$\begin{aligned} |\Phi^\pm\rangle_{ij} &= \frac{1}{\sqrt{2}}(|H\rangle_i|H\rangle_j \pm |V\rangle_i|V\rangle_j), \\ |\Psi^\pm\rangle_{ij} &= \frac{1}{\sqrt{2}}(|H\rangle_i|V\rangle_j \pm |V\rangle_i|H\rangle_j). \end{aligned} \tag{1}$$

Here i and j index the spatial mode of the photons. Then, according to Ref. [10], the nondestructive CNOT gate for photons 2 and 5 can be accomplished, on the condition of detecting a $|-\rangle$ photon in mode $3'$ and a $|H\rangle$ photon in mode $4'$. The logic table of the CNOT operation is given by

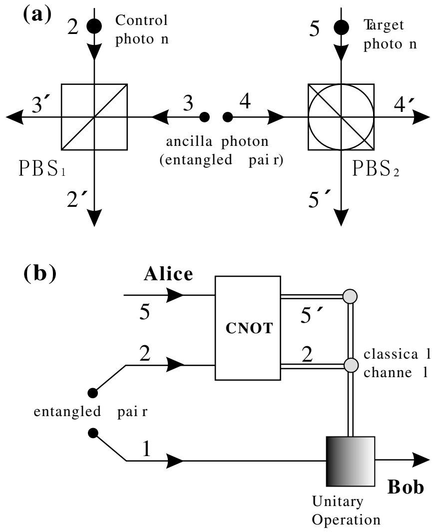

FIG. 1. (a) A nondestructive CNOT gate constructed by polarizing beam splitters (PBS), half-wave plates (HWP), and an ancilla entangled photon pair $|\Psi^-\rangle_{34}$ [10]. Conditioned on detecting a $|-\rangle$ photon in mode $3'$ and a $|H\rangle$ photon in mode $4'$, one can implement the CNOT operation between photons 2 and 5. (b) Quantum circuit for quantum teleportation based on a CNOT gate [22]. By using the CNOT operation, Alice can discriminate the four orthogonal Bell state simultaneously such that a complete teleportation can be achieved.

$|V\rangle|V\rangle \rightarrow |V\rangle|V\rangle$, $|V\rangle|H\rangle \rightarrow |V\rangle|H\rangle$, $|H\rangle|V\rangle \rightarrow |H\rangle|H\rangle$, and $|H\rangle|H\rangle \rightarrow |H\rangle|V\rangle$.

One immediate application of the proposed CNOT gate is that it can be used to generate entanglement between the control qubit and target qubit [17]. For example, by setting the control bit to be in the state $|-\rangle$ and the target qubit in the state $|H\rangle$, one can utilize the nondestructive CNOT gate to prepare photons 2 and 5 in the entangled state $|\Psi^-\rangle_{25}$.

Another important application is that the nondestructive CNOT gate can be used to simultaneously identify all four Bell states [17] in a quantum teleportation protocol [2]. For example, suppose photon 5, which Alice wants to teleport to Bob, is in an unknown polarization state $|\Phi\rangle_5 = \alpha|H\rangle_5 + \beta|V\rangle_5$, and the pair of the photons 1 and 2 shared by Alice and Bob is in the entangled state $|\Psi^-\rangle_{12}$ [Fig. 1(b)]. It is necessary to discriminate the four Bell states of photons 2 and 5, $|\Psi^\pm\rangle_{25}$ and $|\Phi^\pm\rangle_{25}$, in order to realize the complete quantum teleportation [2].

Under the CNOT operation, the four Bell states of photons 2 and 5 will evolve into one of the four orthogonal separable states:

$$|\Psi^\pm\rangle_{25} \rightarrow |\pm\rangle_{2'}|H\rangle_{5'}, \qquad |\Phi^\pm\rangle_{25} \rightarrow |\pm\rangle_{2'}|V\rangle_{5'}. \qquad (2)$$

Therefore, the crucial Bell-state measurement can be accomplished probabilistically by applying a nondestructive CNOT operation and performing a subsequent polarization analysis on photons 2 and 5. Depending on Alice's measurement results, Bob can then perform a unitary transformation, independent of $|\Phi\rangle_5$, on photon 1 to convert its state into the initial state of photon 5.

A schematic of the experimental setup used to demonstrate both the CNOT gate and the identification of the four Bell states required for quantum teleportation is shown in Fig. 2. To realize the CNOT gate, it is necessary to overlap photons 2 and 3 at the PBS_1 and photons 4 and 5 at the PBS_2. In the experiment, the PBS_2, i.e., the desired 45-degree oriented polarizing beam splitter, is accomplished by inserting one half-wave plate (HWP) in each of the two inputs and two outputs of an ordinary polarizing beam splitter. Note that, all four HWP were oriented at 22.5° with respect to the horizontal direction, which corresponds to a 45° polarization rotation. The good temporal overlap

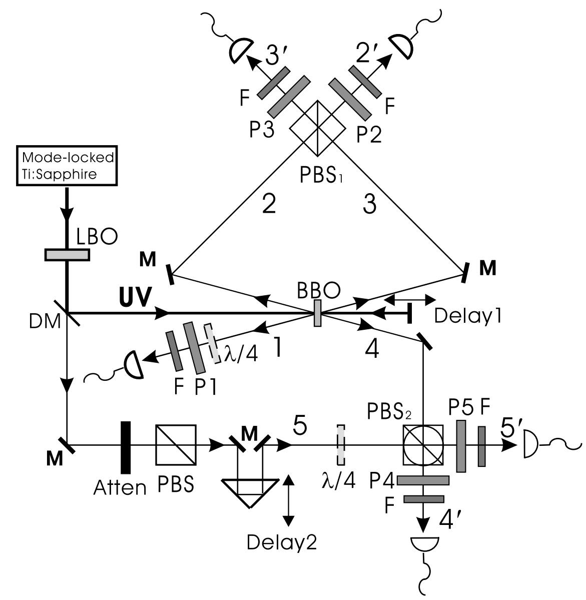

FIG. 2. The experimental setup. Femtosecond laser pulses ($\approx$200 fs, 76 MHz, 788 nm) first pass through a doubler LBO crystal (LiB_3O_5), then the mixed ultraviolet (UV) and infrared components were further separated by a dichroic beam splitter (DM). The reflected UV pulses passing through the BBO crystal twice generate two pairs of entangled photons, one in modes 1 and 2 and the other in modes 3 and 4, where both pairs are prepared in the $|\Psi^-\rangle$ state [23,24]. The observed coincidence rate of entangled photon pairs is about 2.4×10^4/s. The photons in modes 3 and 4 are used as the ancilla pair while the photon in mode 2 as the control qubit. The arbitrary polarization state of the control photon can be readily prepared by performing a polarization projection measurement on photon 1. In the experiment, in order to prepare the target qubit, i.e., photon 5, the transmitted near-infrared pulses are attenuated by penetrating an ultrafast laser output coupler and two polarizers (i.e., the Atten) to a weak coherent beam such that there is only a very small probability of containing a single-photon for each pulse (about 0.05 photon per pulse). The quarter-wave plate ($\lambda/4$) in modes 1 and 5 is used to prepare and analyze the input and output circular polarized states, respectively. The five polarizers P1, P2, …, P5 are used for polarization analysis.

was achieved by adjusting the two delay mirrors, Delay 1 and Delay 2. Experimentally, we first adjust the position of Delay 1 such that photons 2 and 3 arrive at PBS_1 simultaneously, and then adjust the position of Delay 2 to achieve the temporal overlap of photons 4 and 5 at PBS_2. Furthermore, the use of narrow band interference filters (F) with $\Delta\lambda_{FWHM} = 3$ nm for all five photons makes the photons at the same PBS indistinguishable [18]. The temporal overlap between photons 2 and 3 was verified by observing a four-particle interference visibility of 0.82 among photons 1, 2, 3, and 4 by removing the PBS_2; and the temporal overlap of photons 4 and 5 was verified by observing a three-particle interference visibility of 0.68 among photons 3, 4, and 5 by removing the PBS_1 [19,20].

To experimentally demonstrate that the CNOT gate has been successfully implemented, we first prepare the input control and target qubits in the following specific states $|H\rangle_2|H\rangle_5$, $|H\rangle_2|V\rangle_5$, $|V\rangle_2|H\rangle_5$ and $|V\rangle_2|V\rangle_5$. If the CNOT gate works properly, then, on the condition of detecting a $|-\rangle$ polarized photon in mode $3'$ and a $|H\rangle$ polarized photon in mode $4'$, the two qubits in modes $2'$ and $5'$ would be, respectively, in the states $|H\rangle_{2'}|V\rangle_{5'}$, $|H\rangle_{2'}|H\rangle_{5'}$, $|V\rangle_{2'}|H\rangle_{5'}$, and $|V\rangle_{2'}|V\rangle_{5'}$. After the nondestructive CNOT operation, the output components corresponding to the above specific input states, which were measured in the H/V basis, are shown in Fig. 3(a), respectively. The experimental fidelity of achieving the CNOT logic table is estimated to be 0.78 ± 0.05.

Second, to show the CNOT gate also works for an arbitrary superposition of the control qubit, we now prepare the control qubit in the state $\frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ and the target qubit in the state $|H\rangle$ to entangle these two independent photons. Then, after the CNOT operation the two output qubits would be in the state $\frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle)$, i.e., the Bell state $|\Psi^-\rangle$. To verify the expected Bell state has implemented successfully, we first measured the four possible polarization combinations of the control and target qubits in the H/V basis. The signal-to-noise ratio between the desired ($|H\rangle|V\rangle$ and $|V\rangle|H\rangle$) and unwanted ($|H\rangle|H\rangle$ and $|V\rangle|V\rangle$) were measured to be 4.2:1. This confirms that the $|H\rangle|V\rangle$ and $|V\rangle|H\rangle$ terms are the dominant components. Furthermore, to prove the two terms are indeed in a coherent superposition, we also perform a conditional coincidence measurement as a function of the orientation of polarizer P2 as polarizer P5 was fixed at $+45°$. As shown in Fig. 3(b), the experimental results of the polarization correlation exhibit an interference fringe with a visibility of 0.58 ± 0.09, which is consistent with the prediction of the interference fringe for the Bell state $|\Psi^-\rangle$.

The CNOT gate can be used not only to entangle two independent photons, it can also be used to disentangle two entangled photons. To demonstrate the latter, let us now exploit the CNOT gate to simultaneously discriminate all four Bell states in a quantum teleportation experiment. As described in Eq. (2), on the condition of detecting a $|-\rangle$ photon in mode $3'$ and a $|H\rangle$ photon in mode $4'$, the required joint Bell-state measurement can be achieved by performing a polarization measurement both on photon $2'$ in the $\pm$ basis and on photon $5'$ in the H/V basis. For example, registering a $|+\rangle_{2'}|H\rangle_{5'}$ coincidence implies a projection onto the Bell state $|\Psi^+\rangle_{25}$. In this way, we can identify all four Bell states. According to teleportation protocol [2], it is obvious that, if photons 2 and 5 are measured to be in the state $|\Psi^-\rangle_{25}$, then photon 1 will be projected into the state $\alpha|H\rangle + \beta|V\rangle$; if photons 2 and 5 are measured to be in the state $|\Psi^+\rangle_{25}$, then photon 1 will be left in the state $\alpha|H\rangle - \beta|V\rangle$. In these two cases, the two corresponding states of photon 1 would, in general, have a relative phase shift of π [21]. Similarly, for the projections onto the state $|\Phi^+\rangle_{25}$ or $|\Phi^-\rangle_{25}$, photon 1 will be correspondingly left in the state $\alpha|V\rangle - \beta|H\rangle$ or

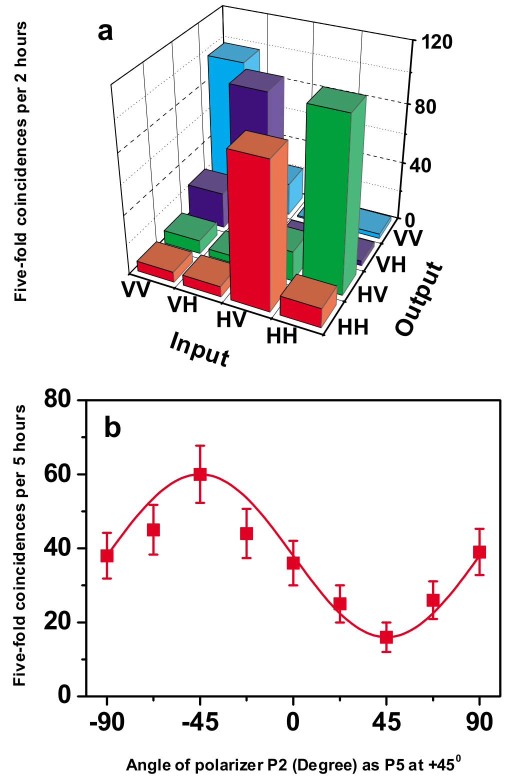

FIG. 3 (color online). (a) Experimental results for the CNOT operation in the H/V basis. When the input control and target qubits are in the following specific states $|H\rangle_2|H\rangle_5$, $|H\rangle_2|V\rangle_5$, $|V\rangle_2|H\rangle_5$, and $|V\rangle_2|V\rangle_5$, the output two qubits in modes $2'$ and $5'$ would be, respectively, in the states $|H\rangle_{2'}|V\rangle_{5'}$, $|H\rangle_{2'}|H\rangle_{5'}$, $|V\rangle_{2'}|H\rangle_{5'}$, and $|V\rangle_{2'}|V\rangle_{5'}$. (b) Experimental results for the CNOT operation when the control qubit was in the state $(1/\sqrt{2}) \times (|H\rangle - |V\rangle)$ and the target qubit in the state $|H\rangle$. After the CNOT operation, the two output qubits would be in the Bell state $|\Psi^-\rangle$. We perform a conditional measurement of polarization as a function of the orientation of polarizer P2 when polarizer P5 was fixed at $+45°$.

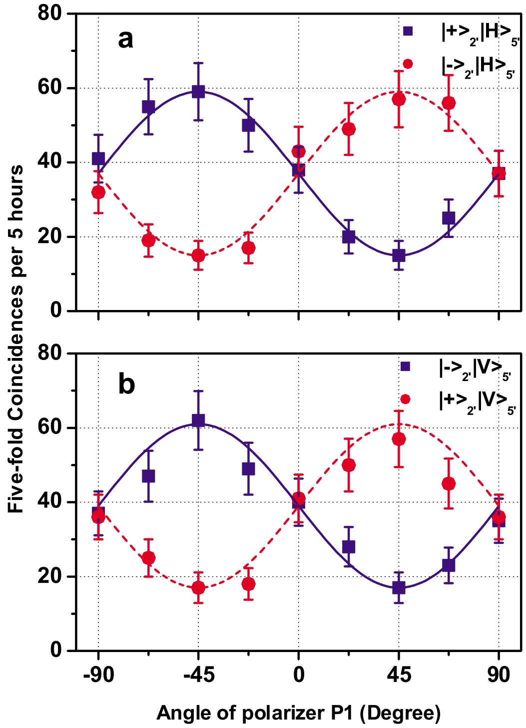

FIG. 4 (color online). The experimental results for quantum teleportation with complete Bell-state analysis. The data clearly confirm the expected π phase shift, hence demonstrating that the four Bell states have been identified successfully in the teleportation experiment.

$\alpha|V\rangle + \beta|H\rangle$. Again, the two states of photon 1 have a relative phase shift of π.

To experimentally verify the above analysis, we decided to teleport the left-hand circular polarization state $\frac{1}{\sqrt{2}} \times (|H\rangle - i|V\rangle)$ from photon 5 to photon 1. The output circular polarization states of photon 1 are analyzed by inserting a quarter-wave plate and a polarizer in front of the detector D1. As shown in Figs. 4(a) and 4(b), the fivefold coincidences are recorded as polarizer 1 was rotated. The experimental results are consistent with the prediction of the π phase shift, hence confirming that the four Bell states have been successfully discriminated. Following the same definition of Ref. [21], our experimental average fidelity of teleportation was estimated to be $F \approx 0.79 \pm 0.05$, which clearly surpasses the classical limit of $2/3$. Note that throughout the whole experiment the fidelities were obtained without performing any background subtraction.

Finally, we emphasize that in the above teleportation experiment we only verified the corresponding relation between the initial state of photon 5 and the final state of photon 1 after the Bell-state analysis is complete, but did not perform a conditional operation on photon 1 to convert its final state into the original state of photon 5. We plan to address this challenging task in a forthcoming experiment.

In summary, we have for the first time experimentally demonstrated a probabilistic nondestructive CNOT gate for two independent photons using only linear optics. Furthermore, we demonstrated that such a device can be used not only to entangle two independent photons, but also to discriminate all four Bell states for quantum teleportation. We believe that the methods developed for this experiment would have various novel applications in quantum information processing with linear optics.

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