Correlated Spontaneous Emission Laser as an Entanglement Amplifier

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We consider a two-photon correlated emission laser as a source of an entangled radiation with a large number of photons in each mode. The system consists of three-level atomic schemes inside a doubly resonant cavity. We study the dynamics of this system in the presence of cavity losses, concluding that the creation of entangled states with photon numbers up to tens of thousands seems achievable.

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Coherent atomic effects lie at the basis of many novel and interesting effects in quantum optics and lasers [1]. In this Letter we show that atomic coherence in a two-mode laser can generate fields that are entangled even in the presence of cavity losses. This leads to an entanglement amplifier.

The generation of macroscopic entangled states is receiving renewed interest. Schemes for entanglement between large atomic ensembles have been demonstrated [2]. However, the generation of macroscopic entangled states of photons remains an open question. Schemes based on the parametric down-conversion process driven by a strong pump pulse have been considered [3–6]. In this Letter we present a new class of entanglement amplifiers based on two-mode correlated *spontaneous* emission lasers (CEL) [7] involving three-level atoms interacting with two modes of the cavity and show that the two lasing modes are entangled.

In order to see clearly how a CEL can lead to an entangled state, we first recall that, in a quantum beat laser [8] or a Hanle-effect laser [9], a beam of three-level atoms in the "V" configuration interacts with two modes of the field. The upper levels $|a\rangle$ and $|b\rangle$ are initially prepared in a coherent superposition or are driven by a coherent field. We consider the simple case when an atom is in a superposition of upper states and there are no photons in the modes associated with the $|a\rangle \rightarrow |c\rangle$ and the $|b\rangle \rightarrow |c\rangle$ transitions; i.e., the initial state of the atom-field system is $(|a\rangle + |b\rangle)/\sqrt{2} \otimes |0, 0\rangle$. An atomic transition to the lower level $|c\rangle$ leads to the entangled state $(|1, 0\rangle + |0, 1\rangle)/\sqrt{2}$ of the field modes. It is thus clear that an amplified entangled state will be generated in a correlated emission laser. In this Letter we discuss different atomic configurations, such as a three-level atomic system in a cascade configuration or Raman configuration. For cascade atoms, the upper and lower levels are prepared in a coherent superposition and the photons are emitted in cascade transitions [10,11].

A question from a practical point of view is whether one can formulate criteria for measuring entanglement in a given system. We recall that formally a system is entangled if it is nonseparable; i.e., the density operator for the state ρ cannot be written as a convex combination of product states

$$\rho = \sum_j p_j \rho_j^{(1)} \otimes \rho_j^{(2)}$$

with $p_j \geq 0$ and $\sum_j p_j = 1$. In this Letter, we use the criterion proposed in [12] to verify the entanglement of the two modes of the field in a correlated emission laser. According to this criterion, a state of the system is entangled if the sum of the quantum fluctuations of two Einstein-Podolsky-Rosen (EPR)-like operators $\hat{u}$ and $\hat{v}$ of the two modes satisfy the inequality

$$(\Delta \hat{u})^2 + (\Delta \hat{v})^2 < 2. \tag{1}$$

Here

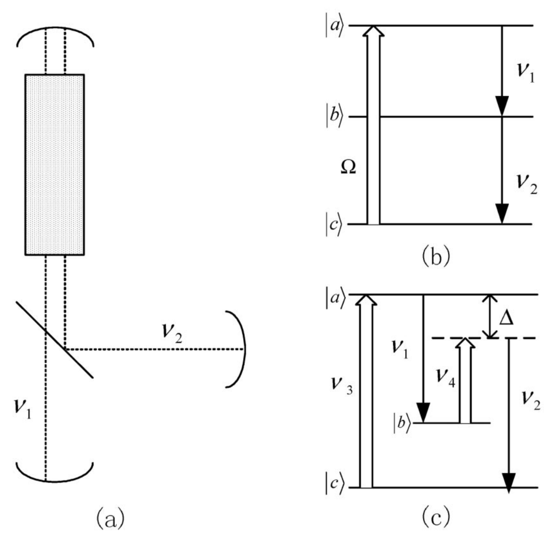

FIG. 1. (a) Schematics for the entanglement amplifier. Atomic medium is placed inside a doubly resonant cavity. (b) A three-level atomic system in a cascade configuration. The transitions between levels $|a\rangle - |b\rangle$ and levels $|b\rangle - |c\rangle$ at frequencies ν_1 and ν_2 are resonant with the cavity. The transition $|a\rangle - |c\rangle$ is dipole forbidden and can be induced by strong magnetic fields. (c) A Raman three-level atomic system where the fields of frequencies ν_3 and ν_4 are strong classical driving fields and the fields at frequencies ν_1 and ν_2 are resonant with the cavity modes.

$$\hat{u} = \hat{x}_1 + \hat{x}_2, \qquad \hat{v} = \hat{p}_1 - \hat{p}_2 \tag{2}$$

and $\hat{x}_j = (a_j + a_j^\dagger)/\sqrt{2}$ and $\hat{p}_j = (a_j - a_j^\dagger)/\sqrt{2}i$ (with $j = 1, 2$) are the quadrature operators for the two modes 1 and 2. For a general state, this is a sufficient criterion for entanglement and as shown in [12], for two-mode continuous variable Gaussian states, this becomes a necessary and sufficient criterion.

We consider a system in which atoms interact with two modes of the field inside a doubly resonant cavity [Fig. 1(a)]. We first consider three-level atoms in a cascade configuration [Fig. 1(b)]. Such a system has been discussed within the context of a correlated spontaneous emission laser. The dipole allowed transitions $|a\rangle - |b\rangle$ and $|b\rangle - |c\rangle$ are resonantly coupled with the two nondegenerate modes ν_1 and ν_2 of the cavity, while the dipole forbidden transition $|a\rangle - |c\rangle$ is induced by a semiclassical field (for example, by applying a strong magnetic field for a magnetic dipole allowed transition). We denote the Rabi frequency of this field by $\Omega e^{-i\phi}$. The interaction Hamiltonian (in the rotating wave approximation) for this system is given by

$$H_I = \hbar g_1(a_1|a\rangle\langle b| + a_1^\dagger|b\rangle\langle a|) + \hbar g_2(a_2|b\rangle\langle c| + a_2^\dagger|c\rangle \times\langle b|) - \frac{1}{2}\hbar\Omega(e^{-i\phi}|a\rangle\langle c| + e^{i\phi}|c\rangle\langle a|), \tag{3}$$

where $a_1(a_1^\dagger)$ and $a_2(a_2^\dagger)$ are the annihilation (creation) operators of the two nondegenerate modes of the cavities and g_1 and g_2 are the associated vacuum Rabi frequencies.

A cascade system may be hard to implement experimentally as the transition between the states $|a\rangle$ and $|c\rangle$ in Fig. 1(b) is dipole forbidden. A more convenient system is depicted in Fig. 1(c). Here atomic levels $|a\rangle$, $|b\rangle$, and $|c\rangle$ are coupled by four fields. The fields at frequencies ν_1 and ν_2 are resonant with the cavity modes and the fields of frequencies ν_3 and ν_4 with Rabi frequencies Ω_3 and Ω_4, respectively, are classical driving fields. The classical field Ω_3 is resonant with the $|a\rangle - |c\rangle$ transition whereas the field Ω_4 is detuned from the $|a\rangle - |c\rangle$ transition by an amount Δ. Similarly, the quantized field at the frequency ν_1 is assumed to be resonant with the $|a\rangle - |b\rangle$ transition and the field at frequency ν_2 is detuned from the $|a\rangle - |c\rangle$ by Δ. This system has recently been demonstrated experimentally and shows promise in applications to quantum memory in atomic systems [13,14]. The Hamiltonian of this system in the interaction picture is

$$H_I = -\frac{\hbar}{2}\Omega_3 e^{-i\phi_3}|a\rangle\langle c| - \frac{\hbar}{2}\Omega_4 e^{-i\phi_4}e^{i\Delta t}|a\rangle\langle b| + \hbar g_1' a_1|a\rangle\langle b| + \hbar g_2' a_2 e^{i\Delta t}|a\rangle\langle c| + \text{H.c.} \tag{4}$$

When the detuning Δ is sufficiently large, the Anti-Stocks Raman transition $|b\rangle - |a\rangle - |c\rangle$ can be effectively estimated as a single transition between levels $|b\rangle$ and $|c\rangle$, and the effective Hamiltonian of the whole system can be written as

$$H_{\text{eff}} = -\frac{\hbar}{2}\Omega_3 e^{-i\phi_3}|a\rangle\langle c| + \hbar g_1' a_1|a\rangle\langle b| + \hbar\frac{g_2'\Omega_4}{2\Delta}e^{i\phi_4}a_2|b\rangle\langle c| + \text{H.c.} \tag{5}$$

Equation (5) is of the same form as the Hamiltonian (3) for the cascade system. It is therefore clear that the atomic system of the form given in Fig. 1(c) can be used to implement a correlated emission laser [10,11] and a noise-free amplifier [15]. Here we discuss its application as an entanglement amplifier.

The master equation of the system in the configuration of Fig. 1(b) can be obtained from the Hamiltonian (3) by using the standard methods of laser theory. We consider only the linear theory. We assume that the atoms are injected in the cavity in the lower level $|c\rangle$ at a rate r_a. The resulting equation for the reduced density operator for the cavity field modes is [11,16].

$$\begin{aligned}\dot{\rho} = &-[\beta_{11}^* a_1 a_1^\dagger \rho + \beta_{11}\rho a_1 a_1^\dagger - (\beta_{11} + \beta_{11}^*)a_1^\dagger \rho a_1 + \beta_{22}^* a_2^\dagger a_2 \rho + \beta_{22}\rho a_2^\dagger a_2 - (\beta_{22} + \beta_{22}^*)a_2 \rho a_2^\dagger] - [\beta_{12}^* a_1 a_2 \rho \\ &+ \beta_{21}\rho a_1 a_2 - (\beta_{12}^* + \beta_{21})a_2 \rho a_1]e^{i\phi} - [\beta_{21}^* a_1^\dagger a_2^\dagger \rho + \beta_{12}\rho a_1^\dagger a_2^\dagger - (\beta_{12} + \beta_{21}^*)a_1^\dagger \rho a_2^\dagger]e^{-i\phi} \\ &- \kappa_1(a_1^\dagger a_1 \rho - 2a_1 \rho a_1^\dagger + \rho a_1^\dagger a_1) - \kappa_2(a_2^\dagger a_2 \rho - 2a_2 \rho a_2^\dagger + \rho a_2^\dagger a_2),\end{aligned} \tag{6}$$

where we have included the cavity damping terms in the usual way (we have assumed that the two cavity modes are coupled to two independant vacuum reservoirs here), with κ_1 and κ_2 being the cavity decay rates of mode 1 and mode 2, respectively. The coefficients β_{11}, β_{22}, β_{12}, and β_{21} are given by

$$\beta_{11} = \frac{g_1^2 r_a}{4}\frac{3\Omega^2}{(\gamma^2 + \Omega^2)(\gamma^2 + \frac{\Omega^2}{4})}, \tag{7}$$

$$\beta_{22} = g_2^2 r_a \frac{1}{\gamma^2 + \Omega^2}, \tag{8}$$

$$\beta_{12} = g_1 g_2 r_a \frac{i\Omega}{\gamma(\gamma^2 + \Omega^2)}, \tag{9}$$

$$\beta_{21} = \frac{g_1 g_2 r_a}{4}\frac{i\Omega(\Omega^2 - 2\gamma^2)}{\gamma(\gamma^2 + \Omega^2)(\gamma^2 + \frac{\Omega^2}{4})}. \tag{10}$$

We have assumed, for simplicity, that the atomic decay rate

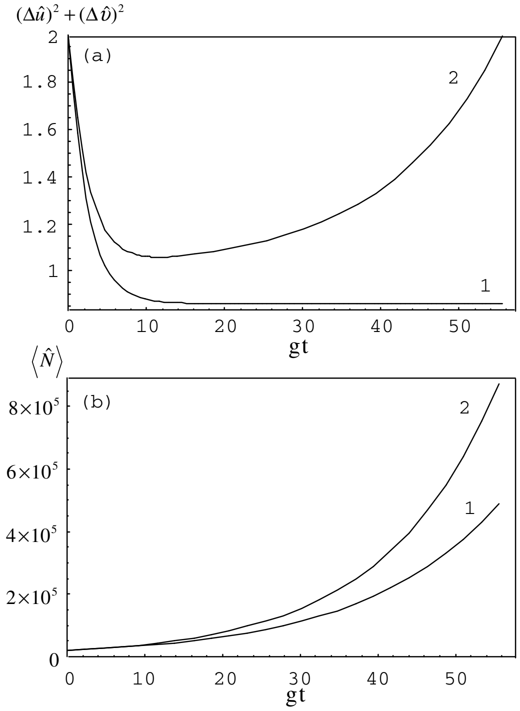

FIG. 2. (a) Time development of $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$, and (b) $\langle\hat{N}\rangle$ for initial coherent states $|100, -100\rangle$ in terms of the normalized time gt. Various parameters are $r_a = 22$ kHz, $g = g_1 = g_2 = 43$ kHz, $\kappa = \kappa_1 = \kappa_2 = 3.85$ kHz, $\gamma = 20$ kHz, and $\Omega = 400$ kHz. In these figures, 1 and 2 represent the results for the parametric case and the general case, respectively. Parameters are chosen such that they correspond to the micromaser experiments [17].

γ is the same for all the three atomic levels. Here the terms proportional to β_{11} and β_{22} correspond to the emission from level $|a\rangle$ and absorption from level $|c\rangle$, respectively, and the terms proportional to β_{12} and β_{21} correspond to atomic coherence generated by the coupling field Ω.

We now discuss how the above system leads to entanglement amplification. We first analyze the case when Ω is much greater than γ and then proceed to the general case with arbitrary Ω.

In the limit when $\Omega \gg \gamma$, we have from Eqs. (7)–(10) that

$$\beta_{11} \sim 0, \beta_{22} \sim 0, \beta_{12} \approx \beta_{21} \sim i g_1 g_2 r_a \frac{1}{\gamma\Omega}. \tag{11}$$

Under these conditions Eq. (6) simplifies considerably and we obtain (with $i\alpha = \beta_{12} = \beta_{21}$)

$$\begin{aligned}\dot{\rho} = & -i\alpha(\rho a_1 a_2 - a_2 \rho a_1)e^{i\phi} - i\alpha(\rho a_1^\dagger a_2^\dagger - a_1^\dagger \rho a_2^\dagger)e^{-i\phi} \\ & + i\alpha(a_1 a_2 \rho - a_2 \rho a_1)e^{i\phi} + i\alpha(a_1^\dagger a_2^\dagger \rho - a_1^\dagger \rho a_2^\dagger)e^{-i\phi} \\ & - \kappa_1(a_1^\dagger a_1 \rho - 2a_1 \rho a_1^\dagger + \rho a_1^\dagger a_1) \\ & - \kappa_2(a_2^\dagger a_2 \rho - 2a_2 \rho a_2^\dagger + \rho a_2^\dagger a_2). \end{aligned} \tag{12}$$

This equation describes a parametric oscillator in the parametric approximation. We can calculate the time evolution of the quantum fluctuations of the EPR operators $\hat{u}$ and $\hat{v}$ and the mean photon numbers from Eq. (12). In particular, we calculate the time evolution of the various moments involved in the quantities $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$ and the total photon numbers $\langle\hat{N}\rangle = \langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle$. The resulting expressions are

$$\begin{aligned}\left[(\Delta\hat{u})^2 + (\Delta\hat{v})^2\right](t) = & \{[(\Delta\hat{u})^2 + (\Delta\hat{v})^2](0) \\ & - \frac{2\kappa}{\alpha+\kappa}\}e^{-2(\alpha+\kappa)t} + \frac{2\kappa}{\alpha+\kappa}\end{aligned} \tag{13}$$

$$\begin{aligned}\langle\hat{N}\rangle(t) = & \left\{\left[\langle\hat{N}\rangle(0) - \frac{\alpha^2}{\kappa^2-\alpha^2}\right]\cosh(2\alpha t)\right. \\ & - \left[\frac{\alpha\kappa}{\kappa^2-\alpha^2} + \langle a_1 a_2 + a_1^\dagger a_2^\dagger\rangle(0)\right] \\ & \left.\times \sinh(2\alpha t)\right\} \times e^{-2\kappa t} + \frac{\alpha^2}{\kappa^2-\alpha^2},\end{aligned} \tag{14}$$

where we have taken the phase of the driven field to be $\phi = -\pi/2$ since only under this special phase the positive exponential terms in $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$ can be canceled out and ensure that this quantity does not grow with time.

It is clear that, for any initial state of the field, the quantity $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$ becomes smaller as time evolves and becomes less than 2 after some time. For large time when $(\alpha+\kappa)t \gg 1$, we have $(\Delta\hat{u})^2 + (\Delta\hat{v})^2 = 2\frac{\kappa}{\alpha+\kappa} < 2$, i.e., the entanglement criterion is satisfied. Thus the system evolves into an entangled state and remains entangled unless the entanglement is destroyed by some other dissipation channels. We show below that the results based on the parametric approximation are valid for small values of gt only and higher order contributions in γ/Ω tend to wipe out the entanglement as time progresses. Thus, for the general case, the entanglement remains only for a limited period of time.

The other important quantity is the mean number of photons in the two modes. If we consider the large time behavior of the total photon number, we can neglect the negative exponent terms in the sinh and cosh functions in Eq. (14). We then have $\langle\hat{N}\rangle(t) = [\langle\hat{N}\rangle(0) - \langle a_1 a_2 + a_1^\dagger a_2^\dagger\rangle(0) + \alpha/(\alpha-\kappa)]\exp[2(\alpha-\kappa)t] - \alpha^2/(\alpha^2-\kappa^2)$. This shows that, for any initial states of the two modes, the total mean photon number increases exponentially for sufficiently large t provided $\alpha > \kappa$. The condition for the growth of mean photon numbers for small t involves the initial states of the field. For example, for the initial coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ for the two modes, this will very much depend on the phase of the coherent amplitude of these two modes. The condition $d\langle\hat{N}\rangle(t)/dt > 0$, for $t \geq 0$, leads us to the following inequality $\alpha\langle a_1 a_2 + a_1^\dagger a_2^\dagger\rangle(0) + \kappa\langle\hat{N}\rangle(0) < 0$ that is $\alpha(\alpha_1\alpha_2 + \alpha_1^*\alpha_2^*) + \kappa(|\alpha_1|^2 + |\alpha_2|^2) < 0$. To satisfy this inequality, the best choice is that, in addition to $\alpha > \kappa$, we also have $\alpha_1\alpha_2 = -|\alpha_1\alpha_2|$.

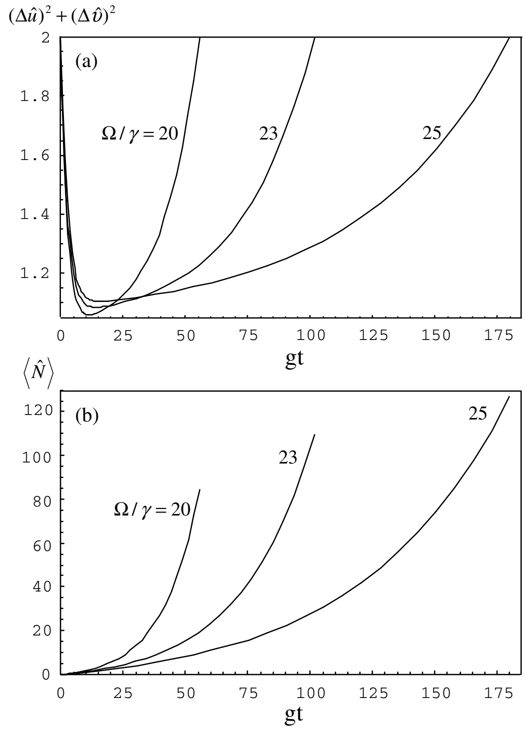

FIG. 3. (a) Time development of $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$ and (b) $\langle\hat{N}\rangle$ for initial vacuum states for the two modes with $\Omega/\gamma = 20$, 23, 25. Curves in (b) are truncated when $(\Delta\hat{u})^2 + (\Delta\hat{v})^2 = 2$ and the state is not necessarily entangled. The chosen parameters are $r_a = g = \gamma$ and $\kappa/g = 0.001$.

We now return to the general case. The various field moments required in the inequality (1) can be obtained from Eq. (6). The resulting expressions are complicated and we do not reproduce them here.

In Figs. 2 and 3 we show the time development of $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$ and $\langle\hat{N}\rangle$ for different Ω/γ and fixed κ/g. In Fig. 2, we plot these quantities for an initial coherent state with 10^4 photons in each mode. The choice of the phase for the coherent amplitude is such that the condition $\alpha_1\alpha_2 = -|\alpha_1\alpha_2|$ is satisfied. The parameter values are such that they correspond to the micromaser experiments in Garching [17]. We find that the two states remain entangled for a long time. The parametric results are valid only for $gt < 10$. The agreement between the parametric results with the exact results for the mean photon number $\langle\hat{N}\rangle$ is valid for a longer range. We also see that an increase in the photon numbers by almost 40 fold is possible. In Fig. 3, we plot $(\Delta\hat{u})^2 + (\Delta\hat{v})^2$ and $\langle\hat{N}\rangle$ for initial vacuum states for the two modes. Again, the entanglement is retained for a large number of photons. The time scale for the two modes to remain entangled increases as the Rabi frequency of the driving field is increased.

In summary, we have studied a correlated emission laser system in which a macroscopic entangled state between two modes of the radiation field can be built. The entanglement does not depend on the initial state of the fields. Our analysis indicates that such macroscopic entangled states can be realized as suggested above by placing the atomic medium inside the doubly resonant cavity. Another possibility is a system wherein atoms with long lived states pass through the cavity one at a time such that there is at most one atom inside the cavity at a given time in the presence of the classical driving fields. This corresponds to experimental arrangements such as those used in the micromaser experiments [17,18].

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