Muon Decay: Measurement of the Transverse Polarization of the Decay Positrons and its Implications for the Fermi Coupling Constant and Time Reversal Invariance

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The two transverse polarization components P_{T_1} and P_{T_2} of the e^+ from the decay of polarized μ^+ have been measured as a function of the e^+ energy. Their energy averaged values are $\langle P_{T_1}\rangle = (6.3 \pm 7.7 \pm 3.4) \times 10^{-3}$ and $\langle P_{T_2}\rangle = (-3.7 \pm 7.7 \pm 3.4) \times 10^{-3}$. From the energy dependence of P_{T_1} and P_{T_2} the decay parameters η, η'' and α'/A, β'/A are derived, respectively. Assuming only one additional coupling besides the dominant $V-A$ interaction one gets improved limits on η, β'/A, and the scalar coupling constant g_{RR}^S: $\eta = (-2.1 \pm 7.0 \pm 1.0) \times 10^{-3}$, $\beta'/A = (-1.3 \pm 3.5 \pm 0.6) \times 10^{-3}$, $\mathrm{Re}\{g_{RR}^S\} = (-4.2 \pm 14.0 \pm 2.0) \times 10^{-3}$, and $\mathrm{Im}\{g_{RR}^S\} = (5.2 \pm 14.0 \pm 2.4) \times 10^{-3}$.

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Muon decay, $\mu^+ \rightarrow \overline{\nu}_\mu e^+ \nu_e$, as a purely leptonic process, provides a precise source of information on the charged current weak interaction. It can be described by the most general, local four-fermion point interaction Hamiltonian [1]. It contains ten complex coupling constants to be determined by experiment. The observables can be expressed in terms of a chiral Hamiltonian in charge-changing form characterized by fields of definite handedness [2]. The matrix element is given by [3,4]

$$\mathcal{M} = \frac{4G_\mathrm{F}}{\sqrt{2}} \sum_{\gamma,\varepsilon,\mu} g_{\varepsilon\mu}^{\gamma} \langle \overline{e}_\varepsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle \overline{\nu}_m | \Gamma_\gamma | (\mu)_\mu \rangle. \quad (1)$$

The index $\gamma = S, V, T$ labels the type of interaction (four-scalar, four-vector, four-tensor), while the indices $\varepsilon, \mu = R, L$ indicate the chirality (R for right-handed, L for left-handed) of the particle spinors. The standard model predicts $g_{LL}^V = 1$, with all other couplings being zero. In fact, with a selected set of μ decay experiments it has been possible to *derive* a lower limit for g_{LL}^V and upper limits for the absolute values of all other nine coupling constants [3].

In a continuing effort to search for interactions beyond the standard model, we have measured the transverse polarization of the e^+ from the decay of polarized μ^+. Although the e^+ are polarized mainly longitudinally ($P_\mathrm{L} = 0.998 \pm 0.045$) [5], the experimental limit ΔP_L still allows for sizeable transverse polarization components $\boldsymbol{P}_{T_1}$ and $\boldsymbol{P}_{T_2}$. $\boldsymbol{P}_{T_1} \equiv P_{T_1} \cdot \hat{\boldsymbol{x}}_1$ lies in the plane defined by the μ polarization $\boldsymbol{P}_\mu$ and the e^+ momentum $\boldsymbol{k}_e$, $\boldsymbol{P}_{T_2} \equiv P_{T_2} \cdot \hat{\boldsymbol{x}}_2$ is perpendicular to this plane. Their unit vectors $\hat{\boldsymbol{x}}_1$ and $\hat{\boldsymbol{x}}_2$, respectively, are given by

$$\hat{\boldsymbol{x}}_2 = \frac{\boldsymbol{k}_e \times \boldsymbol{P}_\mu}{|\boldsymbol{k}_e \times \boldsymbol{P}_\mu|}, \qquad \hat{\boldsymbol{x}}_1 = \hat{\boldsymbol{x}}_2 \times \frac{\boldsymbol{k}_e}{|\boldsymbol{k}_e|}. \quad (2)$$

Both P_{T_1} and P_{T_2} are functions of the e^+ energy E and depend on the decay parameters η, η'' and α'/A, β'/A, respectively[4,6–8]. A precise value of η is needed for a model-independent determination of G_F. P_{T_2}, α'/A, and β'/A all are zero if time reversal invariance holds. These parameters have been determined previously resulting in the energy averaged polarization components $\langle P_{T_1}\rangle = (16 \pm 23) \times 10^{-3}$, $\langle P_{T_2}\rangle = (7 \pm 23) \times 10^{-3}$, and in $\eta = (11 \pm 85) \times 10^{-3}$ [9].

Here we report first results of an improved experiment. Figure 1 shows a schematic view of the experimental setup to measure the transverse polarization from the angular distribution of annihilation photons. All the major parts of

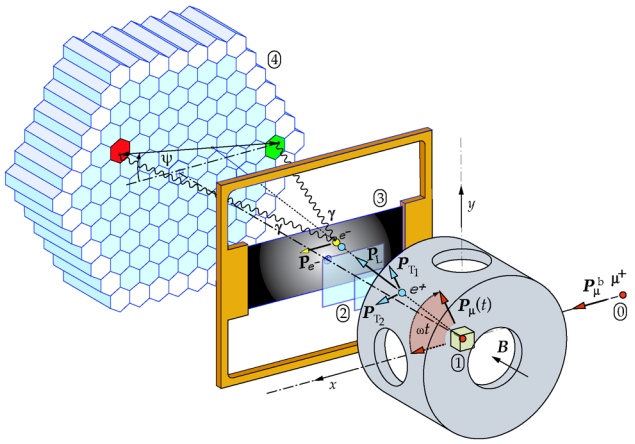

FIG. 1 (color online). Schematic view of the experimental setup. 0: Burst of polarized muons (angular frequency ω, polarization P_μ^b). 1: Be stop target and precession field $\boldsymbol{B}$. 2: Two plastic scintillation counters selecting decay positrons. 3: Magnetized Vacoflux 50™ foil serving as polarization analyzer. 4: Array of 127 BGO scintillators to detect the two γ's from e^+ annihilation-in-flight.

the previous experiment have been replaced by newly designed equipment in order to increase the event rate and reduce the systematic errors. This is described in detail elsewhere [10]. The experiment was performed at the μE1 beamline at the Paul Sherrer Institute (PSI). A longitudinally polarized μ^+ beam ($P_\mu^{\mathrm{b}} \approx 91\%$) enters a beryllium stop target with bunches every 19.75 ns. The polarization $\boldsymbol{P}_\mu(t)$ of the stopped muons precesses in a homogeneous magnetic field ($B = 373.6 \pm 0.4$ mT) with the same angular frequency ω as the accelerator radio frequency. This ensures that $\boldsymbol{P}_\mu(t) \parallel \boldsymbol{P}_\mu^{\mathrm{b}}$ for each newly arriving μ^+ bunch. Because of the burst width of 3.9 ns (FWHM) the polarization $P_\mu(0)$ of the stopped μ^+ is reduced to $(82 \pm 2)\%$. A system of drift chambers (not shown) and two thin plastic scintillator counters T_0 and T_1 select decay e^+ emitted into the direction of $\boldsymbol{B}$. A 1 mm thick magnetized Vacoflux 50$^{\mathrm{TM}}$ foil (49% Fe, 49% Co, 2% V) in the central region with its polarized e^- ($P_{e^-} = 7.2\%$) serves as polarization analyzer. The two γ's from e^+ annihilation in flight with the polarized e^- are selected by an array of 91 interior $Bi_4Ge_3O_{12}$ (BGO) crystals with plastic veto counters in front of them to reject charged particles. The outer layer of 36 BGOs assists in an efficient collection of the deposited energy. Valid events are selected by using the correlation between the γ energies and their opening angle. The intensity distribution of the two γ's has roughly the shape of the figure eight with a maximum in the direction of the bisector of $\boldsymbol{P}_{\mathrm{T}}(t)$ and the e^- polarization $\boldsymbol{P}_{e^-}$ [11]. The precession of $\boldsymbol{P}_\mu(t)$ implies a precession of $\boldsymbol{P}_{\mathrm{T}}(t)$, while $\boldsymbol{P}_{e^-}$ remains constant in time. Thus the intensity distribution of the γ's also precesses with frequency ω. For any given pair ij of BGO detectors we therefore ideally expect a signal for the normalized annihilation rate $N_{ij}(t)$ in the form

$$N_{ij}(t) = 1 + a_{ij}\cos(\omega t + \delta_0) + b_{ij}\sin(\omega t + \delta_0), \quad (3)$$

where t denotes the time the e^+ traverses counter T_0 and δ_0 an instrumental phase common to all time spectra. The events are distributed in a time window of 39.5 ns total width, corresponding to two periods of the accelerator radio frequency. The Fourier coefficients a_{ij} and b_{ij} contain the complete information of the transverse positron polarization. In order to extract these coefficients the following two main obstacles have to be overcome:

(1) The e^+ are emitted with energy E and transverse polarization vector $\{P_{\mathrm{T}_1}(E), P_{\mathrm{T}_2}(E)\}$. During their passage through the Be target, counters, and part of the magnetized foil they lose energy, change their direction, and their polarization precesses. Because of the finite depth of the Be target and due to the statistical nature of the energy loss, e^+ of different original energy and polarization will end up with the same energy E' at the moment of annihilation. This is the energy as detected by measuring the two annihilation quanta. From the resulting transverse polarization $\{P_1(E'), P_2(E')\}$, finally, the four decay parameters η, η'', α'/A, and β'/A have to be deduced.

(2) The harmonic signal modulating the annihilation rate as given in Eq. (3) is accompanied by two other signals of the same frequency, but of different origin: One signal is due to the finite acceptance of the setup. The experiment selects e^+ hitting a fiducial region of the analyzer foil (180×160 mm^2) at a polar angle $\chi < 14°$ and an azimuthal angle α with respect to the z and x axis, respectively. For $\chi > 0°$ there is a small [muon spin rotation (μSR)] decay asymmetry modulated by the precessing $\boldsymbol{P}_\mu(t)$.

The other signal is a residual effect due to the differential nonlinearity of the time-to-digital converter (TDC) and the number of stopped μ^+ during one precession period, which is modulated by the finite width of the muon burst and the decay of a small fraction of the μ^+. These two effects have been combined because they are independent of the rest of the experimental setup, particularly of P_{e^-} and the μSR decay asymmetry mentioned above.

The dependence of the three different contributions on the various observables plays an essential part in extracting the transverse polarization from the measured annihilation rate. The separation of the different contributions proceeds as follows: In order to minimize and control possible fluctuations of the Fourier coefficients the 36.4×10^6 total valid annihilation events were subdivided into ten pairs of data sets of approximately equal size. Each pair λ, ($\lambda = 1, \ldots, 10$), consists of two consecutive measurements with opposite polarization P_{e^-}.

The Fourier coefficients $(a_{\mathrm{res}}^\lambda, b_{\mathrm{res}}^\lambda)$ of the residual effect were calculated by averaging the coefficients of pairs of measurements, which cancels effects of a possible transverse e^+ polarization, as well as averaging over α, which cancels the μSR asymmetry. The coefficients are found to be statistically consistent. Their mean value is $a_{\mathrm{res}} = (+7.44 \pm 0.24) \times 10^{-3}$, $b_{\mathrm{res}} = (-3.62 \pm 0.24) \times 10^{-3}$.

The coefficients $(a_{\mu\mathrm{SR}}, b_{\mu\mathrm{SR}})$ are proportional to $\sin\chi$ and change sign for $\alpha \rightarrow \alpha + 180°$ (see Fig. 2). Therefore, it would be quite easy to eliminate this effect simply by accepting all events without regard to α. It turns out, however, that actually it is essential for the experiment:

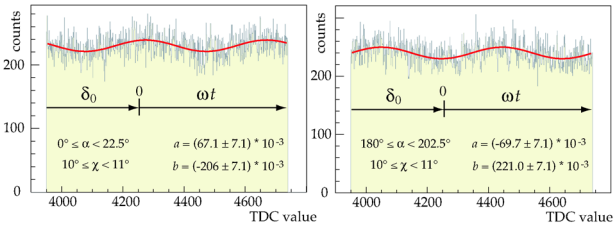

FIG. 2 (color online). Measured time spectra for two selected (α, χ) bins. The polar angular range is equal for both diagrams whereas the azimuthal angles differ by 180°. The solid lines and the coefficients a and b have been obtained from Fourier analysis. The origin of the ωt axis is determined by the maximum of the rate for $\alpha = 0$.

The correlation between the maximum of the e^+ flux at a given angle α with the precessing $\boldsymbol{P}_\mu(t)$ allows us to determine the phase angle δ_0 precisely. This establishes the calibration of the TDC channels with respect to $\boldsymbol{P}_\mu$ which in turn allows to define the two polarization components P_1 and P_2 *at the moment of annihilation*. To achieve the best possible precision on δ_0, events were sorted into 16 sectors α_ν, starting at 0°, and eight bins χ_κ, $4° \leq \chi_\kappa < 12°$. For each (ν, κ) the Fourier coefficients were determined. After subtracting the residual coefficients $(a_{\rm res}, b_{\rm res})$, dividing the result by $\sin\chi_\kappa$, and summing over κ one obtains the coefficients a_ν, b_ν. Figure 3 shows the results without (left) and with correction for the residual effect (right). Also shown is the statistical mean of all 16 coefficients (dots near center). With the properly determined coefficients (right) we obtain $\langle a_{\mu\rm SR}\rangle = (+0.9 \pm 2.0) \times 10^{-3}$, $\langle b_{\mu\rm SR}\rangle = (-1.2 \pm 2.0) \times 10^{-3}$ which demonstrates the high internal symmetry of the experiment. From these coefficients we obtain $\delta_0 = 272.4° \pm 0.5°$.

The transverse polarization, finally, is obtained for each pair λ from the *difference* of the Fourier coefficients with opposite P_{e^-}. This eliminates the residual contribution. Furthermore, we sum over all events from different detector pairs i, j, but identical azimuthal angle of orientation ψ, which averages over α. This cancels the μSR decay asymmetry. The remaining coefficients contain only the signal of the transverse polarization [12]:

$$N_{\gamma\gamma}(t) = 1 + P_\mu P_{e^-}[P_1 G + P_2 H]\cos(\omega t + \delta_0) + P_\mu P_{e^-}[P_1 H - P_2 G]\sin(\omega t + \delta_0) \qquad (4)$$

The analyzing powers G and H were derived from Ref. [13]. In contrast to P_1 and P_2 they depend not only on the sum $u \equiv E_{\gamma_1} + E_{\gamma_2} = E'$ of the gamma energies, but also on $v \equiv |E_{\gamma_1} - E_{\gamma_2}|$ as well as on ψ. Therefore the evaluation was subdivided even further, resulting in coefficients $(a^\lambda_{uv\psi}, b^\lambda_{uv\psi})$. In each of these bins $P_1(E')$ and $P_2(E')$ were determined. For a given (λ, u) it was first verified that the values obtained for the various combinations of (v, ψ) were statistically consistent before combining them. The same procedure was applied with respect to the data sets λ. The resulting two polarization components P_1 and P_2 are shown as a function of E' in Fig. 4. All values are found to be consistent with 0.

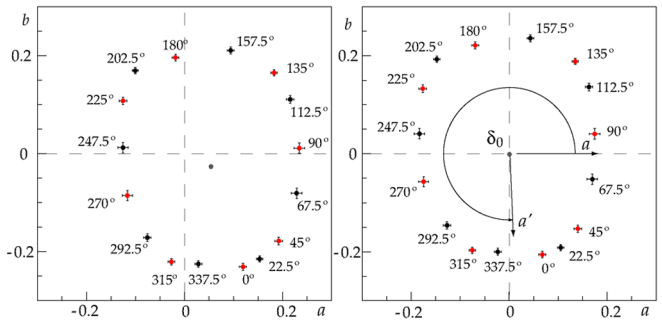

FIG. 3 (color online). Fourier coefficients (a_ν, b_ν) obtained for each of the 16 sectors of the azimuthal angle α. The left picture is without and the right picture is with the correction for the residual effect. The statistical mean of the coefficients of all 16 sectors is also shown as a dot near the center. After the correction it is well centered. The rotation of the axes (a, b) by δ_0 to (a', b') eliminates the arbitrary phase and establishes the correlation of the signal with $\boldsymbol{P}_\mu(t)$.

The polarization components $P_{\rm T_1}$ and $P_{\rm T_2}$ *at the moment of decay* depend linearly on the decay parameters η, η'', and α'/A, β'/A, respectively [4,6–8]. To derive these parameters from the measured distributions of $P_1(E')$ and $P_2(E')$ we have used a novel approach: With a modified GEANT3 [12,14] program we have generated four theoretical distributions $[P_1(E'), P_2(E')]$ in such a way that each was calculated by assigning a nonzero value to one of the four parameters and zero to the other three. Each *individual* e^+ produced was fully polarized with respect to $\boldsymbol{P}_{\rm T_1}$ for η and η'', and to $\boldsymbol{P}_{\rm T_2}$ for α'/A and β'/A, while the complete ensemble of e^+ reproduced just the theoretical polarization as a function of E. The motion of these e^+, the deflection in the foil's magnetic field and the annihilation in the magnetized foil as well as the acceptance by the BGO detector array were then simulated, taking full account of the po-

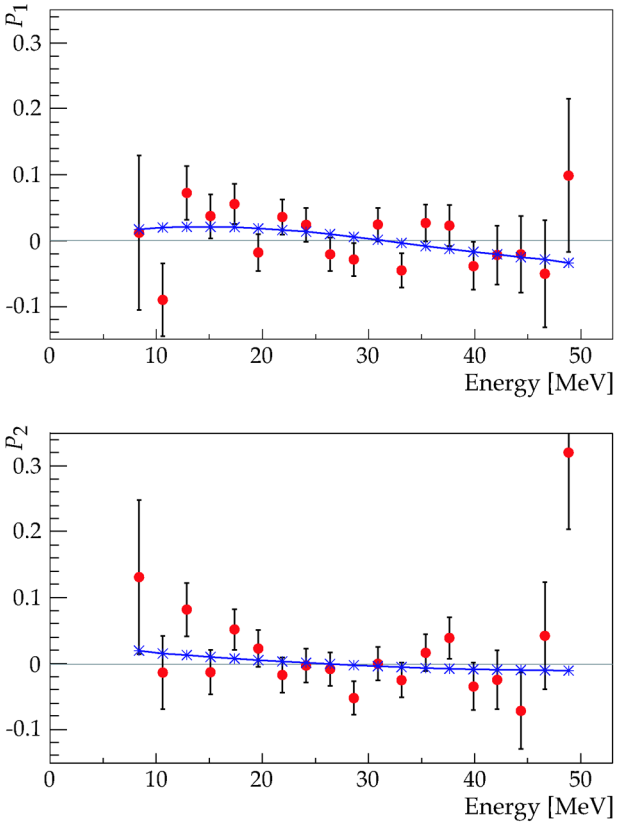

FIG. 4 (color online). Transverse positron polarization components P_1 and P_2 as a function of the e^+ energy E' *at the moment of annihilation*. The curves are the fit to the data with the values obtained for η, η'', α'/A, and β'/A.

TABLE I. $V-A$ values and experimental results. All values, except χ^2/d.o.f., in units of 10^{-3}. The correlation coefficients ρ_{ij} are all compatible with zero except the two coefficients listed. The errors are statistical and systematic.

	$V-A$	General analysis	Restricted analysis
η	0	$71 \pm 37 \pm 5$	$-2.1 \pm 7.0 \pm 1.0$
η''	0	$105 \pm 52 \pm 6$	$\equiv -\eta$
α'/A	0	$-3.4 \pm 21.3 \pm 4.9$	$\equiv 0$
β'/A	0	$-0.5 \pm 7.8 \pm 1.8$	$-1.3 \pm 3.5 \pm 0.6$
$\rho_{\eta\eta''}$		946	...
$\rho_{\alpha'\beta'}$		-893	...
χ^2/d.o.f.		46.2/33	50.3/35
$\mathrm{Re}\{g^S_{RR}\}$	0	...	$-4.2 \pm 14.0 \pm 2.0$
$\mathrm{Im}\{g^S_{RR}\}$	0	...	$5.2 \pm 14.0 \pm 2.4$
$\langle P_{\mathrm{T}_1}\rangle$	-3	$6.3 \pm 7.7 \pm 3.4$	
$\langle P_{\mathrm{T}_2}\rangle$	0	$-3.7 \pm 7.7 \pm 3.4$	

larization dependence. The measured data were then fitted to a *linear* combination of the four simulated distributions. The extraction of the values and errors for η, η'', α'/A, and β'/A is straightforward. The main systematic errors are due to uncertainties in the energy calibration, the energy loss and the background. Radiative corrections to the polarization are small [15,16] except at very low energies and have been neglected.

Table I shows the results of the general and of a restricted analysis. The average polarization components $\langle P_{\mathrm{T}_1}\rangle$ and $\langle P_{\mathrm{T}_2}\rangle$ have been calculated from the values of η, η'', and α'/A, β'/A, respectively.

Based on the most general four-fermion contact interaction (''general analysis'') the parameter η is given by [8]

$$\eta = \frac{1}{2}\mathrm{Re}\{g^V_{LL}g^{S*}_{RR} + g^V_{RR}g^{S*}_{LL} + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) + g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})\}. \quad (5)$$

With $g^V_{LL} \approx 1$, and all other $g^\gamma_{\varepsilon\mu} \approx 0$ [3], one can simplify Eq. (5) considerably by neglecting all terms quadratic in nonstandard couplings. This amounts to assuming one additional coupling beyond $V-A$. Then only two independent parameters remain (''restricted analysis''):

$$\eta = \mathrm{Re}\{g^S_{RR}/2\}, \qquad \beta'/A = -\mathrm{Im}\{g^S_{RR}/4\}. \quad (6)$$

Here, g^S_{RR} is a scalar coupling with right-handed μ and e.

The Fermi coupling constant G_F is generally derived assuming an exclusive $V-A$ interaction, which amounts to setting $\eta = 0$. However, G_F depends on η [7,8]:

$$G_\mathrm{F} \approx G^{V-A}_\mathrm{F}\left(1 - 2\eta\frac{m_e}{m_\mu}\right), \quad (7)$$

where m_e/m_μ is the mass ratio of electron and muon. Taking η into account increases the relative error $\Delta G_\mathrm{F}/G_\mathrm{F}$ from 9×10^{-6} to 360×10^{-6} (general analysis) respective to 68×10^{-6} (restricted analysis). By extending the analysis beyond the general four-fermion local interaction P_{T_1} is found to depend also on a possible tensor interaction with coupling g^T_{RR} [17]. It leads to the same energy dependence of P_{T_1} as η and therefore cannot be distinguished from η by this measurement.

We note that our results on α'/A, β'/A (and deduced from these, $\langle P_{\mathrm{T}_2}\rangle$ and $\mathrm{Im}\{g^S_{RR}\}$) are the only experimental data sensitive to the violation of time reversal invariance for a purely leptonic *reaction*. In contrast to the violation of time reversal invariance in the neutral kaon system [18], a T-odd observable in muon decay would be due to an interference between two couplings with different phase angles and thus be an unambiguous signal of new physics beyond the standard model.

The results of our polarization measurements are in agreement with the standard model. The limits on $\langle P_{\mathrm{T}_1}\rangle$ and $\langle P_{\mathrm{T}_2}\rangle$ have been improved by a factor of 3, the limits on the four decay parameters η, η'', α'/A and β'/A by a factor of 2. No evidence for the violation of time reversal invariance has been found.

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