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Influence of High Magnetic Fields on the Superconducting Transition of One-Dimensional Nb and MoGe Nanowires

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The effects of a strong magnetic field on superconducting Nb and MoGe nanowires with diameter ~10 nm have been studied. We have found that the Langer-Ambegaokar-McCumber-Halperin (LAMH) theory of thermally activated phase slips is applicable in a wide range of magnetic fields and describes well the temperature dependence of the wire resistance, over 11 orders of magnitude. The field dependence of the critical temperature, T_c, extracted from the LAMH fits is in good quantitative agreement with the theory of pair-breaking perturbations that takes into account both spin and orbital contributions. The extracted spin-orbit scattering time agrees with an estimate $\tau_{s.o.} \simeq \tau(\hbar c / Ze^2)^4$, where τ is the elastic scattering time and Z is the atomic number.

The problem of superconductivity in one-dimensional (1D) systems attracts much attentions since it involves such fundamental phenomena as macroscopic quantum tunneling, quantum phase transitions, and environmental effects [1–7]. It is expected that a strong magnetic field can be used to control these phenomena. Indeed, the microscopic theory predicts that a magnetic field, acting on a superconducting condensate, lifts the time reversal symmetry of the spin and orbital states of paired electrons and suppresses the critical temperature, T_c [8,9]. A strong enough field destroys superconductivity. The magnetic field pair-breaking effects were studied in depth in two- and zero-dimensional systems, i.e., thin films [10] and nanograins [11]. However, an experimental verification of the pair-breaking effects in 1D superconductors is long overdue.

A distinct feature of 1D superconductors is the absence of the phase coherence. Because of fluctuations the amplitude of the order parameter has a finite probability to reach zero at some point along the wire, allowing the phase of the order parameter to slip by 2π [12]. The theory of thermally activated phase slips was developed by Langer, Ambegaokar, McCumber, and Halperin (LAMH) [13]. However, the effect of the magnetic field on the phase slippage process is not established. It is also unknown whether the magnetic field can change the relative contributions of quantum and thermally activated phase slips in thin wires [3,4].

In this Letter we study the effects of the magnetic field on the phase slippage rate and the critical temperature of thin wires. It is found that the LAMH provides a good description for 1D superconductors in magnetic fields up to 11 T. The dependence of the critical temperature on the magnetic field, $T_c(B)$, agrees well with the theory of pair-breaking perturbations that takes into account both spin and orbital contributions [8,9]. This is our main result. No

significant contribution of quantum phase slips has been detected in the studied samples.

The samples were made by sputter coating of suspended fluorinated carbon nanotubes with Nb or $Mo_{79}Ge_{21}$. Transport measurements were performed in a He-3 cryostat, as described previously [2,4,5]. The magnetic field was oriented perpendicular to the wire and parallel to the thin film electrodes connected in series with the wire.

A series of resistance versus temperature $R(T)$ curves measured at different magnetic fields is shown in Fig. 1. For each curve, a resistance drop at higher temperature corresponds to a superconducting transition in the film electrodes. The resistance value immediately below the drop is taken as the wire normal resistance R_N. The second resistive transition corresponds to the development of
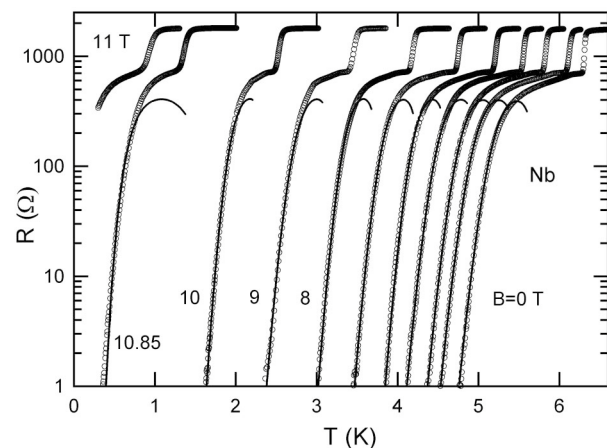

FIG. 1. Resistance versus temperature dependence for a Nb nanowire (the thickness is 8 nm, the normal resistance is $R_N = 700\ \Omega$, and the length is $L = 120$ nm). Each $R(T)$ curve is measured in a fixed magnetic field; some fields are indicated. Solid lines show the fits to the LAMH theory.

superconductivity in the wire. With increasing magnetic field, both transitions shift to lower temperatures.

To analyze these data we employ the LAMH expression for zero-bias resistance:

$$R_{\mathrm{LAMH}}(T) = \frac{\pi\hbar^2\Omega}{2e^2kT}e^{-\Delta F/k_BT}, \qquad (1)$$

where $\Delta F = (8\sqrt{2}/3)(H_c^2/8\pi)A\xi$ is the energy barrier, $\Omega = (L/\xi)(\Delta F/k_BT)^{1/2}(1/\tau_{\mathrm{GL}})$ is the attempt frequency, and $\tau_{\mathrm{GL}} = [\pi\hbar/8k_B(T_c - T)]$ is the Ginzburg-Landau (GL) relaxation time, L is the length of the wire, A is its cross-sectional area, and ξ is the GL coherence length. Following Ref. [3] we express the energy barrier as

$$\Delta F(T) \approx 0.83[L/\xi(0)](R_q/R_N)k_BT_c(1 - T/T_c)^{3/2}, \quad (2)$$

where $R_q = h/4e^2 = 6.45\ \mathrm{k\Omega}$ is the resistance quantum for Cooper pairs and $\xi(0)$ is defined by $\xi(T) = \xi(0)(1 - T/T_c)^{-1/2}$. Taking into account the contribution of quasiparticles, the total resistance is given as $R = (R_N^{-1} + R_{\mathrm{LAMH}}^{-1})^{-1}$. In our fitting procedure we use two adjustable parameters, T_c and $\xi(0)$.

The LAMH fits are shown as solid lines in Fig. 1, for various magnetic fields. Although the LAMH theory is derived for $B = 0$, we find that the resistance agrees with the LAMH fits very well, for both Nb and MoGe samples, even in high fields up to $\sim$11 T.

Our extension of the LAMH theory to high magnetic fields requires an explanation: For a phase slip to occur in a wire, the system needs to overcome an energy barrier that is a product of the condensation energy density $(H_c^2/8\pi)$ in a volume of a phase slip $A\xi$. It can be shown within the GL theory that in magnetic field the condensation energy density goes to zero as $[1 - T/T_c(B)]^2$, where $T_c(B)$ is the field-dependent critical temperature. The coherence length varies as $\xi(0, B)[1 - T/T_c(B)]^{-1/2}$ and diverges at $T_c(B)$. Because the temperature dependence of both the condensation energy and the coherence length has the same form as in zero field, we expect Eqs. (1) and (2) to be applicable in magnetic fields also. The observed agreement with the experiment suggests that the mechanism of the phase slippage in 1D wires is not changed by magnetic field.

To test the LAMH theory in a nonlinear regime, we performed measurements of the voltage-current $V(I)$ dependencies at high bias currents for some MoGe wires [Fig. 2(a)]. We observe that at high currents the wires undergo a transition into the resistive state. This transition is smooth at high temperatures and it is jumpwise, with some hysteresis, at low temperatures [4,14] [Fig. 2(a)]. A closer inspection of the data revealed that even at low temperatures there exists a small nonlinear voltage variation at currents slightly lower than the switching current [Fig. 2(b)]. The $V(I)$ curves remain qualitatively unchanged in magnetic field.
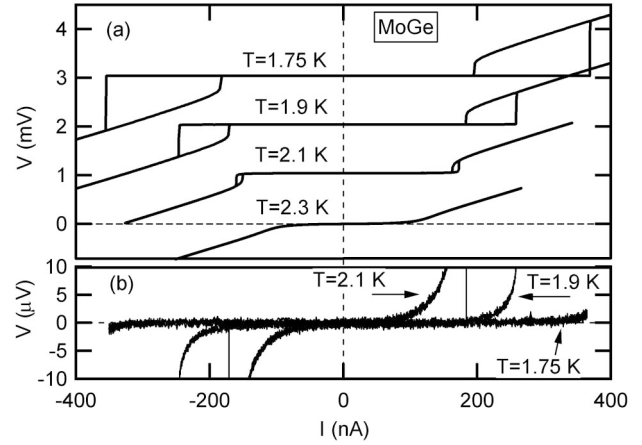

FIG. 2. (a) Voltage versus current dependence at indicated temperatures for a MoGe nanowire (thickness 7 nm, $R_N = 4.3\ \mathrm{k\Omega}$, and $L = 190$ nm) at $B = 0$. The curves are vertically shifted by 1 mV for clarity. (b) The same data at a magnified scale.

In order to investigate in more detail the resistive tails observed slightly below the switching current we measured the differential resistance dV/dI versus bias current I as shown in Fig. 3. It is clear that at all temperatures the dV/dI versus I data follow exponential dependence, which is expected from the LAMH expression $dV/dI = R(T) \times \cosh(I/I_0)$, where $R(T)$ is the zero-bias resistance given by Eq. (1) and $I_0 = 4ekT/h$. We fit the data with the above expression and extract two adjustable parameters $R(T)$ and I_0. Experimental values of I_0 are close to the theoretical value $I_0 = 4ekT/h$ (Fig. 4, inset). We speculate that the observed small upward deviation of I_0 might be due to the fact that some fraction of the bias current is carried by nonequilibrium quasiparticles. Such quasiparticles are
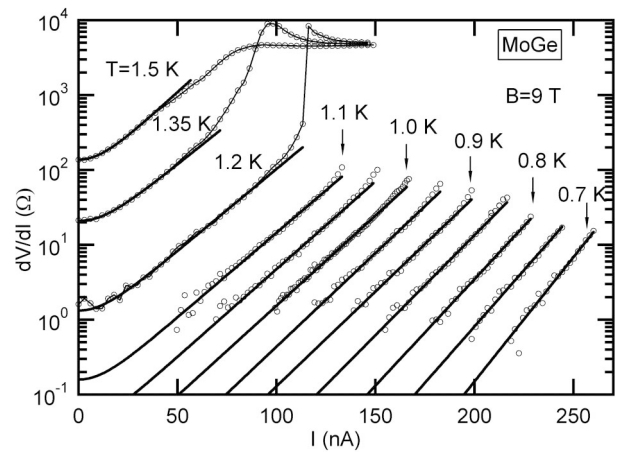

FIG. 3. Differential resistance versus bias current at indicated temperatures for the MoGe nanowire (thickness 7 nm, $R_N = 3.9\ \mathrm{k\Omega}$, and $L = 150$ nm) in a fixed magnetic field $B = 9$ T. Solid lines are fits to the LAMH expression $dV/dI = R(T) \times \cosh(I/I_0)$.
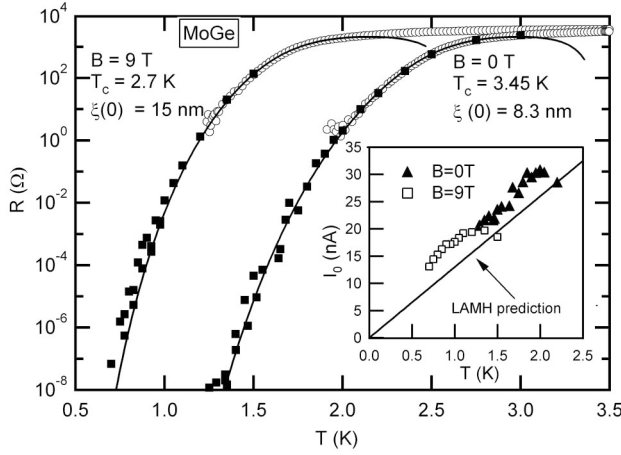
FIG. 4.   Resistance versus temperature for the MoGe nanowire (same as in Fig. 3) in magnetic fields 0 and 9 T. Open circles represent zero-bias-current measurements and black squares indicate the resistance values obtained from the fit of the non-linear portion of dV/dI curves (Fig. 3). Solid lines are the fits to the LAMH theory [Eq. (1)]. Extracted fitting parameters T_c and $\xi(0)$ are indicated. The inset shows the experimental dependence of the parameter I_0 on temperature (solid and open symbols) and the theoretical value $I_0 = 4ekT/h$ (solid line).
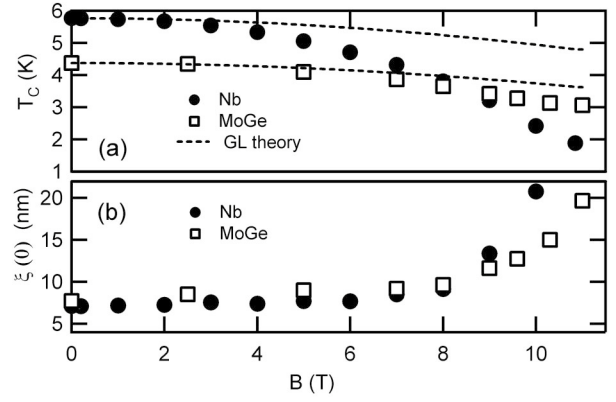

FIG. 5.   Adjustable parameters used in Eq. (1) for the LAMH fits of the type shown in Fig. 1. (a) The critical temperature versus magnetic field $T_c(B)$ is shown for the Nb wire (same as in Fig. 1) and for a MoGe wire. (b) The GL coherence length versus magnetic field. The parameters of the MoGe wire are that thickness is 9 nm, $R_N = 7.5$ kΩ, and $L = 460$ nm. The theoretical Ginzburg-Landau dependence is indicated by dashed lines.

generated by the phase slips and are not taken into account within the LAMH. A detailed theoretical analysis is needed for further understanding of the $I_0(T)$ behavior.

In Fig. 4 we superimpose the zero-bias resistance data and the resistance data obtained from the fits to the non-linear portion of dV/dI vs I curves. These two sets of data appear mutually consistent. The LAMH fit (solid lines in Fig. 4) gives an excellent description of all resistance data in a range of *11 orders of magnitude*. We also find good agreement with the LAMH for data taken in magnetic field $B = 9$ T (Fig. 4). It is therefore concluded that the resistance in studied nanowires is determined by thermally activated phase slips even in high magnetic fields.

In Fig. 5 we plot the extracted parameters $T_c(B)$ and $\xi(0, B)$ versus magnetic field. The coherence length increases very slowly in low fields and starts to grow more rapidly at $B > 8$ T. We consider now the critical temperature $T_c(B)$, which is the main focus of this work. Although the initial decrease of T_c agrees with the variation predicted by the GL theory [Fig. 5(a)], it is clear that the GL phenomenology is not sufficient to account for the observed $T_c(B)$ dependence. One possible reason is that the GL theory does not take into account the spin pair-breaking effect and the coefficients of the GL theory can change in the high field regime. Thus we have to use the exact theory of pair breaking [8]. In both Nb and MoGe nanowires superconductivity persists to magnetic fields that are larger than the paramagnetic limit, B_p [T] $= 1.84T_c$ [K] [9], (10.6 T for Nb and 8.1 T for MoGe). Since the superconductivity is not fully suppressed at such fields, we conclude that the effect of magnetic field on the spin part

of the Cooper pair is reduced by the spin-orbit scattering, as expected for materials with high atomic numbers. If the spin-orbit scattering is sufficiently strong, the transition into the normal sate is continuous. This allows us to take into account both the spin and the orbital effects on T_c in an implicit relation of the theory of pair-breaking perturbations [9]

$$\ln\frac{T_c(B)}{T_c(0)} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\alpha_o + \alpha_s}{2\pi k_B T_c(B)}\right), \quad (3)$$

where $\psi(z)$ is the digamma function and $\alpha_o = 2De^2\langle A^2\rangle/\hbar c^2$ and $\alpha_s \approx \tau_{s.o.}e^2\hbar B^2/2m^2c^2$ are the orbital and the spin pair-breaker strength parameters, D is the diffusion coefficient, and $\tau_{s.o.}$ is the spin-orbit scattering time. To find the vector potential averaged over the cross-sectional area of a wire, $\langle A^2\rangle$, we use the expression for a cylinder in perpendicular magnetic field, $\langle A^2\rangle = B^2d^2/16$, where d is the diameter of the wire.

By solving Eq. (3) numerically the theoretical dependence of the normalized critical temperature versus the normalized magnetic field is obtained. In Fig. 6 this dependence is compared to the experimental values of $T_c(B)/T_c(0)$ [same data as in Fig. 5(a)]. Since both pair breakers, α_o and α_s, have a quadratic dependence on B, only one adjustable parameter is required, which is the critical field of the wire at zero temperature, B_{cw}. The best fits are obtained by choosing $B_{cw} = 11.7$ T for Nb and 16.5 T for MoGe (Fig. 6).

To compare orbital and spin pair-breaking contributions we introduce an orbital critical field (B_{co}) and a spin critical field (B_{cs}), defined as fields needed to suppress superconductivity if only one of the two pair-breaking mechanisms is present. These fields are obtained from
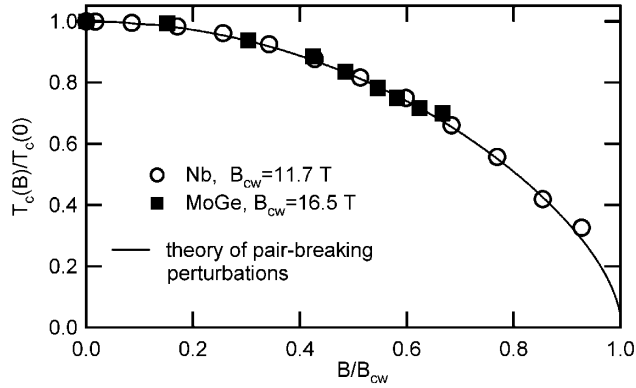
FIG. 6. Normalized critical temperature versus normalized magnetic field for the Nb and MoGe wires. Critical magnetic fields (B_{cw}) for both wires are indicated. The solid line is a fit to the theory of pair-breaking perturbation [Eq. (3)].

equations $2\alpha_o = 2\alpha_s = 1.76kT_c(0)$ [9]. The total critical field is then $B_{cw} = (B_{co}^{-2} + B_{cs}^{-2})^{-1/2}$. Using BCS and GL relations the orbital critical field can be written as $B_{co} = 0.53\Phi_0/d\xi(0)$, where Φ_0 is the flux quantum. The value of the coherence length $\xi(0)$ is known from the LAMH fit in zero field (7.1 nm for Nb and 7.7 nm for MoGe). Taking as d the thickness of the deposited material (corrected for oxidation for the MoGe wire [5]) we estimate $B_{co} \approx 22$ T for both wires. From B_{co} and experimental total critical field B_{cw} we determine the spin critical fields, $B_{cs} \approx 14$ T for Nb and $B_{cs} \approx 25$ T for MoGe. Thus the spin and orbital pair breakers have comparable strengths.

From the B_{cs} we estimate the spin-orbit scattering time as $\tau_{s.o.} = 2.3 \pm 0.5 \times 10^{-13}$ s for the Nb wire and $\tau_{s.o.} = 5 \pm 3 \times 10^{-14}$ s for the MoGe wire. The MoGe result is considerably different from the value $\tau_{s.o.} \simeq 1.3 \times 10^{-12}$ s obtained for thin MoGe films from weak localization measurements [15]. With such a $\tau_{s.o.}$ value the superconductivity in the MoGe wire would be completely suppressed at $B_{cs} \simeq 5$ T, contrary to our observation. On the other hand, we can use the formula $\tau_{s.o.} \simeq \tau(\hbar c/Ze^2)^4$ given in Ref. [16]. With elastic scattering time $\tau \simeq 6 \times 10^{-16}$ s [15] this gives a shorter spin-orbit scattering time 8×10^{-14} s, that agrees with our result. The latter estimate also works well for the Nb nanowire. Here we have elastic time $\tau = \ell/v_F \simeq 1.9 \times 10^{-15}$ s ($v_F = 0.62 \times 10^{-8}$ cm/s [17], $\ell = 1.2$ nm [4]), and so $\tau_{so} \simeq 2.4 \times 10^{-13}$ s, again in agreement with the experimental value. Thus we conclude that the pair-breaking theory combining spin and orbital contributions gives an accurate quantitative prediction for the suppression of the critical temperature of homogeneous 1D superconducting wires.

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