Magnetoelectric Anisotropy in Diffusive Transport

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In this Letter we prove the existence of a new general diffusive transport phenomenon in crossed electric and magnetic fields: magnetoelectric anisotropy. For the specific case of diffusive electrical transport, we present a relativistic model to quantify this effect and present experimental evidence for its existence.

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It is often stated that any two-terminal electrical resistance can have only an even magnetic field dependence [1–4]. These statements are claimed to be based on Onsager's general relations for the symmetry properties of diffusive transport [5]. However, at variance with these claims, we have recently shown both theoretically and experimentally for the special case of chiral electrical conductors that a linear magnetic field contribution to the two-terminal resistance exists, the sign of which depends on the handedness of the conductor [6,7]. In this Letter, we will show by symmetry arguments based on Onsager's relation that generally for any type of diffusive transport in any type of system, under crossed electric and magnetic fields, an anisotropy in the two-terminal resistance exists, in the direction perpendicular to the crossed external fields and the size of which depends *linearly* on the electric and the magnetic field. This new phenomenon represents a transport analogon of an optical effect that was recently observed for the first time: magnetoelectric anisotropy (MEA) [8,9], where a similar anisotropy occurs in the optical absorption. The existence of this effect was predicted in refraction on the basis of symmetry arguments [10,11]. In both cases, the anisotropy is described by a term of the form $\mathbf{k} \cdot \mathbf{E}_0 \times \mathbf{B}_0$, in the diffusion coefficient and in the dielectric constant, respectively ($\mathbf{k}$ being the wave vector of the diffusing particles or the photons, $\mathbf{E}_0$ a static electric field, and $\mathbf{B}_0$ a static magnetic field). We will therefore also refer to the diffusive transport effect described in this Letter as magnetoelectric anisotropy. For the particular case of diffusive electrical transport, we will present a relativistic model that supports our general argument and provides a quantitative estimate of this effect, which agrees well with our experimental observations. The existence of MEA in diffusive electrical transport clearly shows that the aforementioned claims concerning the even magnetic field dependence of electrical two-terminal resistance are not generally true.

Onsager was the first to consider the symmetry properties of kinetic coefficients [5]. (For a discussion, see, e.g., [12–14].) He showed that for a generalized transport coefficient σ_{ij} (e.g., the electrical or thermal conductivity tensor) close to thermodynamic equilibrium one can write

$$\sigma_{ij} = \sigma_{ji}^{\dagger}, \tag{1}$$

where † denotes time reversal. With this relation, one finds for a transport coefficient that depends only on an external magnetic field $\mathbf{B}$

$$\sigma_{ij}(\mathbf{B}) = \sigma_{ji}(-\mathbf{B}). \tag{2}$$

This implies that any two-terminal resistance, proportional to σ_{ii}, cannot have an odd dependence on $\mathbf{B}$. The frequently employed term "linear magnetoresistance" (LMR) [15] refers in practice to a magnetic field dependence where R varies linearly with B for large B, but which is still even in B. Here we consider the implications of Eq. (1) for diffusive transport in systems subject to a static electric $\mathbf{E_0}$ and magnetic field $\mathbf{B}_0$, where we allow also for a dependence on the average wave vector $\mathbf{k}$ of the diffusing particles. As a nonzero average wave vector is intrinsic to transport, such a dependence is quite natural and it is its neglect that leads to conclusions that are not generally true. For this case, Eq. (1) tells us

$$\sigma_{ij}(\mathbf{k}, \mathbf{E_0}, \mathbf{B}_0) = \sigma_{ji}(-\mathbf{k}, \mathbf{E_0} - \mathbf{B}_0). \tag{3}$$

More specifically, we find

$$\sigma_{ij}(\mathbf{k} \cdot \mathbf{E}_0 \times \mathbf{B}_0) = \sigma_{ji}[(-\mathbf{k}) \cdot \mathbf{E}_0 \times (-\mathbf{B_0})] = \sigma_{ji}(\mathbf{k} \cdot \mathbf{E}_0 \times \mathbf{B}_0), \tag{4}$$

and so there are no time-reversal symmetry objections against a linear dependence of σ_{ii}, and therefore of any two-terminal resistance, on $\mathbf{k} \cdot \mathbf{E}_0 \times \mathbf{B}_0$. As the term $\mathbf{k} \cdot \mathbf{E}_0 \times \mathbf{B}_0$ is even under parity and under charge conjugation, we conclude that every diffusive kinetic coefficient for any system subject to crossed electric and magnetic fields can be expanded as

$$\boldsymbol{\sigma}(\mathbf{k}, \mathbf{E}_0, \mathbf{B}_0) = \boldsymbol{\sigma}_0 + \boldsymbol{\sigma}_{\mathrm{MEA}} \mathbf{k} \cdot \mathbf{E}_0 \times \mathbf{B}_0 + \boldsymbol{\sigma}_{\mathrm{MR}} \mathbf{B}_0 \mathbf{B}_0 + \cdots, \tag{5}$$

where the last term on the right-hand side describes the normal magnetodiffusion, which is allowed in all systems (we neglect higher even orders in $\mathbf{B_0}$ that are also allowed). Because of the generality of Onsager's argument, Eq. (5) should apply to all diffusive transport phenomena, such as electrical or thermal conductivity, ionic diffusion, multiple scattering of light, etc. Below we will focus on the case of electrical resistance.

For electrical transport, the electrical current density $\mathbf{J} = ne\langle \mathbf{v} \rangle \propto \mathbf{k}$ (n carrier density, e carrier charge), and we therefore conclude that the two-terminal electrical resistance of any conductor subject to a combined electric and magnetic field is of the form

$$R(\mathbf{I}, \mathbf{E}_0, \mathbf{B}_0) = R_0\{1 + \chi \mathbf{I} \cdot \mathbf{E}_0 \times \mathbf{B}_0 + \beta B_0^2\}, \quad (6)$$

where $\mathbf{I}$ is the electrical current and the parameter β describes the normal quadratic two-terminal magnetoresistance that may result from the longitudinal magnetoresistivity, or from the Hall effect. Charge conjugation symmetry requires that the sign of χ changes with the sign of the charge of the carriers. Note that MEA is not some nonequilibrium effect [3,4], but rather a fundamental contribution to diffusive transport near equilibrium in crossed electric and magnetic fields. The electric field $\mathbf{E_0}$ in Eq. (6) may be externally applied, but could also result from a bulk material polarization, e.g., in noncentrosymmetric crystals, or from a band structure offset at an interface, e.g., in semiconductor heterostructures. Similarly, the magnetic field $\mathbf{B_0}$ could also result from a bulk magnetization. Many situations can therefore be envisaged in which MEA can manifest itself.

The symmetry arguments above that lead to Eq. (6) do not give any information about the magnitude of χ. This magnitude can be estimated by the following simple relativistic argument. Consider the case depicted in Fig. 1(a)

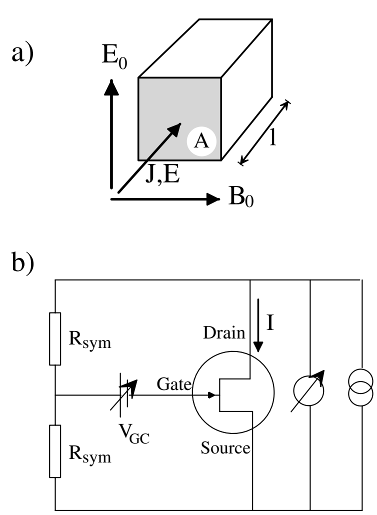

FIG. 1. (a) Definition of geometry. (b) Measurement circuit for the source-drain resistance $R(I, V_{GC}, B_0)$. The two resistors R_{sym} serve to symmetrize the circuit.

where we consider spinless charge carriers. The resistance is given by

$$R = \frac{E}{J}\frac{l}{A} = \frac{E}{ne\langle \mathbf{v} \rangle}\frac{l}{A}. \quad (7)$$

For the remainder, we will assume that $E_0 < B_0$. (The other case can be easily shown to give identical results.) In a reference frame that moves with velocity $\tilde{\mathbf{v}} = c\mathbf{E}_0 \times \mathbf{B}_0/B_0^2$, the crossed fields transform to $\mathbf{E_0'} = 0$ and $\mathbf{B_0'} = \mathbf{B}_0\sqrt{1 + (\tilde{v}/c)^2}$ and the resistance transforms to

$$R' = \frac{E'}{n'e\langle \mathbf{v}' \rangle}\frac{l'}{A}. \quad (8)$$

Using the known transformations for the quantities on the right-hand side of Eq. (8) (see, e.g., [16]) and neglecting the dispersion in the drift velocity, one can easily show that

$$R' = \frac{R}{\sqrt{1 + (\tilde{v}/c)^2}}\frac{1}{1 - [(\langle v \rangle \tilde{v} + \tilde{v}^2)/c^2]}. \quad (9)$$

In this moving reference frame, only a transverse static magnetic field exists, and the resistance must therefore have the following field dependence:

$$R' = R_0'(1 + \beta' B_0'^2). \quad (10)$$

Combining Eqs. (9) and (10) and retaining only terms to first order in v/c, one readily finds

$$R \simeq R_0\left(1 - \frac{\beta' \langle v \rangle E_0 B_0}{c} + \beta' B_0^2\right), \quad (11)$$

and upon comparison between Eq. (11) and Eq. (6), we obtain

$$\chi \simeq \frac{\beta}{Anec}. \quad (12)$$

MEA thus appears as a relativistic correction to the normal quadratic magnetoresistance, reminiscent of the Rashba and Dresselhaus spin-orbit couplings in noncentrosymmetric structures or materials [17]. Recent theoretical work has predicted that such spin-orbit coupling should lead to contributions to the two-terminal resistance that are odd in the perpendicular magnetic field, in line with Eq. (6) [18]. That prediction can be regarded as a special case of our more general treatment. Here we have neglected spin, and by incorporating also the spin of the charge carriers, additional contributions to Eq. (12) will arise. We also have not explicitly incorporated the Hall effect. Although the Hall electrical field in finite conductors is intrinsically perpendicular to the current and the external magnetic field, it does not lead to measurable MEA, as it inherently changes sign with the magnetic field or the current.

According to our model above, conductors with a large drift velocity and a large magnetoresistance are beneficial for the experimental observation of MEA. Large values for these parameters are typically found in semiconductors. This, and the need for a strong electric field perpendicular to the current, makes field effect transistors (FETs) [19] the obvious candidates in an experimental search for MEA. The MEA of commercial FET devices was experimentally determined as $\Delta R(I, V_{GC}, B_0) \equiv \frac{1}{2}\{R(I, V_{GC}, B_0) - R(-I, V_{GC}, B_0)\}$, where R is the drain-source resistance, by means of standard phase-sensitive detection techniques, in the configuration shown in Fig. 1(b). The gate-channel voltage V_{GC} provides the electric field E_0, inherently perpendicular to the channel current I. Upon normalizing by the zero field resistance, we obtain $\Delta R(I, V_{GC}, B_0)/R(I, V_{GC}, B_0 = 0) = \chi I E_0 B_0$ [Eq. (6)]. Although, in principle, the drain and source of a FET are identical, many devices are asymmetrical to optimize device performance and therefore show rectifying behavior $R(I) \neq R(-I)$. We have limited ourselves to symmetrical n-channel FET types where drain and source are interchangeable and have connected these contacts in a symmetrical way, so $R(I, V_{GC}, B_0 = 0) \simeq R(-I, V_{GC}, B_0 = 0)$. MEA was observed for all investigated FET types (see Table I). The largest values were obtained on BF245C junction FETs (JFETs) [20] and will be discussed below.

Figure 2 shows the elementary dependencies of MEA on current, magnetic field, and electric field for a BF245C. Figure 2(a) shows that $\Delta R/R$ has the linear current dependence predicted in Eq. (6). Figure 2(b) shows that $\Delta R/R$ has the linear B_0 dependence predicted in Eq. (6). In addition, $\Delta R/R$ also has a small quadratic contribution, which we attribute to the B_0^2 dependence of the small residual zero field rectification. This quadratic contribution explains the slightly different slopes in Fig. 2(a). It is also this contribution that hinders clear observation with our current method of MEA in asymmetrical FETs, where this contribution is many orders of magnitude larger. Figure 2(c) shows that the dependence of $\Delta R/R$ on V_{GC} resembles strongly that of R on V_{GC}. The latter finds its origin in the reduction of the height of the drain-source channel because of the increasing junction depletion layer thickness with increasing V_{GC}. The resemblance therefore suggests that the dependence of $\Delta R/R$ on V_{GC} is dominated by the dependence of χ on the effective channel section A [as suggested by Eq. (12)] and not directly by the electric field. Note that even at $V_{GC} = 0$ the built-in junction potential provides an electric field in the channel, and a finite MEA should be observed, as seen in Fig. 2(c). Figure 3(a) shows the dependence of $\Delta R/R$ on the angle θ

TABLE I. All investigated symmetrical n-channel silicon JFET types.

Type	Manufacturer
BF245A	Philips Semiconductors
BF245B	Fairchild Semiconductors
BF245C	Philips Semiconductors
BFR30	Philips Semiconductors
J211	Siliconix

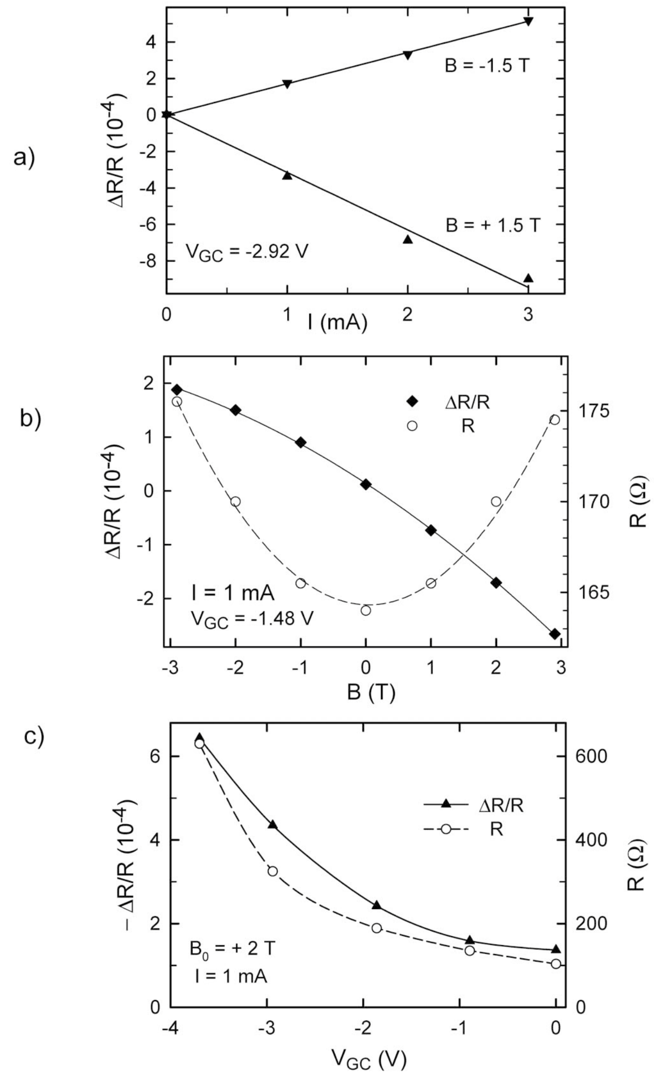

FIG. 2. (a) MEA of a BF245C JFET as a function of the channel current I. Solid lines are linear fits. (b) Channel resistance and MEA of a BF245C JFET as a function of the magnetic field B_0. Dashed line is a fit to a quadratic dependence. Solid line is a fit to a linear plus quadratic dependence. (c) Channel resistance and MEA of a BF245C JFET as a function of the gate-channel voltage V_{GC}. Lines are meant only to guide the eye.

between the magnetic field and the gate-channel electric field, both perpendicular to the current. The solid line is a fit to a $\cos\theta$ dependence, as predicted by Eq. (6). The good agreement again confirms the validity of Eq. (6). We also checked that no MEA occurred for $\mathbf{I} \| \mathbf{B}_0$. The proportionality between MEA and quadratic magnetoresistance, as predicted by Eq. (12), can be evaluated by measuring for a given FET both effects simultaneously, at several temperatures. Figure 3(b) shows $\Delta R/RB = \chi I E_0$ as a function of β, determined at temperatures in the range 77–293 K. An approximately linear relation is observed, proving that $\chi \propto \beta$, as predicted by Eq. (12). Here we have assumed that all other parameters, such as the junction electric field and

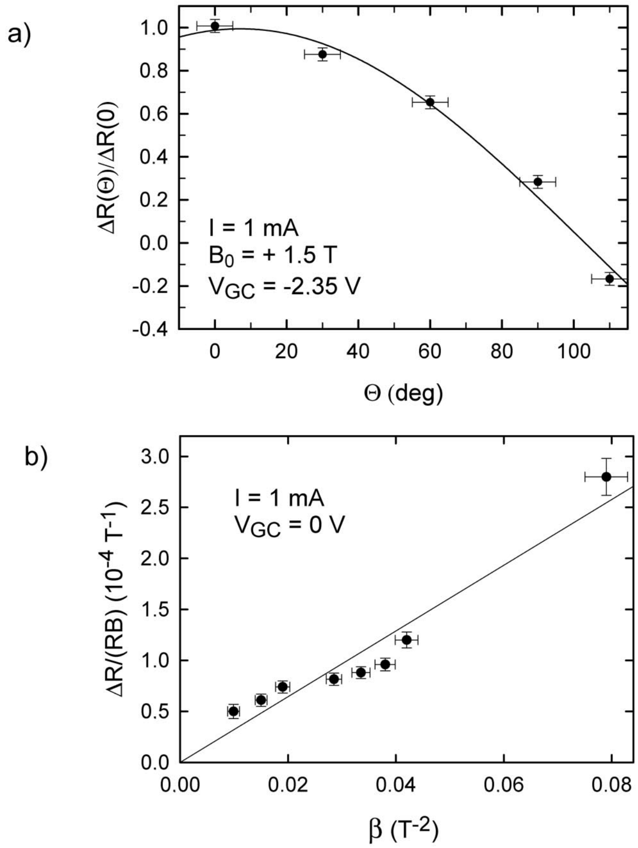

FIG. 3. (a) Angular dependence of MEA of a BF245C JFET. Solid line is a fit to a $\cos\theta$ dependence. (b) Normalized MEA $\Delta R/RB$ as a function of quadratic magnetoresistance parameter β, obtained at temperatures in the range 77–293 K on a BF245C JFET. Solid line is a linear fit through the origin.

carrier concentration, remain constant in the temperature range studied.

We can now evaluate the quantitative validity of Eqs. (6) and (12). The observed $\beta = 1 \times 10^{-2}\ \mathrm{T}^{-2}$ corresponds to a mobility of $0.1\ \mathrm{m}^2/\mathrm{V\,s}$. Together with the observed channel resistance and an estimated channel length of 100 μm, we deduce $Anec \approx 3 \times 10^3$ A/s. With typical channel electric fields of 10^5 V/m [19], we calculate with Eqs. (6) and (12) $\Delta R/R \approx 3 \times 10^{-4}$ at $B_0 = 1$ T and $I = 1$ mA, reasonably close to the observed values. A more detailed modeling would be necessary to determine the accuracy of Eq. (12). At this stage we cannot determine whether a spin effect significantly contributes to the observed MEA.

So far we have only considered diffusive conductors. For ballistic conductors, one can easily show by direct application of time- and parity-reversal symmetry arguments that the carrier transmission probability, and therefore the electrical resistance, may also show MEA. Equation (6) will therefore also apply to ballistic conductors. In fact, the ballistic case is even closer to the optical MEA than the diffusive case discussed above. However, Eq. (12) does not apply, as no uniform drift velocity can be expected in ballistic conductors.

In summary, our experimental results prove the existence of magnetoelectric anisotropy in electrical resistance and are in reasonable agreement with our simple relativistic model for this new effect. They thereby support our general prediction of MEA in all forms of diffusive transport in any system and at the same time present a striking new transport analogon of an optical effect. Our findings also show that often-heard claims concerning the even magnetic field dependence of electrical two-terminal resistance are not generally true. Although the effect observed so far is quite small, it may in principle be interesting for applications such as sensors or data storage.

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