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D-Term Inflation, Cosmic Strings, and Consistency with Cosmic Microwave Background Measurements

Jonathan Rocher*

Institut d'Astrophysique de Paris, GReCO, FRE 2435-CNRS, 98bis boulevard Arago, 75014 Paris, France

Mairi Sakellariadou†

*Division of Astrophysics, Astronomy, and Mechanics, Department of Physics, University of Athens,
Panepistimiopolis, GR-15784 Zografos, Greece*
Département de Physique Théorique, Université de Genève, 24 quai E. Ansermet, CH-1211 Genève 4, Switzerland

Standard D-term inflation is studied in the framework of supergravity. D-term inflation produces cosmic strings; however, it can still be compatible with cosmic microwave background (CMB) measurements without invoking any new physics. The cosmic strings contribution to the CMB data is not constant, nor dominant, contrary to some previous results. Using current CMB measurements, the free parameters (gauge and superpotential couplings, as well as the Fayet-Iliopoulos term) of D-term inflation are constrained.

Introduction.—The inflationary paradigm [1] offers simple answers to the shortcomings of the standard hot big bang model. In addition, simple inflationary models offer successful candidates for the initial density fluctuations leading to the observed structure formation. One crucial question though is to answer how generic is the onset of inflation [2] and to find consistent and natural models of inflation from the point of view of particle physics. One can argue that the initial conditions which favor successful inflationary models are the likely outcome of the quantum era before inflation [2]. It is more difficult, however, to find natural ways to guarantee the flatness of the inflaton potential.

The early history of the Universe at energies below the Planck scale is described by an effective $N = 1$ supergravity theory. Since inflation should have taken place at an energy scale $V^{1/4} \lesssim 4 \times 10^{16}$ GeV, this implies that inflationary models should be constructed in the framework of supergravity. Here is where the problem arises: it is difficult to implement slow-roll inflation within supergravity. The positive false vacuum of the inflaton field breaks spontaneously global supersymmetry, which gets restored after the end of inflation (when V disappears). In supergravity theories, the supersymmetry breaking is transmitted to all fields by gravity, and thus any scalar field, which could play the role of the inflaton, gets an effective mass $\sim \sqrt{8\pi V}/M_{\mathrm{Pl}} \sim H$, where H stands for the expansion rate during inflation, and M_{Pl} denotes the reduced Planck mass. This problem, known as the problem of "Hubble-induced mass," originates from F-term interactions, and thus it is resolved if we consider the vacuum energy as being dominated by nonzero D terms of some superfields [3]. This result led to a dramatic interest in D-term inflation, since in addition this model can easily be implemented in string

theory. However, later on D-term inflation in its turn was thought to be plagued with problems.

In D-term inflation, the inflationary era ends when a U(1) gauge symmetry is spontaneously broken by a vacuum expectation value of some scalar field, leading to the formation of gauge cosmic strings. As it was explicitly shown in Ref. [4], cosmic strings are generically expected to be formed at the end of a hybrid inflation phase, in the context of supersymmetric grand unified theories. It was claimed [5] that the cosmic strings contribution to the angular power spectrum of the Cosmic Microwave Background (CMB) temperature anisotropies is constant and dominant (75%). From the observational point of view, however, strong constraints [6] are placed on the allowed cosmic strings contribution to the CMB: it cannot exceed $\sim 10\%$. Thus, standard D-term inflation was thought to be inconsistent with cosmology. To rescue D-term inflation there have been proposed different mechanisms which either consider more complicated models or require additional ingredients so that cosmic strings are not produced at the end of hybrid inflation. For example [7], one can add a nonrenormalizable term in the potential, or add an additional discrete symmetry, or consider grand unified theory (GUT) models based on nonsimple groups. More recently, a new pair of charged superfields has been introduced in D-term inflation so that cosmic strings formation is avoided [8].

The aim of our study is to show that standard D-term inflation leading to the production of cosmic strings is still compatible with cosmological data, and, in particular, CMB, without invoking any new physical mechanisms. We find that in D-term inflation the cosmic strings contribution to the CMB data depends on the free parameters, as for F-term inflation [9]. The maximum allowed cosmic

strings contribution to the CMB measurements places upper limits on the inflationary scale (which is also the cosmic string energy scale), or equivalently on the coupling of the superpotential.

We first review the results for F-term hybrid inflation, in which case the supersymmetric renormalizable superpotential reads

$$W_{\text{infl}}^F = \kappa S(\Phi_+ \Phi_- - M^2), \tag{1}$$

where S, Φ_+, Φ_- are three chiral superfields, and κ, M are two constants. The cosmic strings contribution to the CMB is a function of the coupling κ, or equivalently of the mass scale M. It can be consistent with the most recent measurements, which require that it is at most equal to 9% [6], provided [9]

$$M \lesssim 2 \times 10^{15} \text{ GeV} \Leftrightarrow \kappa \lesssim 7 \times 10^{-7}. \tag{2}$$

The above limit was obtained in the context of the SO(10) gauge group. Upper limits of the same order of magnitude are found for other gauge groups [9].

This result implies that F-term inflation leading to the production of cosmic strings of the GUT scale can be compatible with measurements, provided the coupling is sufficiently small. Thus, hybrid supersymmetric inflation loses some of its appeal since it requires some amount of fine-tuning of its free parameter; κ should be of the order of 10^{-6} or smaller. This constraint on κ is in agreement with the one given in Ref. [10]. The parameter κ is also subject to the gravitino constraint that imposes an upper limit to the reheating temperature to avoid gravitino overproduction. Within supersymmetric GUTs, and assuming a seesaw mechanism to give rise to massive neutrinos, the inflaton field will decay during reheating into pairs of right-handed neutrinos. Using the constraints on the seesaw mechanism, it is possible [9,11] to convert the constraint on the reheating temperature to a constraint on the coupling parameter κ, namely, $\kappa \lesssim 8 \times 10^{-3}$, which is clearly a weaker constraint.

The superpotential coupling κ is allowed to get higher values, namely, it can approach the upper limit permitted by the gravitino constraint, if one employs the curvaton mechanism [12]. Such a mechanism can easily be accommodated within supersymmetric theories, where one expects to have a number of scalar fields. For fixed κ, the cosmic strings contribution decreases rapidly as the initial value of the curvaton field, ψ_{init}, decreases. Thus, the Wilkinson Microwave Anisotropy Probe (WMAP) measurements lead to an upper limit on ψ_{init}, namely, $\psi_{\text{init}} \lesssim 5 \times 10^{13}(\kappa/10^{-2})$ GeV [9]. This limit holds for κ in the range $[5 \times 10^{-5}, 1]$; for lower values of κ, the cosmic strings contribution is always suppressed and thus lower than the WMAP limit.

The above results hold also if one includes supergravity corrections. This is expected since the value of the inflaton field is several orders of magnitude below the Planck scale.

D-term inflation.—D-term inflation is derived from the superpotential

$$W_{\text{infl}}^D = \lambda S \Phi_+ \Phi_-, \tag{3}$$

where S, Φ_-, Φ_+ are three chiral superfields and λ is the superpotential coupling. D-term inflation requires the existence of a nonzero Fayet-Iliopoulos term ξ, permitted only if an extra U(1) symmetry beyond the GUT framework is introduced. In the context of supersymmetry we calculate the radiative corrections leading to the effective potential,

$$V_{\text{eff}}^{D\text{-SUSY}}(|S|) = \frac{g^2 \xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[2\ln\frac{|S|^2 \lambda^2}{\Lambda^2} + (z+1)^2 \right. \right.$$
$$\left. \left. \times \ln(1 + z^{-1}) + (z-1)^2 \ln(1 - z^{-1}) \right] \right\}, \tag{4}$$

where $z = \lambda^2 |S|^2/(g^2 \xi)$, with g the gauge coupling of the U(1) symmetry and ξ the Fayet-Iliopoulos term, are chosen to be positive; Λ stands for a renormalization scale. In the absence of the curvaton mechanism, the quadrupole anisotropy is the sum of the inflaton field (scalar and tensor parts) and cosmic strings contributions, and we normalize it to the Cosmic Background Explorer (COBE) data.

We compute the mass scale of the symmetry breaking, given by $\sqrt{\xi}$, and we find that it increases with λ. We then calculate the cosmic strings contribution to the temperature anisotropies. We find that within supersymmetry D-term inflation is consistent with CMB data provided the superpotential coupling λ is quite small, namely, $\lambda \lesssim 3 \times 10^{-5}$.

However, the dependence of z_Q (the index Q denotes the scale corresponding to the quadrupole anisotropy) on the superpotential coupling λ results in values of the inflaton field S_Q above the Planck mass. This implies that the correct analysis has to be done in the framework of supergravity. For small values of the gauge coupling g, the study in the context of supergravity becomes just the analysis within supersymmetry. Some previous studies [5,13] found in the literature kept only the first term of the radiative corrections. We find that it is necessary to perform the analysis using the *full* effective potential, which we calculated for minimal supergravity. More precisely, using a minimal Kähler potential $(K = |\phi_-|^2 + |\phi_+|^2 + |S|^2)$ and a minimal gauge kinetic function $[f(\Phi_i) = 1]$, the scalar potential reads [9]

$$V_{\text{eff}}^{D\text{-SUGRA}} = \frac{g^2 \xi^2}{2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[2\ln\frac{|S|^2 \lambda^2}{\Lambda^2} \exp\left(\frac{|S|^2}{M_{\text{Pl}}^2}\right) \right. \right.$$
$$\left. \left. + (z+1)^2 \ln(1 + z^{-1}) + (z-1)^2 \ln(1 - z^{-1}) \right] \right\}, \tag{5}$$

where $z = [\lambda^2 |S|^2/(g^2 \xi)] \exp(|S|^2/M_{\text{Pl}}^2)$. The number of e

foldings is

$$N_Q = \frac{2\pi^2}{g^2} \int_1^{z_Q} \frac{W(c\tilde{z})}{\tilde{z}^2 f(\tilde{z})[1 + W(c\tilde{z})]^2} \, d\tilde{z}, \qquad (6)$$

where $W(x)$ denotes the "W-Lambert function" defined by $W(x)\exp[W(x)] = x$, and $c \equiv (g^2\xi)/(\lambda^2 M_{\rm Pl}^2)$. The number of e foldings N_Q is thus a function of c and z_Q, for g fixed. Setting $N_Q = 60$, we obtain a numerical relation between c and z_Q which allows us to construct a function $z_Q(\xi)$ and express the three contributions to the CMB only as a function of ξ. The total $(\delta T/T)$ is given by

$$\left[\left(\frac{\delta T}{T}\right)_{Q\text{-tot}}\right]^2 \sim \left(\frac{\xi}{M_{\rm Pl}^2}\right)^2 \left\{\frac{\pi^2}{90g^2} z_Q^{-2} f^{-2}(z_Q) \frac{W(cz_Q)}{[1 + W(cz_Q)]^2} \right.$$
$$\left. + \left(\frac{0.77g}{8\sqrt{2}\pi}\right)^2 + \left(\frac{9\pi}{4}\right)^2 \right\}, \qquad (7)$$

where the three contributions come from the scalar and tensor parts of the inflaton field, and the cosmic strings, respectively. We normalize the left-hand side of Eq. (7) to the COBE data, i.e., $(\delta T/T)_Q^{\rm COBE} \sim 6.3 \times 10^{-6}$, and we solve it numerically to obtain ξ and, thus, the three contributions for given values of g and λ.

The cosmic strings contribution to the CMB data is found to be an increasing function of the mass scale $\sqrt{\xi}$, as shown in Fig. 1 below.

Our results, summarized in Fig. 2, differ from the results obtained in the framework of supersymmetry unless $\lambda \gtrsim 10^{-3}$ or $g \lesssim 10^{-4}$. The cosmic strings contribution to the CMB turns out to be dependent on the free parameters, with, however, the robust result that the cosmic strings contribution is not constant, nor is it always dominant, in contradiction to Ref. [5]. This implies that contrary to what is often assumed, the simplest D-term inflation is still an open possibility and one does not need to consider more complicated models. Our analysis shows that $g \gtrsim 1$ necessitates multiple-stage inflation, since otherwise we cannot have sufficient e foldings to resolve the horizon problem of standard cosmology, while $g \gtrsim 2 \times 10^{-2}$ is incompatible with the WMAP measurements. For $g \lesssim 2 \times 10^{-2}$, we can also constrain the superpotential coupling λ and get $\lambda \lesssim 3 \times 10^{-5}$. This limit was already found in the framework
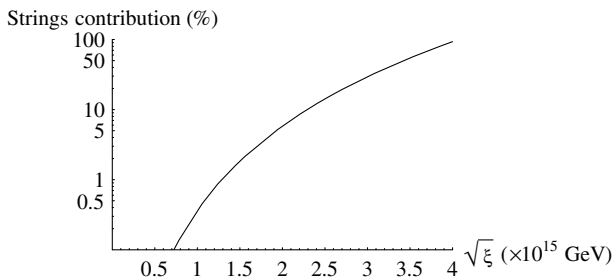
of supersymmetry [9], and it is in agreement with the constraint $\lambda \lesssim \mathcal{O}(10^{-4}$–$10^{-5})$ of Ref. [13]. Supergravity corrections impose, in addition, a lower limit to the coupling λ. If, for example, $g = 10^{-2}$, the cosmic strings contribution imposes $10^{-8} \lesssim \lambda \lesssim 3 \times 10^{-5}$. The constraint induced by CMB measurements is expressed as a single constraint on the Fayet-Iliopoulos term ξ, namely, $\sqrt{\xi} \lesssim 2 \times 10^{15}$ GeV.

As a next step we examine whether there is a mechanism to allow more natural values of the couplings. Assuming the existence of a scalar field that is subdominant during inflation as well as at the beginning of the radiation dominated era, such a field (the curvaton) gives an additional contribution to the temperature anisotropies, which we calculate below for the case of D-term inflation. The curvaton contribution, in terms of the metric perturbation, reads [14]

$$\left(\frac{\delta T}{T}\right)_{\rm curv} \equiv \frac{\Psi_{\rm curv}}{3} = \frac{4}{9} \frac{\delta\psi_{\rm init}}{\psi_{\rm init}}. \qquad (8)$$

The initial quantum fluctuations of the curvaton field are given by $\delta\psi_{\rm init} = H_{\rm inf}/(2\pi)$. The expansion rate during inflation, $H_{\rm infl}$, is a function of the inflaton field, and it is given by the Friedmann equation: $H_{\rm infl}^2(\varphi) = (8\pi/3)V(\varphi)$. Thus, for the D-term tree-level effective potential, the additional curvaton contribution to the total temperature anisotropies is given by [13]

$$\left[\left(\frac{\delta T}{T}\right)_{\rm curv}\right]^2 = \frac{1}{6}\left(\frac{2}{27\pi}\right)^2 \left(\frac{g\xi}{M_{\rm Pl}\psi_{\rm init}}\right)^2. \qquad (9)$$

Normalizing the total $(\delta T/T)$ to COBE, we then obtain the contributions of the different sources (inflaton field, split into scalar and tensor parts, cosmic strings, curvaton field) to the CMB as a function of one of the three parameters $\psi_{\rm init}, \lambda, g$, keeping the other two fixed. We show in Fig. 3 the three contributions as a function of $\psi_{\rm init}$, for $\lambda = 10^{-1}$ and $g = 10^{-1}$. Clearly, there are values of $\psi_{\rm init}$ that allow bigger values of the superpotential coupling λ and of the

Strings contribution (%)

FIG. 1. The cosmic strings contribution to the CMB data, as a function of the mass scale $\sqrt{\xi}$ in units of 10^{15} GeV.
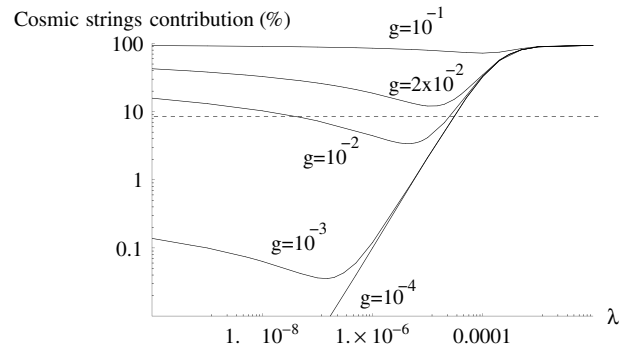
Cosmic strings contribution (%)

FIG. 2. Cosmic strings contribution to the CMB temperature anisotropies as a function of the superpotential coupling λ for different values of the gauge coupling g. The maximal contribution allowed by WMAP is represented by a dotted line.
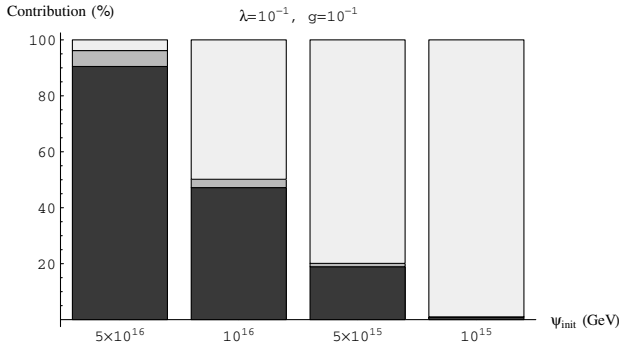
FIG. 3.   The cosmic strings (dark gray), curvaton (light gray), and inflaton (gray) contributions to the CMB temperature anisotropies as a function of the initial value of the curvaton field ψ_{init}, for $\lambda = 10^{-1}$ and $g = 10^{-1}$.

gauge coupling g than the upper bounds obtained in the absence of a curvaton field.

More explicitly, the fine-tuning on the couplings can be avoided provided

$$\psi_{\text{init}} \lesssim 3 \times 10^{14} \left(\frac{g}{10^{-2}}\right) \text{GeV} \quad \text{for } \lambda \in [10^{-1}, 10^{-4}].$$

(10)

Clearly, for smaller values of λ, the curvaton mechanism is not necessary.

We bring to the attention of the reader that in the above study we have neglected the quantum gravitational effects, which would lead to a contribution to the effective potential, even though $S_Q \sim \mathcal{O}(10 M_{\text{Pl}})$. Our analysis is, however, still valid, since the effective potential given in Eq. (5) satisfies the conditions [15] $V(|S|) \ll M_{\text{Pl}}^4$ and $m_S^2 = d^2V/dS^2 \ll M_{\text{Pl}}^2$, and thus the quantum gravitational corrections $[\Delta V(|S|)]_{QG}$ are negligible when compared to the effective potential $V_{\text{eff}}^{D\text{-SUGRA}}$.

Conclusions.—D-term inflation gained a lot of interest since it was shown that it avoids the problem of ''Hubble-induced mass,'' but it was later thought to be plagued with an inconsistency with the data. Standard D-term inflation ends with the formation of cosmic strings which was claimed to lead to a constant and dominant contribution to the CMB data, much higher than the one allowed by measurements. In this study, we show that this is not the case, and, therefore, standard D-term hybrid inflation can still be compatible with cosmological data.

We consider standard D-term inflation in its simplest form and without any additional ingredients. We perform our analysis in the framework of supergravity, since we reach scales above the Planck scale, and we consider all one-loop radiative corrections. We show that the cosmic strings produced at the end of D-term inflation can lead to a contribution to the CMB data which is allowed by the measurements. The price to be paid is that the couplings must be small. However, this constraint can be less severe if one invokes the curvaton mechanism.

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*Electronic address: rocher@iap.fr
†Electronic addresses: msakel@cc.uoa.gr; mairi@mpej.unige.ch

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