Rabi Oscillations Revival Induced by Time Reversal: A Test of Mesoscopic Quantum Coherence

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Using an echo technique proposed by Morigi *et al.* [Phys. Rev. A **65**, 040102 (2002)], we have time-reversed the atom-field interaction in a cavity quantum electrodynamics experiment. The collapse of the atomic Rabi oscillation in a coherent field is reversed, resulting in an induced revival signal. The amplitude of this ''echo'' is sensitive to nonunitary decoherence processes. Its observation demonstrates the existence of a mesoscopic quantum superposition of field states in the cavity between the collapse and the revival times.

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The investigation of a coherent atom-photon interaction in high quality microwave [1] or optical [2] cavities provides deep insights in fundamental quantum phenomena and in quantum information procedures [3]. In these latter studies, it is essential to manipulate the atom-field system in a coherent and reversible unitary way, minimizing the adverse influence of nonunitary irreversible decoherence processes.

Morigi *et al.* [4] have described a clever way to distinguish the contributions from coherent and incoherent processes in the system's evolution. Borrowing from NMR refocusing techniques [5], they proposed to let the system evolve for a given time T and then submit the atom to a short electromagnetic pulse leading to a reversal of the system's unitary evolution. At time $2T$, the system is expected to ''come back'' exactly in the initial state, in the absence of decoherence.

We have applied this scheme to the study of the atomic Rabi oscillation in a mesoscopic coherent field, with about 13 photons on the average. This oscillation undergoes a collapse due to the dispersion of the Rabi frequencies associated to different photon numbers. This effect is unitary and reversible. After a time proportional to the field amplitude, the oscillation is expected to ''revive'' spontaneously [6]. Applying a π-phase shift to the atomic coherence after the collapse time, we have obtained an early induced Rabi revival.

The collapse and revival phenomena are manifestations of complementarity. The Rabi oscillation is an interference effect between two probability amplitudes associated to different atomic states. At a fundamental level, the collapse of Rabi oscillations and the observation of revivals reveals atom-field entanglement. Information about the atomic state is imprinted onto the field. The system evolves into an entangled superposition involving coherent states with different phases [7–9]. The induced revival results from the erasure of the atomic imprint onto the field and from the unitary deconstruction of the atom-field entanglement. Observing the echo proves the unitarity of the whole process and is evidence of the transient generation of a mesoscopic quantum superposition state in the cavity.

The method of Morigi *et al.* [4] applies to a two-level atom (atomic levels $|e\rangle$ and $|g\rangle$) resonantly coupled to a quantized field mode (creation and annihilation operators $a^\dagger$ and a and photon number states $|n\rangle$). In the interaction picture, the system's evolution is ruled by the Jaynes-Cummings (JC) Hamiltonian [10]:

$$H_{\mathrm{JC}} = \hbar \frac{\Omega_0}{2} (a|e\rangle\langle g| + a^\dagger |g\rangle\langle e|), \tag{1}$$

where Ω_0 is the vacuum Rabi frequency. In an echo sequence, the atom first evolves from time 0 to T under the action of the unitary evolution operator $U_1 = \exp(-iH_{\mathrm{JC}}T/\hbar)$. The atom then undergoes, at time T, a percussional controlled phase kick corresponding to the unitary operation $U_\pi = (|e\rangle\langle e| - |g\rangle\langle g|) = \sigma_z$ (σ_z: Pauli matrix). The Jaynes-Cummings evolution then resumes for the remaining time $\tau - T$, with the evolution operator $U_2 = \exp[-iH_{\mathrm{JC}}(\tau - T)/\hbar]$. The overall evolution operator U is

$$U = U_2\sigma_z U_1 = \sigma_z^2 U_2 \sigma_z U_1 = \sigma_z e^{-iH_{\mathrm{JC}}(2T-\tau)/\hbar}, \tag{2}$$

where we have used the identities $\sigma_z^2 = 1$ and $\sigma_z H_{\mathrm{JC}} \sigma_z = -H_{\mathrm{JC}}$. This latter identity means that the evolution after the phase kick is the time-mirror image of the evolution between 0 and T. Equation (2) shows that, at time $\tau = 2T$, the unitary evolution brings the system back to its initial state, up to a global π-phase shift between the amplitudes associated to levels e and g (σ_z term).

We have applied this method to the study of the Rabi oscillation, with, as initial conditions, an atom in e and a mesoscopic field in the coherent state $|\alpha\rangle = \sum_n c_n |n\rangle$, with $c_n = \exp(-|\alpha|^2/2)\alpha^n/\sqrt{n!}$. The mean photon number is $\overline{n} = |\alpha|^2$ and its variance is $\Delta n = \sqrt{\overline{n}}$. The probability, $P_g(t_i)$, for finding the atom in g after the interaction time t_i (without kick and in the absence of decoherence) is

$$P_g(t_i) = \sum_n p_n \sin^2\left(\frac{\Omega_0\sqrt{n+1}t_i}{2}\right), \quad (3)$$

where $p_n = |c_n|^2$ is the Poisson distribution of the photon number. This signal, plotted in Fig. 1(a) for $\overline{n} = 13.4$, exhibits the collapse and revival features. It is convenient to express the characteristic times of the evolution with the Rabi oscillation period $t_R = 2\pi/\Omega_0\sqrt{\overline{n}}$ as the time unit. The collapse ($1/e$ decay of the contrast) occurs after a time $T_c = 2\sqrt{2}/\Omega_0 = \sqrt{2\overline{n}}t_R/\pi$, after the order of $\sqrt{\overline{n}}$ oscillations [9]. This collapse corresponds to the dispersion of the Rabi oscillation frequencies, of the order of Ω_0 around the average value $\Omega_0\sqrt{\overline{n}} = 2\pi/t_R$. The spontaneous revival occurs at time $T_r = 4\pi\sqrt{\overline{n}}/\Omega_0 = 2\overline{n}t_R$. At this time, the Rabi oscillations in n and $n+1$ photons have accumulated, to first order in $n - \overline{n}$, a 2π phase difference and sum up constructively. This refocusing effect is only partial, due to higher order contributions in $n - \overline{n}$ in the expansion of the Rabi frequencies.

Figure 1(b) shows the Rabi signal for the same coherent field when a phase kick is applied at time $T = 5.5t_R$. The echo signal comes back to $P_g = 0$ at time $t_i = 2T$, retracing back the collapse stage. After time $t_i = 2T$, the signal decays again, retracing, so to speak, the evolution of the system in the *past* of the initial condition. A new collapse occurs, with a spontaneous revival at a later time. Note the difference between the induced revival, which has a unit contrast, and the spontaneous one, which is only partial.

The spontaneous and induced revivals are specific features of a unitary evolution. They are strongly affected by decoherence processes whose time constant is set by the cavity field energy damping time T_{cav}. Qualitatively, on a quantum trajectory starting from a Fock state $|n\rangle$, quantum jumps between $|n\rangle$ and $|n-1\rangle$ occur at a rate n/T_{cav}, randomly changing the Rabi frequency. This results in a randomization of the phase of the Rabi oscillation, which is no longer exactly reversed by the echo technique.

Figure 1(c) presents the same echo signal as in Fig. 1(b), with a finite cavity damping time $T_{\mathrm{cav}} = 152t_R$. This value corresponds to the experimental conditions described below. The signal is obtained by a numerical integration of the atom-cavity field master equation [11], fully equivalent to an average over the quantum trajectories [12]. The stimulated echo occurring at time $t_i = 2T = 11t_R = T_{\mathrm{cav}}/16$ has a contrast reduced by a factor of 1.1. The spontaneous revival occurring much later (at time $35t_R \approx T_{\mathrm{cav}}/6$) is reduced by a much larger factor (6.6). Note that the contrast decay cannot be described in terms of a single time constant, which would lead to a reduction of the spontaneous revival contrast of 1.3 instead of 6.6. We come back later to this ''nonlinear'' feature of the decoherence process.

We have implemented the echo method with the cavity QED (CQED) setup sketched in Fig. 2(a), described in detail in [1]. ^{85}Rb atoms, effusing from oven O, are velocity selected by laser optical pumping and excited in B into the circular Rydberg state with principal quantum number 51 (level e), whose lifetime (30 ms) is much longer that the duration of an experimental sequence. The atomic preparation is pulsed, so that the position of each atom is known along the beam within ± 0.4 mm. The atoms cross one at a time the cavity C sustaining a Gaussian field mode M_a (waist $w = 6$ mm). The cavity frequency, ν_{cav}, is close to resonance with the $e \rightarrow g$ transition at 51.1 GHz. Note that C also sustains a mode M_b at $\nu_{\mathrm{cav}} - 88$ kHz. Its influence on the resonant interaction of the atom with M_a can be neglected in a first qualitative analysis.

An electric field applied across the cavity mirrors is used to tune the atomic frequency ν_{at} through the Stark effect and hence to control the atom-mode M_a resonance condi-

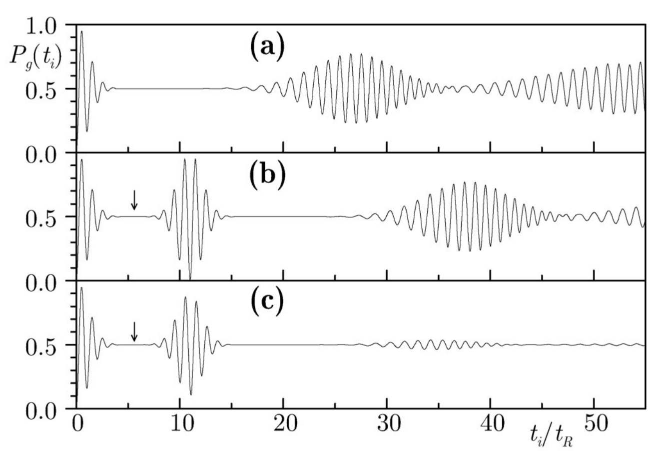

FIG. 1. Quantum Rabi signals. Probability $P_g(t_i)$ for finding the atom in g at time t_i, expressed in units of the Rabi period t_R. (a) Initial coherent field with $\overline{n} = 13.4$ photons on the average. (b) Same as in (a), with a kicking pulse applied at $t_i = 5.5t_R$ (depicted by the vertical arrow). (c) Same as (b) with a finite cavity damping time $T_{\mathrm{cav}} = 152t_R$.

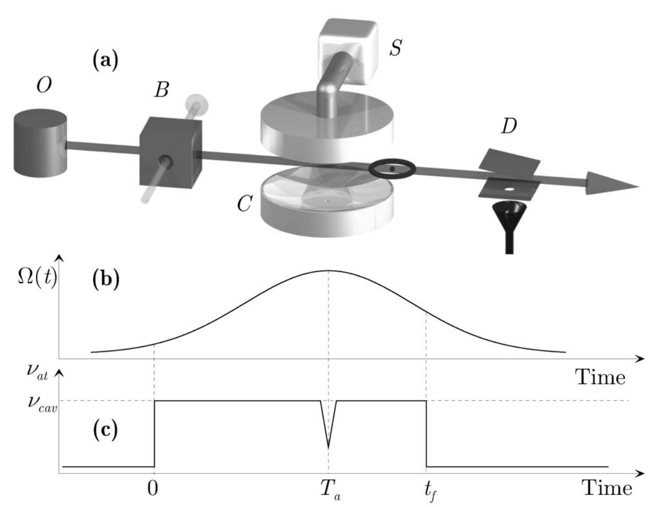

FIG. 2. (a) Scheme of the experimental apparatus. (b) Atom-cavity coupling $\Omega(t)$ versus time. (c) Atomic frequency ν_{at} versus time.

tion. The vacuum Rabi frequency at cavity center is $\Omega_0/2\pi = 49$ kHz. The cavity, cooled to 1.4 K, is made up of two superconducting niobium mirrors. The energy damping time is $T_{\rm cav} = 850\ \mu$s. The atoms are detected after C by a field-ionization detector D (quantum efficiency ~70%) discriminating levels e and g.

A coherent field, produced by a pulsed microwave source S, is injected in mode M_a of C through a small hole in one of the cavity mirrors. The average photon number $\overline{n}$ is controlled with attenuators. An independent measurement of $\overline{n}$ (with a precision of $\pm 10\%$) is performed by detecting the dispersive light shift produced by the field on a Rydberg atom [13].

A complete experimental sequence occurs while an atom crosses the cavity. Two essential parameters are plotted, versus time, in Figs. 2(b) and 2(c). Figure 2(b) shows the variation of the Rabi frequency due to the atomic motion across the Gaussian cavity mode, $\Omega(t) = \Omega_0 \exp[-v^2(t - T_a)^2/w^2]$, where v is the selected atomic velocity and T_a is the time when the atom crosses the cavity axis. Figure 2(c) shows the variation of $\nu_{\rm at}$ versus time. The atom enters the cavity mode while being far off resonance. The starting point of the Rabi oscillation, at time $t = 0$, corresponds to the sudden tuning of the atom into resonance with $\nu_{\rm cav}$. Note that M_b is transiently resonant with the atom, for an interaction time so short that it results in no sizable effect.

For a direct comparison with the theoretical discussion, we must define the effective interaction time t_i as

$$t_i(t) = \frac{1}{\Omega_0}\int_0^t \Omega(t')dt'. \tag{4}$$

The kick is realized at time T_a, when the atom reaches the cavity axis, by a fast (0.4 μs) electric field pulse, corresponding to a π relative phase shift accumulation between e and g. The effective kicking time T is thus simply $T = t_i(T_a)$. The echo occurs when the atom is still inside the mode, at a position mirroring the one at $t = 0$ with respect to the axis.

In order to probe the Rabi oscillation, the atom-field interaction is frozen at time t_f by detuning the atom from the cavity. The atomic populations remain then constant until the detection occurs in D. Time t_f can be varied between zero and a time larger than the cavity crossing time (the kicking electric field pulse is not applied for the observation of the signal with $t_f < T_a$).

Figure 3 presents experimental Rabi signals $P_g(t_i)$, versus the effective interaction time t_i, for an atom at $v = 154$ m/s and an average photon number $\overline{n} = 13.4$, corresponding to a Rabi period $t_R = 5.6\ \mu$s. This signal is obtained by averaging 20 000 realizations of the sequence for each t_i value. The Rabi signal without the kicking pulse [Fig. 3(a)] exhibits a collapse slightly faster than in Fig. 1(a). This is due to a small residual thermal field in the cavity (mean photon number $n_{\rm th} = 0.4$), which is displaced in phase space by the injection of the coherent field. The photon number distribution is accordingly broadened. Note that the spontaneous revival is outside the experimentally accessible time window. Figures 3(b) and 3(c) show the Rabi signal when a kicking pulse is applied at $T = 18$ and 22 μs, respectively. The first part of the data (open circles) is independent of the kicking pulse. It has been reproduced from Fig. 3(a) for visual convenience.

The solid lines in Figs. 3(a)–3(c) result from a numerical simulation of the atom-field master equation taking into account decoherence and experimental imperfections. This simulation is in good agreement with the observed echo contrast [33% in Fig. 3(c)]. It includes decoherence and the effect of the residual thermal field. The equilibrium thermal field in C contains one photon on the average. It is reduced down to $n_{\rm th} = 0.4$ at the beginning of each sequence by an erasure procedure using auxiliary atomic samples [1]. It relaxes back towards equilibrium, with the time constant $T_{\rm cav}$. The echo contrast is accordingly reduced by a 0.88 factor. A larger contrast reduction is due to other imperfections (presence of mode M_b, detection errors, spatial dispersion along the atomic beam direction, etc.).

This echo signal reveals the existence of a mesoscopic coherence in the atom-field system between the collapse and the induced revival times. As shown in [9] the initial coherent field splits into two components $|\Psi_c^\pm\rangle = |\alpha \exp[\pm i\Phi(t_i)]\rangle$, which, before the kicking pulse, rotate slowly in the opposite direction in phase space by an angle $\Phi(t_i) = \Omega_0 t_i/4\sqrt{\overline{n}}$. Each field component is correlated

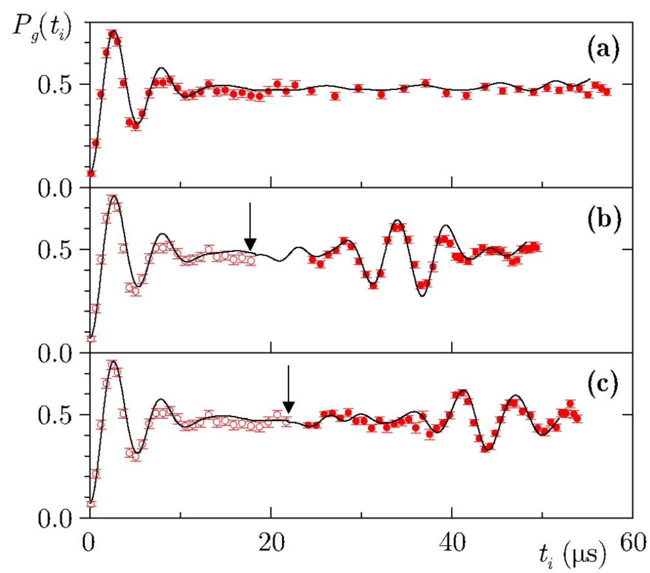

FIG. 3 (color online). Experimental Rabi oscillation $P_g(t_i)$. The solid circles are experimental points; the error bars reflect the statistical variance of the data. The solid lines result from a numerical integration of the atom-field evolution (see the text). (a) Oscillation without kicking pulse. Collapse occurs around $t_i = 9\ \mu$s. (b),(c) Kicking pulse (vertical arrow) at $T = 18$ and $T = 22\ \mu$s, respectively. The Rabi collapse signal (open circles) is reproduced from (a) for visual convenience.

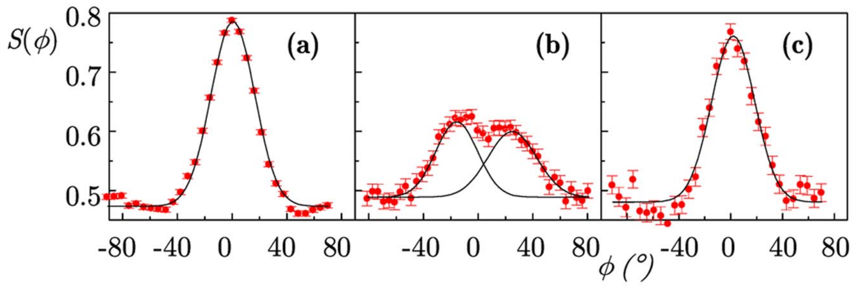

FIG. 4 (color online). Field phase distribution $S(\phi)$. (a) Initial coherent field. (b) Phase distribution at the kicking pulse time $t_i = T = 22$ μs. (c) Effective interaction time $t_i = 2T$ (induced revival time): reconstruction of the initial coherent state. Solid lines are Gaussian fits.

with an atomic state $|\Psi_a^\pm\rangle = \{\exp[\mp i\Phi(t_i)]|e\rangle \pm |g\rangle\}/\sqrt{2}$. The Rabi oscillation results from an interference between the probability amplitudes associated to these two atomic states. The atom-field system evolves into the superposition $(|\Psi_c^+\rangle|\Psi_a^+\rangle + |\Psi_c^-\rangle|\Psi_a^-\rangle)/\sqrt{2}$, which describes an entanglement as soon as $\langle\Psi_c^+|\Psi_c^-\rangle \approx 0$; i.e., $\Phi(t_i) \geq 1/\sqrt{\bar{n}}$ or $t_i \geq T_c$. The collapse of the Rabi oscillation thus reveals atom-field entanglement.

After the kicking pulse, the evolution of $\Phi(t_i)$ is reversed. The two field components merge again into a single state at time $2T$ and the atom-field entanglement vanishes, restoring the Rabi oscillation. In this way, the collapse and revival phenomena can be explained as a complementarity effect. The amplitude of the echo measures the degree of coherence of the whole process. At the kicking time, the maximum separation of the two field components reaches $\Phi(T) = 26°$. We have directly probed the evolution of $\Phi(t_i)$ by recording the field phase distribution $S(\phi)$ using the homodyne method described in [9]. Note that, ideally, the experimental signal $S(\phi)$ varies between 0.5 and 1 [9].

Figures 4(a)–4(c) show the phase distributions observed at the effective interaction times $t_i = 0$, T, and $2T$. The initial Gaussian distribution has a 23° width, in good agreement with the expected value $1/\sqrt{\bar{n}/(n_{\rm th} + 1)} \times (180/\pi) = 19°$. The splitting at the kicking time is clearly apparent [Fig. 4(b)] as is the recombination at time $t_i = 2T$ [Fig. 4(c)]. The maximum separation $(40.5 \pm 5.5°)$ is close to the value predicted by the simple model above $2\Phi(T) = 52°$ and in fair agreement with the result (49°) of a numerical simulation.

The decoherence rate of a mesoscopic superposition is proportional to the square of the distance D of the two field components in phase space. In this experiment, it remains moderate during the echo sequence, the maximum separation being $D = 2.6$. For the spontaneous revival, however, the two field components have to go through a much larger maximum distance $D' = 2\sqrt{\bar{n}} = 7.3$. This explains why the spontaneous revival contrast is strongly reduced [see Fig. 1(c)], making its observation impossible in our setup.

We have implemented a time-reversal echo method to study coherent atom-field processes in CQED. Applying it to the study of Rabi oscillations in a mesoscopic coherent field, we have induced an early quantum Rabi revival, less sensitive to decoherence than the spontaneous one. The observation of this echo signal demonstrates the coherence of a mesoscopic superposition of field states produced between the collapse and the revival times. With slower atoms, providing longer interaction times, we could use induced and spontaneous revival signals to investigate quantitatively the decoherence of these superpositions and explore in this way the quantum-classical boundary.

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