Comment on "Quantum Vacuum Contribution to the Momentum of Dielectric Media"

In Ref. [1] Feigel obtains a material contribution to the momentum density of a homogeneous dielectric medium that is claimed to solve the longstanding Abraham-Minkowski (AM) controversy on the momentum of light. This approach even reveals a nonzero matter momentum induced by vacuum fluctuations in homogeneous magneto-electric (ME) media.

The intention of this Comment is first to argue that the Lagrangian approach to moving *macroscopic* media in [1], although original, gives no new clue on the interpretation of the *macroscopic* Maxwell equations [2], and for which a Lagrangian method including matter has been proposed [3]. Secondly we contest the conclusion of [1] that vacuum fluctuations induce nonzero momentum in ME matter.

We adopt the macroscopic Maxwell equations ($c_0 = 1$) without free charges and currents, supplied by the ME (dispersion-free) relations (15). The approach by Jackson [4] is valid here and gives the conservation law,

$$\partial_t \mathbf{G} = \nabla \cdot \mathbf{T} - \mathbf{f}^M, \tag{1}$$

with $\mathbf{G} = (4\pi)^{-1}\mathbf{D} \times \mathbf{B}$ and $\mathbf{f}^M = -\frac{1}{8\pi}E_i E_j \nabla \varepsilon_{ij} - \frac{1}{8\pi}H_i H_j \nabla \mu_{ij} - \frac{1}{4\pi}E_i H_j \nabla \chi_{ij}$ the M-force density. The (asymmetric) M-stress tensor is $T_{ij} = (4\pi)^{-1} \times [H_i B_j + E_i D_j - \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H})\delta_{ij}]$. Newton's second law in the presence of the Lorentz force density $\rho_q \mathbf{E} + \mathbf{J}_q \times \mathbf{B}$, with $\rho_q = -\nabla \cdot \mathbf{P}$ and $\mathbf{J}_q = \partial_t \mathbf{P} + \nabla \times \mathbf{M}$, gives

$$\partial_t(-\rho_m \mathbf{v} + \mathbf{P} \times \mathbf{B}) = \nabla \cdot (\rho_m \mathbf{v}\mathbf{v} + \mathbf{U}) - \mathbf{f}^M, \tag{2}$$

with $U_{ij} = E_i P_j - B_i M_j - \frac{1}{2}[(\mathbf{E}\cdot\mathbf{P} - \mathbf{B}\cdot\mathbf{M})\delta_{ij}]$. In *homogeneous* media $\mathbf{f}^M = 0$, and Eq. (2) is equivalent to the conservation of "pseudomomentum" $\partial L/\partial \mathbf{v}$, found in (13) of [1] and Eq. (45) of [3]. Both equations combine to $\partial_t[\rho_m \mathbf{v} + (4\pi)^{-1}\mathbf{E} \times \mathbf{B}] = \nabla \cdot [\mathbf{T}_0 - \rho_m \mathbf{v}\mathbf{v}]$, with $\mathbf{T}_0$ the symmetric vacuum Maxwell stress tensor. This result, confirmed by more microscopic approaches [3,4], compares to the prediction (11) of Noether's theorem derived in [1] (where, however, $(4\pi)^{-1}\mathbf{E} \times \mathbf{H}$ seems to appear), and could invite us to identify $(4\pi)^{-1}\mathbf{E} \times \mathbf{B}$ and $\rho_m \mathbf{v}$ as the momentum densities of radiation field and matter [3]. Neither this radiation momentum nor $\mathbf{G}$ obey Planck's principle of inertia. The conclusive argument to prefer the M (Minkowski) set $(\mathbf{G}, \mathbf{T}, \mathbf{f}^M)$ over, e.g., the A (Abraham) set [2] remains unrevealed, here and in [1].

In homogeneous media is pseudomomentum $\int d\mathbf{r}(-\rho_m \mathbf{v} + \mathbf{P} \times \mathbf{B})$ conserved in time. For homogeneous fields the pseudomomentum density itself is conserved. Reference [1] concludes from this in Eqs. (14, 20) that $\rho_m \mathbf{v} \equiv \mathbf{P} \times \mathbf{B}$, a relation allowed by parity and time reversal but Lorentz-*variant*, even when $v \ll 1$ and apparently appealing to no motion and no field in some distant past. But sources would violate the conservation laws [2] as well as the homogeneity assumption. If one *assumes*, like in [1], a stationary homogeneous vacuum field, then $\partial_t \langle 0|\mathbf{P} \times \mathbf{B}|0\rangle = 0$, so $\partial_t \rho_m \mathbf{v} = 0$. No special reason exists for the matter to achieve a momentum density $\rho_m \mathbf{v} = \langle 0|\mathbf{P} \times \mathbf{B}|0\rangle$. For a ME medium this is nonzero but constant, so no contradiction exists when the matter is and stays at rest.

This conclusion follows from scattering theory in a different way. Consider a *finite* medium, obeying Eq. (15) of Ref. [1]. Photons scatter with cross-section $d\sigma(\mathbf{k}_{\rm in}, \mathbf{k}_{\rm out}, \mathbf{E}^{\rm ext}, \mathbf{B}^{\rm ext})/d\Omega$ with $|\mathbf{k}_{\rm in}| = |\mathbf{k}_{\rm out}| = \omega$ for if the medium is at rest no Doppler effect occurs. The *external* vacuum radiation momentum density at frequency ω is stationary and isotropic: $\mathbf{E} \times \mathbf{B} \sim \mathbf{k}\omega^{-1}E(\mathbf{k})^2 \sim \hbar\omega^2 \mathbf{k}\, d\hat{\mathbf{k}}_{\rm in} d\omega$. Their momentum transfer in time Δt is

$$\Delta \mathbf{p} \sim \hbar\omega^2 d\omega \Delta t \int d\hat{\mathbf{k}}_{\rm in} \int d\hat{\mathbf{k}}_{\rm out} \frac{d\sigma(\mathbf{k}_{\rm in}, \mathbf{k}_{\rm out})}{d\Omega}(\mathbf{k}_{\rm in} - \mathbf{k}_{\rm out}).$$

Simultaneous PT symmetry guarantees that $d\sigma(\mathbf{k}_{\rm in}, \mathbf{k}_{\rm out}, \mathbf{E}^{\rm ext}, \mathbf{B}^{\rm ext})/d\Omega = d\sigma(\mathbf{k}_{\rm out}, \mathbf{k}_{\rm in}, -\mathbf{E}^{\rm ext}, -\mathbf{B}^{\rm ext})/d\Omega$. Since the ME effect is bilinear in $\mathbf{E}^{\rm ext}$ and $\mathbf{B}^{\rm ext}$, we have $\Delta \mathbf{p} = 0$ and the object thus stays at rest.

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