Comment on "Quantum Vacuum Contribution to the Momentum of Dielectric Media"

In a recent Letter [1], the author investigates the back reaction of electromagnetic quantum fluctuations onto a (nonlinear) medium and predicts a potentially measurable effect. Although this is an interesting idea, we would like to point out difficulties and limitations of the derivation and arguments presented in Ref. [1], which cast serious doubts on the prediction.

First of all, the Lagrangian in Eq. (9) of Ref. [1] with $\boldsymbol{V} = \dot{\mathbf{R}}$ cf. Eq. (13), does not describe fluid dynamics. (It could correspond to the motion of a rigid body—but in that case, $\boldsymbol{V}$ and $\mathbf{R}$ would not be field variables.) The nonlinear term $(\boldsymbol{V} \cdot \nabla)\boldsymbol{V}$ in the equations of motion stemming from the comoving derivative $d/dt = \partial/\partial t + \boldsymbol{V} \cdot \nabla$ is missing, for instance.

For example, a nonrelativistic, irrotational, barotropic, and viscosity-free fluid can be described by the Lagrangian $\mathcal{L}_{\rm fluid} = -[\varrho\dot{\phi} + \varrho(\nabla\phi)^2/2 + \mu(\varrho)]$, where ϱ is the density of the fluid, ϕ the velocity potential with $\boldsymbol{V} = \nabla\phi$, and $\mu(\varrho)$ the specific enthalpy (determining the pressure). Variation with respect to the fluid's degrees of freedom ϕ and ϱ yields the equation of continuity and Bernoulli's equation, respectively, and the Hamiltonian is just the energy density $\mathcal{H} = \varrho\boldsymbol{V}^2/2 + \mu(\varrho)$.

After adding the same coupling term as in Eq. (9), i.e., $\mathcal{L}_{\rm int} = \xi(\varrho)[\mathbf{E} \times \mathbf{B}] \cdot \nabla\phi$ with $4\pi c\mu\xi = 1 - \varepsilon\mu$, we obtain (for $d\xi/d\varrho \neq 0$) an additional term $\propto [\mathbf{E} \times \mathbf{B}] \cdot \boldsymbol{V}$ in the Bernoulli equation and, consequently, the influence (back reaction) of the electromagnetic field manifests itself via a force density $\propto \nabla(\boldsymbol{V} \cdot [\mathbf{E} \times \mathbf{B}]d\xi/d\varrho)$ inducing an acceleration $\dot{\boldsymbol{V}}$ of the fluid (as one would expect) instead of an instantaneously generated velocity—as in Ref. [1] cf. Eq. (14). [If Eq. (14) is supposed to represent a conservation law, the corresponding integration constant (e.g., describing the initial conditions) is missing.]

However, one should be extremely careful with such constructions which are not based on an explicit microscopic model. First, the nonrelativistic limit is a bit tricky—e.g., consider the additional term $\delta\mathcal{L}_{\rm int}/\delta\phi$ in the equation of continuity. Second, one has to identify the relevant degrees of freedom of the fluid in terms of which the interaction Lagrangian is valid (e.g., whether an irrotational flow is consistent). Third, a possible dependence of ε and μ on degrees of freedom of the fluid (e.g., ϱ) must be taken into account too in order to describe effects such as the induced force when a charged capacitor is dipped into a dielectric fluid, etc.

Similar problems arise in the argumentation based on the Noether theorem in Eqs. (10) and (11). Without taking into account possible dependencies like $\varepsilon(\varrho)$, the assumed translational invariance is valid for strictly homogeneous media only. According to the Noether theorem, the (pseudo)momentum of the photons within homogeneous media is conserved and, hence, there is no back-reaction force acting on the medium at all. Therefore, the subsequent discussion (e.g., bending of an interface due to radiation forces) is not directly applicable.

The derivation of Eq. (22) via Eq. (19) and the subsequent discussion is rather incomprehensive. E.g., specifying Eq. (19) for the case $\chi_{xy} \to \chi \neq 0$ and $\chi_{yx} = 0$ (cf. Ref. [1]), the modes propagating in z direction with the relevant polarization can be described by the effective Lagrangian (in natural units with $\varepsilon_0 = \mu_0 = c^2 = 1$) $\mathcal{L}_{\rm eff} = [\varepsilon(\partial_t A)^2 - (\partial_z A)^2/\mu]/2 + \chi(\partial_t A)(\partial_z A)/\mu$. This Lagrangian is completely equivalent to that of an ordinary moving medium with the velocity being determined by χ. In a moving (ordinary) medium, the vacuum expectation value of the energy flux $\langle(\partial_t A)(\partial_z A)\rangle$ corresponding to the term $\langle\mathbf{E} \times \mathbf{B}\rangle$ occurring in Eq. (20) does not vanish analytically but diverges formally $\propto \boldsymbol{V}\langle\mathbf{E}^2 + \mathbf{B}^2\rangle$. However, it is clear that the vacuum fluctuations of the electromagnetic field do not induce any change of the motion of the (assumed to be homogeneous) medium in this situation. A calculation of an effect of quantum fluctuations with a resulting $\omega_{\rm cut}^4$ behavior as in Eqs. (21) and (22) cannot be trusted without proper regularization and renormalization (cf. the cutoff-independent Casimir effect or Lamb shift) or the investigation of a microscopic model (cf. the above points).

In view of the symmetries of nature (e.g., Lorentz invariance), a medium with the unusual properties required by the author of Ref. [1] can only exist under some external influence (providing a preferred reference frame), e.g., electromagnetic fields. Now, quantum fluctuations on top of these external (classical) fields in nonlinear media could have potentially measurable effects (e.g., in inhomogeneous media), but a reliable derivation (prediction) should be based on a realistic microscopic model (facilitating the identification of the effective degrees of freedom of the medium) and either a fully relativistic treatment or a consistent nonrelativistic expansion.

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[1] A. Feigel, Phys. Rev. Lett. **92**, 020404 (2004); see also Phys. Rev. Focus **13**, story 3, 23 January 2004, http://focus.aps.org/story/v13/st3

