Dipolar Superfluidity in Electron-Hole Bilayer Systems

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Bilayer electron-hole systems, where the electrons and holes are created via doping and are confined to separate layers, undergo excitonic condensation when the distance between the layers is smaller than the typical distance between the particles within the layer. We argue that the excitonic condensate is a novel dipolar superfluid in which the phase of the condensate couples to the *gradient* of the vector potential. We predict the existence of a dipolar supercurrent which can be tuned by an in-plane magnetic field. Thus the dipolar superfluid offers an example of excitonic condensate in which the *composite* nature of its constituent excitons is manifest in the macroscopic superfluid state. We also discuss various properties of this superfluid including the role of vortices.

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Introduction.—Superfluidity was discovered by Kapitsa in 1938 as the ability of liquid Helium (He^4) to carry momentum without dissipation [1]. Its phenomenological theory was developed by Landau and collaborators, who introduced the notion of the condensate wave function as an order parameter that describes the superfluid component of the liquid [2]. This superfluid component can carry momentum with no dissipation. The cornerstone of the Landau theory of superfluidity is the notion that the phase of the superfluid condensate couples to the gauge potential and that the condensate current is given by $\mathbf{J} = \rho_s(\nabla\phi - e\mathbf{A})$ where ρ_s is a measure of the superfluid density, ϕ is the phase of the condensate wave function, and $\mathbf{A}$ is the gauge potential (we use units such that $c = 1 = \hbar$). He^4, being neutral, does not carry electrical current, although it can produce nontrivial gauge potentials by rotation. Another example of a neutral superfluid is an excitonic condensate. Electrons and holes in semiconductors can form (metastable) bound states called excitons which are expected to behave as neutral bosons at low densities, and therefore can undergo Bose-Einstein condensation [3–7]. The condensate, being neutral, possesses properties similar to superfluidity. During the past few years electron-hole plasmas have been created by optically exciting the electrons from a valence band to the conduction band and then spatially confining the resulting electrons and holes to different quantum wells using a static electric field [8]. The investigation of their properties has been limited to photoluminescence measurements: processes that can probe phase coherence but not superfluidity. In particular, they have not been investigated via transport measurements which can provide a *direct* signature of their superfluid properties.

It has been argued that quantum Hall *electron-electron* bilayers at total filling factor $\nu = 1$ provide a realization of excitonic superfluid. These systems have been studied via transport; they show, among spectacular transport properties, a small but finite dissipation in the counterflow channel [9]. Here we focus on *electron-hole* systems in zero magnetic field perpendicular to the layers, discussing effects due to the presence of a physical dipole.

Recent developments in heterojunction fabrications open up an exciting possibility of electron-hole bilayer systems where the electrons reside in one layer and holes in the other layer separated by a distance d (~ 200 Å), and where the density of electrons or holes in individual layers can be adjusted using independent gates [10]. These systems have very weak but *nonzero* interlayer tunneling which allows the electrons and holes to couple with an in-plane magnetic field applied between the two layers. As a result of Coulomb attraction between electrons and holes, the excitonic condensate is expected to occur when the typical distance r_s between electrons (holes) within a layer exceeds the distance d between the two layers [11–13]. (In contrast, bilayer quantum Hall systems can be mapped onto excitonic superfluids only near specific filling factors such as $\nu = 1$.) These systems offer an alternate view of the electron-hole excitonic condensate where the condensate is neutral, yet has a well defined *dipole moment* associated with each exciton. In this Letter, we argue that this excitonic condensate represents a qualitatively new kind of superfluid where the condensate is neutral and carries no momentum density. We call this nominally neutral superfluid a *dipolar superfluid.* The dipole moment associated with each exciton in the condensate allows this liquid to couple to electromagnetic fields in a nontrivial fashion. We find that the phase of the dipolar superfluid couples to the *gradient of the gauge potential*. As a result, we predict that it will exhibit a neutral persistent dipolar current, consisting of equal and oppositely directed currents in the two layers, upon application of an in-plane magnetic field $\mathbf{B}_{||}$. Thus, the composite structure of excitons is manifest in the macroscopic superfluid state. In the following paragraphs, we present the hydrodynamics of such a superfluid based on the Ginzburg-Landau (GL) energy functional, state various predictions which follow from it (including the existence of critical field $\mathbf{B}_{||}^c$ above which the dipolar superfluidity is destroyed), and then briefly outline the derivation of the GL functional from a microscopic Hamiltonian. Because there is no long-range order in two dimensions, photoluminescence from a coherent exciton droplet is suppressed when the thermal length for phase fluctuations reaches the size of the droplet [14]. On the other hand, the phase stiffness, which determines superfluidity, persists up to the Kosterlitz-Thouless (KT) transition temperature which is much higher. Therefore, a direct probe of phase stiffness can provide a better signature of excitonic condensation. Although the true nature of the superfluid phase transition (being KT-type) cannot be described by the standard mean-field theory, we concentrate on temperatures much smaller than the KT-transition temperature, $T \ll T_{\rm KT}$, where the Hartree-Fock mean-field description is applicable [15].

GL energy functional and effective action.—Let us consider a bilayer system with electrons in the top layer and holes in the bottom layer. We introduce a notation where the $\pm$ subscript corresponds to the top and the bottom layers, respectively. With the coordinate system shown in Fig. 1 we get $\mathbf{r}_\pm = \mathbf{r} \pm \mathbf{d}/2$ where $\mathbf{d} = d\hat{z}$ is a vector normal to the layers and $\mathbf{r}$ is a two-dimensional vector within the layer. We define the excitonic condensate order parameter as

$$\Delta(\mathbf{r}) = \langle c_+^\dagger(\mathbf{r}) c_-(\mathbf{r})\rangle = |\Delta(\mathbf{r})| \exp[i\Phi(\mathbf{r})], \qquad (1)$$

where $c_\pm^\dagger(\mathbf{r})$ creates an electron in the top (bottom) layer at position $\mathbf{r}$, and we have used the fact that $c_-(\mathbf{r}) = c_h^\dagger(\mathbf{r})$ where $c_h^\dagger(\mathbf{r})$ creates a *hole* at position $\mathbf{r}$ in the bottom layer. Upon a gauge transformation $c_\pm(\mathbf{r}) \to \exp[ie\varphi_\pm(\mathbf{r})]c_\pm(\mathbf{r})$, the order parameter $\Delta(\mathbf{r})$ transforms as $\Delta(\mathbf{r}) \to \exp[i\Phi(\mathbf{r})]\Delta(\mathbf{r})$ with

$$\Phi(\mathbf{r}) \to \Phi(\mathbf{r}) - e\varphi_+(\mathbf{r}) + e\varphi_-(\mathbf{r}), \qquad (2)$$

where $-e < 0$ is the electron charge. We call the phase of the order parameter, $\Phi(\mathbf{r})$, the *dipolar phase.* It is "approximately" charge neutral since the condensate is an electron-hole condensate. However, the electron and the hole operators are always spatially separated: electrons are in the top layer and the holes are in the bottom layer.

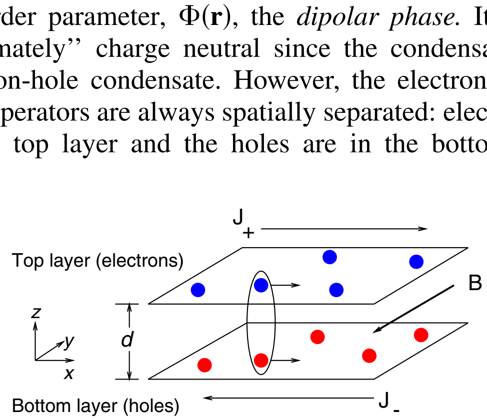

FIG. 1 (color online). Schematic bilayer electron-hole system. The electrons and holes form bound excitons which condense at low densities. We predict that an in-plane field $\mathbf{B}_{||}$ will produce equal but opposite currents $\mathbf{J}_+$ and $\mathbf{J}_-$ in the two layers.

Therefore as one winds the phases of $c_+(\mathbf{r})$ and $c_-(\mathbf{r})$ in the same direction, since $\varphi_\pm(\mathbf{r}) = \varphi(\mathbf{r} \pm \mathbf{d}/2)$, the phases that enter into the shift of the dipolar phase $\Phi(\mathbf{r})$ *are not fully compensated.* This phase shift that enters in the gauge transformation of a nominally charge-neutral dipolar phase, Eq. (2), is crucial for the hydrodynamics of the dipolar superfluid. Now we determine the coupling of the dipolar phase $\Phi(\mathbf{r})$ to external gauge potentials in the top and bottom layers, $\mathbf{A}_\pm(\mathbf{r}) = \mathbf{A}(\mathbf{r} \pm \mathbf{d}/2)$. To be consistent with U(1) gauge transformation $\mathbf{A}_\pm(\mathbf{r}) \to \mathbf{A}_\pm(\mathbf{r}) + \nabla\varphi_\pm(\mathbf{r})$ the dipolar phase must transform as

$$\nabla\Phi(\mathbf{r}) \to \nabla\Phi(\mathbf{r}) - e\mathbf{A}_+(\mathbf{r}) + e\mathbf{A}_-(\mathbf{r}). \qquad (3)$$

The gauge potentials in top and bottom layers enter with opposite signs since they couple to oppositely charged electrons and holes, respectively. Equation (3) shows that, in contrast to an ordinary neutral superfluid, the phase of the dipolar superfluid couples to the difference of the gauge potentials between the two layers.

The GL energy functional for the dipolar superfluid depends on the order parameter $\Delta(\mathbf{r})$, and for an inhomogeneous state should depend only on the gauge-invariant combinations (3) involving the gradient of the dipolar phase $\Phi(\mathbf{r})$. In particular, the gradient part of free energy is given by

$$F = \frac{1}{2}\rho_d \int_{\mathbf{r}} [\nabla\Phi(\mathbf{r}) - e\mathbf{A}_+(\mathbf{r}) + e\mathbf{A}_-(\mathbf{r})]^2, \qquad (4)$$

where ρ_d is the dipolar superfluid density [16]. It follows that the currents in the top and bottom layers are

$$\mathbf{J}_\pm(\mathbf{r}) = -\frac{\delta F}{\delta \mathbf{A}_\pm(\mathbf{r})} = \pm e\rho_d[\nabla\Phi(\mathbf{r}) - e\mathbf{a}(\mathbf{r})]. \qquad (5)$$

Here we have introduced the antisymmetric and symmetric combinations of gauge potentials, $\mathbf{a} \equiv (\mathbf{A}_+ - \mathbf{A}_-)$ and $\mathcal{A} \equiv (\mathbf{A}_+ + \mathbf{A}_-)$, which couple to the dipolar phase $\Phi(\mathbf{r})$ and the total phase $\varphi_T = \varphi_+ + \varphi_-$, respectively. Equation (5) implies that, as far as the dipolar condensate is concerned, there is no net electric current, $\mathbf{J} = (\mathbf{J}_+ + \mathbf{J}_-) = 0$. This result is naturally expected since excitons do not carry any electric or mass current unless one breaks them apart. On the other hand, the excitonic condensate *does* carry a net dipolar current

$$\mathbf{J}_d(\mathbf{r}) = 2e\rho_d[\nabla\Phi(\mathbf{r}) - e\mathbf{a}(\mathbf{r})]. \qquad (6)$$

Equation (6) is the central result of this Letter. It has exactly the same form as a supercurrent in a superconductor, $\mathbf{J} = e\rho_s(\nabla\phi - 2e\mathbf{A})$. Therefore, in analogy with a superconductor, we expect persistent dipolar currents produced by the external gauge potential $\mathbf{a}$. For a smoothly varying gauge potential, the antisymmetric combination $\mathbf{a}(\mathbf{r}) = \mathbf{A}(\mathbf{r} + \mathbf{d}/2) - \mathbf{A}(\mathbf{r} - \mathbf{d}/2) \approx d\partial_z\mathbf{A}(\mathbf{r})$ can be tuned by varying a uniform in-plane magnetic field. To be specific, let us consider magnetic field $\mathbf{B}_{||} = -B_{||}\hat{y}$ between the two layers, generated by gauge potential

$\mathbf{A}(\mathbf{r}, z) = (-B_{||}z, 0, 0)$. In this case, assuming the dipolar condensate phase is uniform, we get

$$\mathbf{J}_d = 2e^2\rho_d d B_{||}\hat{x}. \quad (7)$$

Thus, we predict that a uniform in-plane magnetic field will induce persistent and opposite currents in the top and the bottom layers in the direction perpendicular to the magnetic field. This is a direct consequence of the noncompensation of the gauge potential acting on the electron and hole parts of the dipolar condensate. Alternatively, one can view the dipolar persistent current $\mathbf{J}_d$ as arising from "perfect diamagnetism" of electrons and holes. Indeed, turning on the magnetic field $\mathbf{B}_{||}$ produces in-plane electric fields $\mathbf{E}_\pm$ which are equal and opposite. These electric fields accelerate electrons and holes *in the same direction* in respective layers. We emphasize that these time-dependent electric fields are physical and they cannot be gauged away in a simply connected geometry. The condensate phase stiffness does not allow the resulting current to decay, thus giving rise to persistent dipolar supercurrent. Since the dipolar phase couples to $\mathbf{a} \sim \mathbf{B}_{||}$, it follows that the curl of the gauge potential $\nabla \times \mathbf{a}$, which introduces vortices in the dipolar phase, is determined by gradients in $\mathbf{B}_{||}$ over the length scale d and is necessarily small for externally applied fields. In addition, the constraint $\nabla \cdot \mathbf{B}_{||} = 0$ necessitates other gradients in the magnetic field to compensate for the gradients that induce vorticity in the gauge potential $\mathbf{a}$. Thus the creation of vortices in the dipolar phase requires a nontrivial texture in the external magnetic field over very short length scales. In this sense, the dipolar superfluid is robust against the creation of vortices. This feature might make the particle-hole condensate more robust against vortex-induced dissipation in the counterflow channel.

It is straightforward to obtain the effective action for a dipolar superfluid from the GL energy functional,

$$S = \int_{\mathbf{r},t}\left[n(\partial_t\Phi - ea_0) - \frac{\rho_d}{2}(\nabla\Phi - e\mathbf{a})^2 - \frac{n^2}{2C}\right], \quad (8)$$

where $a_0 = (A_{0+} - A_{0-})$ is the difference between electrical potentials in the two layers, we have approximated the potential energy by a quadratic term with mass $1/C$ for the number fluctuations, and we have confined ourselves to a long-wavelength description [17]. The first term in the action is standard and results from the commutation relation between the dipolar condensate number and the phase, $[n, \Phi] = i$. Integrating out the massive fluctuations leads to the standard effective action for the phase

$$S_\Phi = \int_{\mathbf{r},t}\left[\frac{C}{2}(\partial_t\Phi)^2 - \frac{\rho_d}{2}(\nabla\Phi)^2\right]. \quad (9)$$

Equation (9) implies that the phase fluctuation mode (superfluid sound mode) is a gapless collective mode with dispersion $\omega_c = v_c k$ where $v_c = \sqrt{\rho_d/C}$ is the collective mode velocity. It follows from the analogy with a neutral superfluid that a state with dipolar supercurrent $\mathbf{J}_d$ is stable only if the superfluid velocity, defined by $\mathbf{J}_d = 2en_d\mathbf{v}_s = 2e\mathbf{v}_s/(\pi r_s^2)$, is smaller than the collective mode velocity v_c. Since the dipolar supercurrent is linearly proportional to $B_{||}$, we predict that for magnetic fields greater than a critical field $B_{||}^c = n_d/(ed\sqrt{\rho_d C})$, the dipolar superfluid state is destroyed by collective phase fluctuations. For typical bilayer parameters ($n_d \sim 10^{11}/\text{cm}^2$) we get the critical field $B_{||}^c \sim 100$ T much larger than typical experimental field values. We can also define critical current as the dipolar current which leads to pair-breaking effects in which an exciton gives rise to an electron and a hole. This current is given by $J_d^c = e\rho_d|\Delta|/v_\text{F}$ where v_F is the Fermi velocity. Since the dipolar current is proportional to $B_{||}$, this criterion provides another bound on the maximum value of the in-plane field (for typical parameters we get $B_{||}^c \sim 10$ T) [17].

Microscopic theory.—We briefly describe the derivation of the dipolar current, Eq. (6), from a microscopic model. The one-body Hamiltonian for an electron-hole bilayer system in the presence of gauge potentials is

$$H_1 = \sum_{\mathbf{k}\mathbf{k}'}\left[c_{+\mathbf{k}'}^\dagger\frac{1}{2m_e}(\mathbf{p} - e\mathbf{A}_+)^2_{\mathbf{k}'\mathbf{k}}c_{+\mathbf{k}} + c_{-\mathbf{k}'}\frac{1}{2m_h}(\mathbf{p} + e\mathbf{A}_-)^2_{\mathbf{k}'\mathbf{k}}c_{-\mathbf{k}}^\dagger\right], \quad (10)$$

where m_e (m_h) is the electron (hole) band mass. The interaction term in the microscopic Hamiltonian is a sum of the intralayer repulsive interaction, $V_A(\mathbf{k}) = 2\pi e^2/k$, and interlayer attractive interaction, $V_E(\mathbf{k}) = -V_A(\mathbf{k})e^{-kd}$. We use Hubbard-Stratonovich transformation to introduce the order-parameter fields and obtain mean-field equations [18]. The attractive interaction leads to the pairing of electrons and holes near the Fermi surface ($|\mathbf{k}| \approx k_F$) and a nonzero dipolar condensate order parameter $\Delta = \langle c_+^\dagger c_-\rangle = \langle c_e^\dagger c_h^\dagger\rangle$. In the absence of external gauge potentials, the mean-field Hamiltonian is given by

$$H_0 = \sum_{\mathbf{k}\sigma\sigma'}c_{\mathbf{k}\sigma}^\dagger[\epsilon_{3\mathbf{k}}\tau^3 + \text{Re}\,\Delta_\mathbf{k}\tau^1 + \text{Im}\,\Delta_\mathbf{k}\tau^2]_{\sigma\sigma'}c_{\mathbf{k}\sigma'}, \quad (11)$$

where $\sigma, \sigma' = \pm$ is the layer index, $\epsilon_3 = (\epsilon_e + \epsilon_h)/2$ is the mean-band energy, and $\epsilon_{e(h)\mathbf{k}} = \hbar^2k^2/2m_{e(h)}$ is the electron (hole) dispersion. In Eq. (11) we have neglected a constant term proportional to $\epsilon_0 = (\epsilon_e - \epsilon_h)/2$ since it vanishes for symmetric electron-hole bilayers, $m_e = m_h = m^*$, and does not qualitatively change the conclusions. The eigenvalues of the mean-field Hamiltonian are given by $E_\pm(\mathbf{k}) = \epsilon_{0\mathbf{k}} \pm \sqrt{\epsilon_{3\mathbf{k}}^2 + |\Delta_\mathbf{k}|^2}$ and the corresponding mean-field Green's function is $G_0^{-1}(\mathbf{k}, i\omega_n) = (i\omega_n - H_0)$. Now we consider the effect of small gauge potential perturbations on the condensate state. To linear order, the change in the Hamiltonian is $\delta\epsilon_{e(h)} = \mp e[\mathbf{p}\cdot\mathbf{A}_\pm + \mathbf{A}_\pm\cdot\mathbf{p}]/(2m_{e(h)})$, which leads to the change in the Green's function, $G^{-1} = G_0^{-1} - \delta G^{-1}$, where

$$\delta G^{-1} = -\frac{e}{2}\left\{\frac{1}{m_e}[\mathbf{p}, \mathbf{A}_+]_+ - \frac{1}{m_h}[\mathbf{p}, \mathbf{A}_-]_+\right\}. \tag{12}$$

Note that the gauge potentials in the two layers have a relative minus sign because the charges of the carriers in the two layers are opposite. This is also consistent with the GL energy functional description in which the dipolar phase is coupled to the *difference* of gauge potentials in the two layers. The change in the free energy due to a change in the gauge potential is given by $\delta F = -\mathrm{Tr}(G_0\delta G^{-1}G_0\delta G^{-1})/2$ where "Tr" denotes the trace over space-time and layer-index degrees of freedom [18]. For symmetric electron-hole bilayers, in the long-wavelength limit, the matrix δG^{-1} in layer-index space is given by $\delta G^{-1} = -e\mathbf{k}_F \cdot [\mathbf{a}\mathbf{1} + \mathcal{A}\tau^3]/(2m^*)$. Evaluating the trace over space-time degrees leads to the dipolar superfluid current $\mathbf{J}_d$

$$\mathbf{J}_d = \frac{e^2 v_F^2}{4}\mathrm{tr}[G_0 \cdot \mathbf{1} \cdot G_0(\mathbf{a}\mathbf{1} + \mathcal{A}\tau^3)], \tag{13}$$

with a similar expression for the total mass current $\mathbf{J}$. Note that only the term proportional to $\mathbf{a}\mathbf{1}$ survives in the trace for the dipolar current $\mathbf{J}_d$. This reflects the fact that the dipolar condensate phase Φ responds to the antisymmetric combination of gauge potentials. Since we are expanding around $\mathbf{A} = 0$ and since the effect of $e\mathbf{a}$ is the same as that of $\nabla\Phi$, only the gauge-invariant combination enters the expression for the dipolar current,

$$\mathbf{J}_d = \frac{2ev_F^2}{8}\mathrm{tr}[G_0\mathbf{1}G_0\mathbf{1}](\nabla\Phi - e\mathbf{a}) = 2e\rho_d[\nabla\Phi - e\mathbf{a}]. \tag{14}$$

Thus we recover Eq. (6) with the appropriate definition of dipolar superfluid density from a microscopic calculation. In general, if we expand around a nonzero $\mathcal{A}$, which corresponds to nonuniform fields, we will also get a nonzero total mass current $\mathbf{J}$.

Discussion.—The possibility of excitonic condensation in semiconductors has been discussed in the literature for a long time. Recent claims of observations of such condensates depend primarily on photoluminescence measurements and do not directly probe their superfluid properties. We have argued that excitonic condensates in electron-hole bilayer systems present a qualitatively different superfluid in which the phase of the condensate is coupled to the *gradient* of the gauge potential. As a result we predict that such systems will develop a persistent dipolar supercurrent with the application of an in-plane magnetic field. This supercurrent can be detected via separate contacts to the electron and the hole layers and will provide a *direct* signature of the superfluid properties of excitonic condensates. As a possible realization of this experiment in available samples, we propose the study of induced charges and voltages in each layer in response to an ac in-plane magnetic field with frequency ω. In the absence of the excitonic condensate, Faraday induction will lead to a dipolar current $J_d \sim \omega B$ which in turn will induce equal and opposite charges $Q_\pm \sim \pm B$ in the two layers. In contrast, in the presence of condensate, the dipolar current will be $J_d \sim B$ and therefore the induced charges will be given by $Q_\pm \sim \pm B/\omega$, leading to a very different frequency dependence. In the current experimental setups, where the excitons are weakly confined in a parallel trap and recombine by optical emission, our analysis implies that the position of the spot, where recombinations take place, will oscillate with an applied oscillating in-plane magnetic field.

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