Resummed QCD Power Corrections to Nuclear Shadowing

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We calculate and resum a perturbative expansion of nuclear enhanced power corrections to the structure functions measured in deeply inelastic scattering of leptons on a nuclear target. Our results for the Bjorken x, Q^2 and A dependence of nuclear shadowing in $F_2^A(x, Q^2)$ and the nuclear modification to $F_L^A(x, Q^2)$, obtained in terms of the QCD factorization approach, are consistent with the existing data. We demonstrate that the low Q^2 behavior of these data and the measured large longitudinal structure function point to a critical role for the power corrections when compared to other theoretical approaches.

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In order to understand the overwhelming data from the BNL Relativistic Heavy Ion Collider (RHIC) and make predictions for the future Electron Ion Collider (EIC) and CERN Large Hadron Collider (LHC) in terms of the successful perturbative QCD factorization approach [1], we need precise information of the nuclear parton distribution functions (NPDFs) $\phi_f^A(x, \mu^2)$ of flavor f, atomic weight A, momentum fraction x, and factorization scale μ. Although the μ dependence of leading twist NPDFs can be calculated in terms of perturbative QCD (PQCD) evolution equations, the x- and A-dependent boundary condition at a scale μ_0 must be fixed from existing measurements, mostly of the structure function $F_2^A(x, \mu^2)$ in lepton-nucleus deeply inelastic scattering (DIS) [2]. It was pointed out recently [3] that the available fixed-target data contain significant higher twist effects hindering the extraction of the nuclear parton distributions.

On the other hand, it has been argued that for physical processes where the effective x is very small and the typical momentum exchange of the collision $Q \sim \mu$ is not large the number of soft partons in a nucleus may saturate [4]. Qualitatively, the unknown boundary of this novel regime in (x, Q) is where the conventional PQCD factorization approach should fail [5].

In this Letter we present a PQCD calculation of the resummed power corrections to the DIS nuclear structure functions and demonstrate their importance in extracting NPDFs. From our results we *quantitatively* identify the characteristic scale of these power corrections $\xi^2 \ll m_N^2$, with nucleon mass $m_N = 0.94$ GeV. We conclude that for $Q^2 \geq m_N^2$ inclusive lepton-nucleus DIS is above the saturation boundary and can be treated systematically in the framework of the PQCD factorization approach with resummed high twist contributions.

Under the approximation of one-photon exchange, the lepton-hadron DIS cross section $d\sigma_{\ell h}/dxdQ^2 \propto L_{\mu\nu}W^{\mu\nu}(x, Q^2)$, with Bjorken variable $x = Q^2/(2p \cdot q)$ and virtual photon's invariant mass $q^2 = -Q^2$. The leptonic tensor $L_{\mu\nu}$ and hadronic tensor $W^{\mu\nu}$ are defined in [6]. The hadronic tensor can be expressed in terms of structure functions based on the polarization states of the exchange virtual photon: $W^{\mu\nu}(x, Q^2) = \epsilon_T^{\mu\nu}F_T(x, Q^2) + \epsilon_L^{\mu\nu}F_L(x, Q^2)$, where $\epsilon_L^{\mu\nu}, \epsilon_T^{\mu\nu}$ are given in [7]. The transverse and longitudinal structure functions are related to the standard DIS structure functions F_1, F_2 as follows: $F_T(x, Q^2) = F_1(x, Q^2)$, $F_L(x, Q^2) = F_2(x, Q^2)/(2x) - F_1(x, Q^2)$ if $4x^2m_N^2 \ll Q^2$ [6]. In DIS the exchange photon γ^* of virtuality Q^2 and energy $\nu = Q^2/(2xm_N)$ probes an effective volume of transverse area $1/Q^2$ and longitudinal extent $\Delta z_N \times x_N/x$, where Δz_N is the nucleon size, $x_N = 1/(2r_0m_N) \sim 0.1$ and $r_0 \sim 1.2$ fm [5]. When Bjorken $x \ll x_N$ the lepton-nucleus DIS covers several nucleons in longitudinal direction while it is localized in the transverse plane. Although a hard interaction involving more than one nucleon is suppressed by powers of $1/Q^2$ [5], it is amplified by the nuclear size if $x \ll x_N$. In the collinear factorization approach we evaluate the nuclear enhanced power corrections to the structure functions to any power in the quantity $(\xi^2/Q^2)(A^{1/3} - 1)$ and keep ξ^2 to the leading order in α_s.

In terms of collinear QCD factorization, the structure functions at the lowest order in α_s are given by [6]

$$F_T^{(\mathrm{LT})}(x, Q^2) = \frac{1}{2}\sum_f Q_f^2 \phi_f(x, Q^2) + \mathcal{O}(\alpha_s), \qquad (1)$$

$$F_L^{(\mathrm{LT})}(x, Q^2) = \mathcal{O}(\alpha_s), \qquad (2)$$

where (LT) indicates the leading twist contribution, $\sum_f$ runs over the (anti)quark flavors, Q_f is their fractional charge and ϕ_f is the leading twist quark distribution

$$\phi_f(x, \mu^2) = \int \frac{d\lambda_0}{2\pi} e^{ix\lambda_0} \langle p|\bar{\psi}_f(0)\frac{\gamma^+}{2p^+}\psi_f(\lambda_0)|p\rangle \qquad (3)$$

in the light cone $A^+ = n^\mu A_\mu = 0$ gauge for hadron momentum $p^\mu = p^+\bar{n}^\mu$, where $\bar{n}^\mu = [1, 0, 0_\perp]$ and $n^\mu = [0, 1, 0_\perp]$ specify the "+" and "−" light cone directions, respectively. In Eq. (3) the parameter $\lambda_0 = p^+y_0^-$.

To compute the nuclear enhanced power corrections we choose the $A^+ = 0$ gauge and a frame in which the nucleus is moving in the "+" direction, $p^\mu \equiv P_A^\mu/A = p^+\bar{n}^\mu$, and the exchange virtual photon momentum is $q^\mu = -xp^+\bar{n}^\mu + Q^2/(2xp^+)n^\mu$. In this frame the struck quark propagates along the "−" direction and interacts with the "remnants" of the nucleus. The leading tree level contributions to $W^{\mu\nu}$ in lepton-nucleus DIS are given by the Feynman diagrams in Fig. 1(a). The cutline represents the final-state quark and the blob connected to γ^*—the lowest order coupling between the photon and the active partons. For transversely polarized photons Fig. 1(b) gives the leading twist contribution in Eq. (1). For longitudinally polarized γ^* the leading tree coupling Fig. 1(c) results in the first power correction to F_L,

$$F_L(x, Q^2) = F^{(\mathrm{LT})}(x, Q^2) + \frac{1}{2}\sum_f Q_f^2 4\left(\frac{4\pi^2\alpha_s}{3Q^2}\right) \times \int \frac{d\lambda_0}{2\pi} e^{ix\lambda_0}\langle p|\bar{\psi}_f(0)\frac{\gamma^+}{2p^+}\psi_f(\lambda_0)\hat{F}^2_{\lambda_0}|p\rangle, \tag{4}$$

where the $\mathcal{O}(\alpha_s)F_L^{(\mathrm{LT})}$ is given in [8] and the operator

$$\hat{F}^2_{\lambda_0} = \int \frac{d\tilde{\lambda}_n d\tilde{\lambda}_0}{2\pi}\frac{F^{+\alpha}(\tilde{\lambda}_n)F^+_\alpha(\tilde{\lambda}_0)}{(p^+)^2}\theta(\tilde{\lambda}_n)\theta(\tilde{\lambda}_0 - \lambda_0). \tag{5}$$

Since the effective γ_L^* coupling is made of short-distance contact terms of quark propagators [9], the integrals of $\tilde{\lambda}_0$ and $\tilde{\lambda}_n$ in Eq. (5) should be localized.

To derive the gauge invariant and nuclear enhanced power corrections at the tree level, we add gluon interactions to the struck quark and convert the gluon field operators in the hadronic matrix element of $W^{\mu\nu}$ to the corresponding field strength [5,7]. We note that there must be an *even* number of interactions between the γ^* coupling and the final-state cut because the quark propagator of momentum $x_i p + q$ has only two types of terms [9], $i(\gamma^+/2p^+)/(x_i - x \pm i\epsilon)$ (pole term $\rightarrow\!\!\times\!\!\rightarrow$) and $i(xp^+/Q^2)\gamma^-$ (contact term $\rightarrow\!\!|\!\!\rightarrow$), and the gluons are transversely polarized. Consequently, the summation of the coherent multiple scattering can be achieved by sequential insertion of a basic unit consisting of a pair of gluon interactions, leftside of Fig. 1(d), connected by a quark contact term, and a quark pole term to the left (L) or right (R) if the unit is to the left or right of the final-state cut. Integrating over the loop momentum fraction $\tilde{x}_i$, we can replace this unit by an effective scalar interaction, rightside of Fig. 1(d), with a rule,

$$\left(x\frac{4\pi^2\alpha_s}{3Q^2}\right)\int\frac{d\lambda_i}{2\pi}\frac{e^{i(x_i-x_{i-1})\lambda_i}}{x_i - x_{i-1} - i\epsilon}\begin{cases}\frac{\gamma^-\gamma^+}{2}\frac{-i\hat{F}^2(\lambda_i)}{x_{i-1}-x+i\epsilon} & \mathrm{L}\\ \frac{\gamma^+\gamma^-}{2}\frac{-i\hat{F}^2(\lambda_i)}{x_i-x-i\epsilon} & \mathrm{R}\end{cases}. \tag{6}$$

In Eq. (6) the boost-invariant operator $\hat{F}^2(\lambda_i)$ is different from $\hat{F}^2_{\lambda_0}$ in Eq. (5) and is defined as

$$\hat{F}^2(\lambda_i) \equiv \int\frac{d\tilde{\lambda}_i}{2\pi}\frac{F^{+\alpha}(\lambda_i)F^+_\alpha(\tilde{\lambda}_i)}{(p^+)^2}\theta(\lambda_i - \tilde{\lambda}_i). \tag{7}$$

Its expectation value can be related to the small-x limit of the gluon distribution, $\langle p|\hat{F}^2(\lambda_i)|p\rangle \approx \lim_{x\to 0}\frac{1}{2}xG(x, Q^2)$, and is independent of λ_i. It is the $\int d\lambda_i$ in Eq. (6) that gives the $A^{1/3}$-type nuclear enhancement [5]. Assuming that the interaction leaves the nucleons in a color singlet state, taking all possible final-state cuts [10] and performing the integrals over the incoming partons' momentum fractions we obtain the twist $2 + 2n$ contribution to the structure functions:

$$\delta F_T^{(n)} \approx \frac{1}{2}\sum_f Q_f^2\left[x\frac{4\pi^2\alpha_s}{3Q^2}\right]^n \times \int\frac{d\lambda_0}{2\pi}e^{ix\lambda_0}\frac{(i\lambda_0)^n}{n!}\langle P_A|\bar{\psi}_f(0)\frac{\gamma^+}{2p^+}\psi_f(\lambda_0) \times \prod_{i=1}^{n}\left[\int d\lambda_i\theta(\lambda_i)\hat{F}^2(\lambda_i)\right]|P_A\rangle, \tag{8}$$

$$\delta F_L^{(n)} \approx \frac{1}{2}\sum_f Q_f^2\frac{4}{x}\left[x\frac{4\pi^2\alpha_s}{3Q^2}\right]^n \times \int\frac{d\lambda_0}{2\pi}e^{ix\lambda_0}\frac{(i\lambda_0)^{n-1}}{(n-1)!}\langle P_A|\bar{\psi}_f(0)\frac{\gamma^+}{2p^+}\psi_f(\lambda_0)\hat{F}^2_{\lambda_0} \times \prod_{i=1}^{n-1}\left[\int d\lambda_i\theta(\lambda_i)\hat{F}^2(\lambda_i)\right]|P_A\rangle, \tag{9}$$

with corrections down by powers of the nuclear size.

We evaluate the multifield matrix elements in Eqs. (8) and (9) in a model of a nucleus of constant lab frame

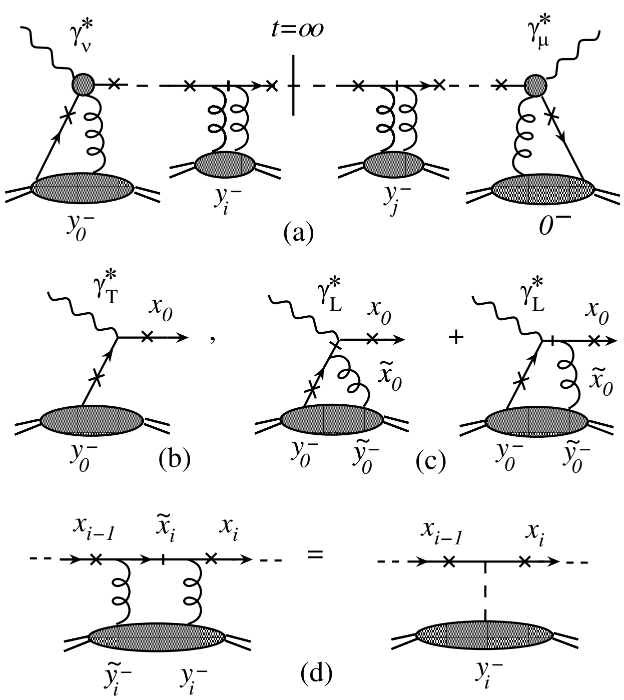

FIG. 1. (a) Tree level multiple final-state scattering of the struck quark in DIS on nuclei; (b) and (c) leading coupling for transversely and longitudinally polarized photons to the active partons, respectively; (d) effective scalar vertex for the basic two-gluon interactions.

density $\rho(r) = 3/(4\pi r_0^3)$ and approximate the expectation value of the product of operators to be a product of expectation values of the basic operator units in a nucleon state of momentum $p = P_A/A$:

$$\langle P_A|\hat{O}_0 \prod_{i=1}^{n} \hat{O}_i|P_A\rangle = A\langle p|\hat{O}_0|p\rangle \prod_{i=1}^{n}[N_p\langle p|\hat{O}_i|p\rangle],$$

with the normalization $N_p = 3/(8\pi r_0^3 m_N)$. The integrals $\int d\lambda_i \theta(\lambda_i) = (3r_0 m_N/4)(A^{1/3} - 1)$ are taken such that the nuclear effect vanishes for $A = 1$. Resumming the $A^{1/3}$-enhanced power corrections Eqs. (8) and (9), we find:

$$F_T^A(x, Q^2) \approx \sum_{n=0}^{N} \frac{A}{n!}\left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n \frac{d^n F_T^{(\mathrm{LT})}(x, Q^2)}{d^n x}$$
$$\approx AF_T^{(\mathrm{LT})}\left(x + \frac{x\xi^2(A^{1/3}-1)}{Q^2}, Q^2\right), \quad (10)$$

$$F_L^A(x, Q^2) \approx AF_L^{(\mathrm{LT})}(x, Q^2) + \sum_{n=0}^{N} \frac{A}{n!}\left(\frac{4\xi^2}{Q^2}\right)$$
$$\times \left[\frac{\xi^2(A^{1/3}-1)}{Q^2}\right]^n x^n \frac{d^n F_T^{(\mathrm{LT})}(x, Q^2)}{d^n x}$$
$$\approx AF_L^{(\mathrm{LT})}(x, Q^2) + \frac{4\xi^2}{Q^2} F_T^A(x, Q^2), \quad (11)$$

where N is the upper limit on the number of quark-nucleon interactions and ξ^2 represents the characteristic scale of quark-initiated power corrections to the leading order in α_s

$$\xi^2 = \frac{3\pi\alpha_s(Q^2)}{8r_0^2}\langle p|\hat{F}^2(\lambda_i)|p\rangle.$$

In deriving Eqs. (10) and (11), we have taken $\langle p|\hat{F}^2_{\lambda_0}|p\rangle \approx (3r_0 m_N/4)\langle p|\hat{F}^2(\lambda_i)|p\rangle$ and $N \approx \infty$ because the effective value of ξ^2 is relatively small, as shown below.

Eqations (10) and (11), are the main result of this Letter. Important applications to other QCD processes and observables that naturally follow from this new approach are given in [11]. The overall factor A takes into account the leading dependence on the atomic weight and the isospin average over the protons and neutrons in the nucleus is implicit. We emphasize the simplicity of the end result, which amounts to a shift of the Bjorken x by $\Delta x = x\xi^2(A^{1/3} - 1)/Q^2$ with only one parameter $\xi^2 \propto \lim_{x\to 0} xG(x, Q^2)$. In the following numerical evaluation we use the lowest order CTEQ6 PDFs [12].

Figure 2 shows a point by point in (x, Q^2) calculation of the process-dependent modification to $F_2(A)/F_2(D)$ (per nucleon) in the shadowing $x < 0.1$ region compared to NA37 and E665 data [13,14]. We find that a value of $\xi^2 = 0.09$-$0.12\ \mathrm{GeV}^2$, which is compatible with the range from previous analysis [15] of Drell-Yan transverse momentum broadening ($\xi^2 \sim 0.04\ \mathrm{GeV}^2$) and momentum imbalance in dijet photoproduction ($\xi^2 \sim 0.2\ \mathrm{GeV}^2$), makes our calculations consistent with the both x- and A-dependence of the data. Our calculations might have overestimated the shift in the region x close to x_N where the $N \approx \infty$ should fail [11]. In Fig. 2, we impose $Q^2 = m_N^2$ for virtualities smaller than the nucleon mass, below which high order corrections in $\alpha_s(Q)$ need to be included and the conventional factorization approach might not be valid. Our result is comparable to the EKS98 scale-dependent parametrization [2] of existing data on the nuclear modification to $F_2^A(x, Q^2)$, as seen in the Δ_{D-T} = Data-Theory panels of Fig. 2. We emphasize, however, that the physical interpretation is different: in [2] the effect is attributed to the modification of the input parton distributions at $\mu_0 = 1.5$ GeV in a nucleus and its subsequent leading twist scale dependence. In contrast, our resummed QCD power corrections to the structure functions systematically cover higher twist for all values of $Q \geq \mu_0$.

With ξ^2 fixed, Fig. 3 shows the predicted Q^2 dependence of $F_2(Sn)/F_2(C)$. The Q^2 behavior of our result, distinctly

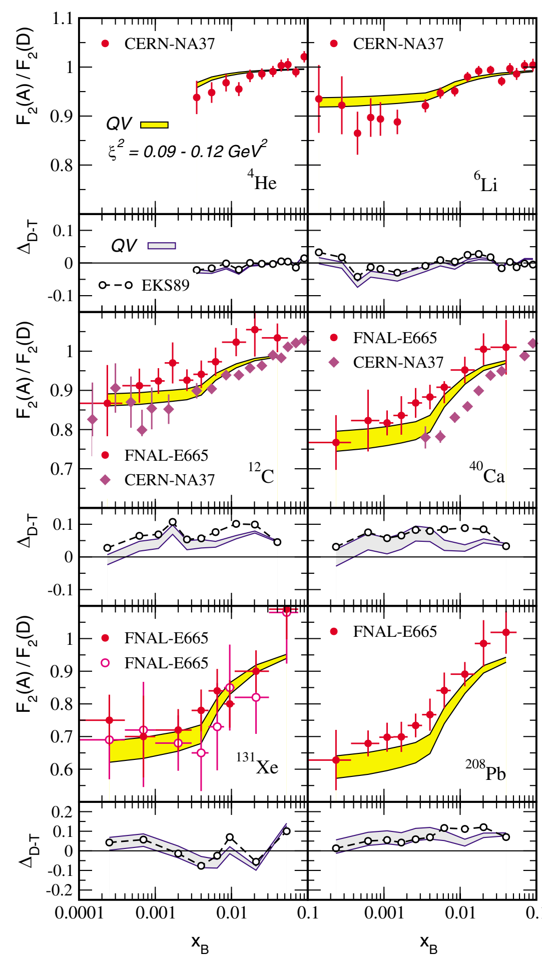

FIG. 2 (color online). All-twist resummed $F_2(A)/F_2(D)$ calculation from Eqs. (10) and (11) vs CERN-NA37 [14] and Fermilab E665 [13] data on DIS on nuclei. The band corresponds to the choice $\xi^2 = 0.09$-$0.12\ \mathrm{GeV}^2$. Data-Theory, where Δ_{D-T} is computed for the set presented by circles, also shows comparison to the Eskola-Kolhinen-Salgado 1998 scale-dependent shadowing parametrization [2].

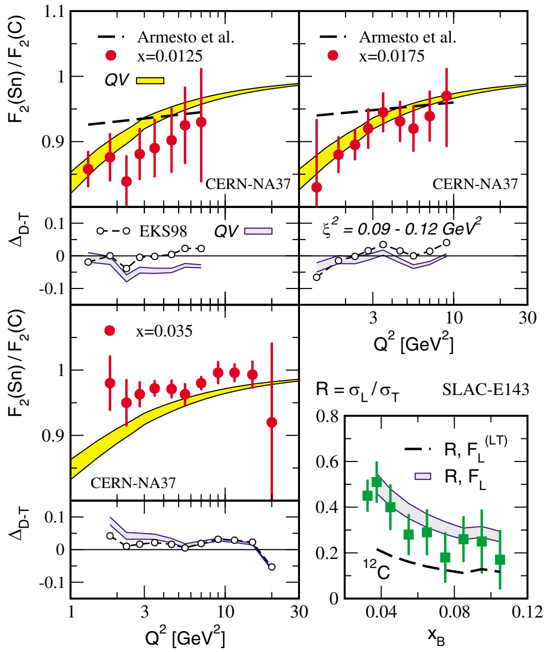

FIG. 3 (color online). CERN-NA37 data [17] on $F_2(Sn)/F_2(C)$ show evidence for a power-law in $1/Q^2$ behavior consistent with the all-twist resummed calculation. Comparison to Gribov-Regge model by Armesto *et al.* [16] and the EKS98 shadowing parametrization [2] (in Δ_{D-T}) is also presented. The bottom right panel illustrates the role of higher twist contributions to F_L on the example of $R = \sigma_L/\sigma_T$. Data is from SLAC-E143 [18].

different from a Gribov-Regge model calculation by Armesto *et al.* [16], highlights the important role of resummed QCD power corrections, when presented versus NA37 data [17]. Data-Theory panels again show comparison to the EKS98 parameterization [2]. The QCD power corrections at small and moderate Q^2 to the leading twist structure functions can also be tested via the ratio of the cross sections for longitudinally and transversely polarized virtual photons

$$R(x, Q^2; A) = \frac{\sigma_L}{\sigma_T} = \frac{F_L^A(x, Q^2)}{F_T^A(x, Q^2)}. \tag{12}$$

Comparison to E143 data for ^{12}C [18] is given in the bottom right panel of Fig. 3. The dashed curve includes only the bremsstrahlung and gluon splitting $F_L^{(LT)}$ from [8] and is insensitive to modifications of the nuclear parton distributions. The gray band represents a calculation of R from Eqs. (10) and (11).

In conclusion, for $\xi^2 = 0.09$-0.12 GeV^2 our calculated and resummed power corrections are consistent with the x-, Q^2- and A-dependence of existing data on the small-x nuclear structure functions without any leading twist shadowing. Our results, therefore, give an *upper limit* for the characteristic scale ξ^2 of these power corrections in DIS on cold nuclear matter. Furthermore, the enhancement of the longitudinal structure function $F_L^A(x, Q^2)$ strongly favors a critical role for the higher twist in the presently accessible (small $x < 0.1$, $Q^2 \sim$ few GeV^2) kinematic regime. Less leading twist shadowing and correspondingly less antishadowing than currently anticipated will have an important impact on the interpretation of the $d + A$ and $Au + Au$ data from RHIC [19]. The predictions of this systematic approach to the process-dependent low Q^2 nuclear modification in QCD processes will also be soon confronted by copious new data from RHIC, EIC and the LHC.

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