Relativistic Generalization of the Landau Criterion as a New Foundation of the Vavilov-Cherenkov Radiation Theory

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The relativistic generalization of the Landau criterion is obtained for the determination of threshold conversion of medium elementary Bose-condensed excitation into Cherenkov's photon unbremsstrahlung radiation. In contraposition to classic Vavilov-Cherenkov radiation (VCR) theory, the new VCR theory admits the conditions for effective and direct VCR realization even for high-frequency transverse electromagnetic waves in isotropic plasma.

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The Vavilov-Cherenkov radiation (VCR) phenomenon has justly become an inherent part of modern physics [1]. The VCR in a refractive medium was experimentally discovered by Cherenkov and Vavilov [2] more that half a century ago. This was also the time when Tamm and Frank developed the electromagnetic macroscopical theory of this phenomenon, which, as well as the VCR discovery, was marked later by a Nobel prize [2–4]. The Tamm-Frank theory appeared to be very similar to the Heaviside theory, which had been forgotten for a century [5].

The Heaviside-Tamm-Frank theory (HTF) demonstrated that the cylindrically symmetrical electromagnetic field, created in a medium by an electron, which moves rectilinearly with the constant velocity v_0, does not exponentially reduce only in the case of superthreshold electron velocity $v_0 \geq c/n$, where c is the light speed in vacuum and n is the medium refractive index. According to the HTF theory, this field must be identical to the VCR field, observed in the experiment.

However, such direct identification is not in agreement with the basic microscopical conception, that VCR photons are radiated by a medium, and not by an electron itself [3,6]. The latter can serve only for the initiation of such radiation by the medium. The phenomenological quantum theory of the VCR, developed by Ginzburg [1,7,8], as well as macroscopical HTF theory, still does not take into consideration the changes of the radiating medium energy state, which might be necessary for the VCR realization. The quantum VCR theory [1,7,8] reflects only on the changes of some medium momentum (but not in energy) state in an implicit, rather special and debatable [9,10] form. This is due to the fact that the quantum theory [1,7,8] uses the Minkovsky presentation for the photon momentum in a medium: $\mathbf{p}_m = \frac{\varepsilon_p n \mathbf{k}}{c}$, where $\mathbf{k} = \frac{\mathbf{v}_p}{v_p}$, c is the light speed in vacuum, and ε_p and $\mathbf{v}_p$ are, respectively. the energy and velocity of a photon in a medium with $n > 1$. From the other side, indeed, $\mathbf{p}_m = \mathbf{p}_a + \mathbf{p}_c$, where $\mathbf{p}_a = \frac{\varepsilon_p}{cn}\mathbf{k}$ is the Abraham form of the momentum of the photon itself and $\mathbf{p}_c = \frac{\varepsilon_p (n^2-1)\mathbf{k}}{nc}$ is the Abraham force impulse, which is on average transferred to the medium by a moving photon [8–12].

Thus, the classic theory of the VCR phenomenon leaves a question of the energy mechanism of the VCR effect realization in a medium open. Indeed, to elaborate this mechanism we need to find out the necessary possible changes of the energy state of the medium itself, which ensure the VCR effect realization.

On the basis of the statement of the relativistic generalization form of the Landau criterion [13], we obtained here new necessary conditions of the realization of the VCR phenomenon. In contraposition to classic VCR theory, these conditions extend the region of possible manifestations of the VCR effect, since we have a presentation for the threshold velocity in the form of $v_{\mathrm{th}} = c/n_*(n)$, where $n_* > n$ for any n and $n_* > 1$ even when $n < 1$, as for plasma. In this new VCR theory the VCR phenomenon has the same nature as for numerous physical systems with dissipative instability, in which the realization of corresponding excitations in a medium becomes energetically favorable [13–18], when some conditions become superthreshold. The suggested theory is based on directly using the Abraham momentum of photon $\mathbf{p}_a$ in a medium with $n \neq 1$ and on the consideration of nonzero real photon rest mass m_p, which is connected with this photon momentum $\mathbf{p}_a$ by the relativistic equation [19] $m_p^2 c^2 = \frac{\varepsilon_p^2}{c^2} - p_a^2$. The energy $m_p c^2$ may correspond to the energy of the extremely long wave (Bose-condensed) elementary excitation of medium, which can transformed into a VCR photon. In the last part of the work the comparison of the new VCR theory with the results of Cherenkov's (1959) experiments [2] is made and it gives a better correspondence, than for classic VCR theory [1,3,4,6–8] in the describing of threshold edge of VCR cone of rays.

1. *New quantum VCR theory.*—In the laboratory coordinate system, let us consider a resting unbounded medium with the mass M and a particle (electron) with the

mass m_e, which moves rectilinearly through this medium with the constant velocity $\mathbf{v}_0$. Dissipation, which is due to the polarization interaction between the fast moving electron and the medium, affects the energy state of the system and, thus, may give an energetically favorable possibility of the birth of a photon in the medium, as well as rotons born in superfluid helium [13], when it moves through a capillary vessel with some superthreshold velocity.

To determine the analogous threshold value of $\mathbf{v}_0$ in the modified quantum theory, we may use the following equations of energy and momentum conservation laws:

$$m_e c^2 \Gamma_0 + Mc^2 = (M - \Delta M)c^2 \Gamma_2 + \varepsilon_p + m_e c^2 \Gamma_1, \quad (1)$$

$$m_e \mathbf{v}_0 \Gamma_0 = (M - \Delta M)\mathbf{v_2}\Gamma_2 + \frac{\varepsilon_p \mathbf{v}_p}{c^2} + m_e \mathbf{v}_1 \Gamma_1, \quad (2)$$

where $\Gamma_\alpha = 1/\sqrt{1 - \frac{v_\alpha^2}{c^2}}$ ($\alpha = 0, 1, 2$), $\Delta M \ll M$, and ΔM is the medium mass loss, connected with the energy of Bose-condensed excitation of the medium which is due to the emission just from the medium [3,6] of the photon with its rest mass m_p, energy $\varepsilon_p = \hbar\omega$, and momentum $\mathbf{p}_a = \frac{\varepsilon_p \mathbf{v}_p}{c^2}$. Here $v_p \equiv |\mathbf{v}_p| = \frac{c}{n}$ for $n > 1$ and $\mathrm{v_p} = cn$ if $n < 1$, where v_p is the magnitude of photon velocity in a nondissipative medium with the refractive index $n \neq 1$. Actually vector $\mathbf{v}_p$ is the velocity of energy transfer and for nondissipative media the meaning of the $\mathbf{v}_p$ also must coincide with the group velocity $\mathbf{v}_{\rm gr}$ of electromagnetic wave, where $v_{\rm gr} = |\mathbf{v}_{\rm gr}| = c/n$ for $n > 1$ ([8], p. 142) and $v_{\rm gr} = cn$ for $n < 1$ ([8], p. 151). This well-known presentation of $\mathbf{v}_{gr}$ agrees with the Maxwell equation in media [8]. Thus, the dependence of the Abraham momentum $\mathbf{p}_a$ on ε_p satisfies the relativistic equation for the momentum of a free particle having the velocity $\mathbf{v}_p$ and energy ε_p [19]. The Abraham form of photon momentum in the medium is also consistent with the famous Plank relation between the energy flux and the corresponding electromagnetic field momentum [6,12]. If $\varepsilon_p = 0$ and the radiation of the photon is impossible, then $\Delta M = 0$ and $v_0 = \mathbf{v}_1$, $\mathbf{v}_2 = 0$ in (1) and (2). Thus, in consistency with (1) and (2), it is possible to evaluate ΔM from the following equation (only if $\Delta M = m_p$, because it is possibly a more common case with $\Delta M > m_p$ [20]):

$$\Delta M = m_p = \frac{\varepsilon_p}{c^2}\sqrt{1 - \frac{v_p^2}{c^2}} = \begin{cases} \frac{\hbar\omega}{c^2}\frac{\sqrt{n^2-1}}{n}, & n > 1, \\ \frac{\hbar\omega}{c^2}\sqrt{1 - n^2}, & n < 1. \end{cases} \quad (3)$$

For the VCR with the optical wavelengths, Eq. (3) yields $m_p/m_e \approx 10^{-5}$ (for water with $n = 1, 33$).

This estimate is many orders greater than the upper limit of a photon mass in vacuum, and therefore it may be interesting to obtain the generalization of Maxwell-Proca theory [21–23] for electromagnetic fields in media with $n \neq 1$.

Equation (3) for $n < 1$ exactly coincides with the well-known [6,8] dispersion equation for high-frequency transverse electromagnetic waves in isotropic noncollision plasma if $m_p = \Delta M$ is fixed by equation $m_p = \frac{\hbar\omega_p}{c^2}$, where ω_p is the plasma frequency. Thus, ΔMc^2 must have a physical meaning as some potentially accessible energy, which determines the possibility of the VCR realization in a certain frequency region, due to the interaction between the given medium and a fast moving electron. For example, the VCR realization in isotropic plasma ($n < 1$) takes place only due to the conversion of longitude, extremely long wave plasmon with energy ΔMc^2 in transverse Cherenkov's photon for superthreshold velocity $\mathbf{v}_0$ of electron. In plasma physics the initiation by a fast particle of the conversion process of longitude plasmon into a transverse high frequency photon was also considered in [24,25], but without any connection with the VCR phenomenon.

Equations (1) and (2) may coincide with the corresponding equations of the Ginzburg quantum theory [1,7,8], if $n > 1$, $\Delta M = 0$, and if the very special medium momentum $\tilde{\mathbf{p}}_c$ ($\tilde{\mathbf{p}}_c = M\mathbf{v}_2\Gamma_2$) presentation $\tilde{\mathbf{p}}_c = \mathbf{p}_c$ is used. Obviously, in the general case, the latter equation may be violated.

Let us consider (1)–(3) for $n > 1$ in the limit $\mathbf{v}_2 \to 0$ and $\tilde{\mathbf{p}}_c = M\mathbf{v}_2\Gamma_2 \to 0$, when $\Delta M \neq 0$. By using some simple 4-vector algebra, it is possible to obtain the next condition of the VCR effect realization [for $n < 1$, we must replace n by $1/n$ in (4)]:

$$\cos\theta = \frac{c}{v_0 n_*}\left(1 + \frac{\varepsilon}{\Gamma_0}\frac{\sqrt{n^2 - 1}}{n}\right), \quad n_* = n + \sqrt{n^2 - 1}, \quad (4)$$

where $\cos\theta = \frac{\mathbf{v}_0 \cdot \mathbf{v}_p}{v_0 \cdot v_p}$, $\varepsilon = \frac{\varepsilon_p}{m_e c^2}$. Then, even for transverse electromagnetic waves in plasma with $n < 1$, the VCR effect may be also realized because for $n < 1$ in (4) we have $n_* = \frac{n}{1 - \sqrt{1 - n^2}} > 1$.

Thus, condition (4) only for the limit $n \to 1$ (when $\Delta M \to 0$) is equivalent to the results of the theory [1,7,8] and in the other cases with $n \neq 1$ we obtained the new VCR quantum theory which for $n < 1$ embrace the results of [24] on long wave plasmon transformation [20]. In this Letter, for simplicity, we consider the VCR theory only in the limit of Bose-condensed excitation which has zero momentum $\tilde{\mathbf{p}} = \Delta M\mathbf{v}_2\Gamma_2 \to 0$ in (1) and (2). For example, in plasma with $n < 1$ this VCR theory describing the Cherenkov radiation as a result of the longitudinal Bose-condensed plasmon transformation into a high-frequency transverse electromagnetic wave due to the scattering of this plasmon on the relativistic electron (see also [20,24]). In the present VCR theory for $n > 1$ the thermal phonons of the medium may be considered also as possible Bose excitations, which may transform in the Cherenkov photon during VCR realization. The corresponding temperature dependence of the

VCR phenomenon may be determined by the temperature dependence of refracting index n. It is also possible to obtain some other version of the VCR theory in the case of finite momentum $\tilde{\mathbf{p}}$ in (1) and (2) for the elementary Bose excitations of medium.

2. *Relativistic form of Landau criterion for VCR.*—To obtain a relativistic generalization of the Landau criterion for the VCR realization, it is necessary to use the energy balance equation (1) in the coordinate system moving with the initial electron velocity $\mathbf{v}_0$. In this system, the medium has the velocity $-\mathbf{v}_0$, while the electron rests in its initial state. From (1), in the limit $\mathbf{v}_2 \to 0$, it is possible to write the following presentation of the energy law conservation equation in this coordinate system:

$$m_e c^2\left[1-\Gamma_0\Gamma_1\left(1-\frac{(\mathbf{v}_0\mathbf{v}_1)}{c^2}\right)\right]=\varepsilon_p\Gamma_0\left[1-\frac{(\mathbf{v}_0\mathbf{v}_p)}{c^2}-\sqrt{1-\frac{v_p^2}{c^2}}\right]. \quad (5)$$

The left-hand side of Eq. (5) is negative for all the values $\mathbf{v}_1 \neq \mathbf{v}_0$ (and equal zero only for $\mathbf{v}_1 = \mathbf{v}_0$), because, in its final state, the electron has some nonzero velocity and its energy becomes higher than that in its initial state with zero velocity. For this reason, the following threshold condition (6) [obtained for the VCR phenomenon realization with $\varepsilon_p > 0$, when the right-hand side of (5) is also negative],

$$\cos\theta > \frac{c}{v_0 n_*}, \quad (6)$$

coincides with the condition of the Doppler abnormal effect (DAE) phenomenon realization. Actually, in this coordinate system, even for a point electron with no internal degrees of freedom, the emission of photon radiation with energy $\varepsilon_p > 0$ is due to an increase in the electron's energy level, which is characteristic for the DAE phenomenon in different physical systems [1,17]. From (6) and the necessary condition $|\cos\theta| \leq 1$, the following threshold velocity of electron for the VCR and DAE phenomena realization [the same as from (4) if $\varepsilon \to 0$] may be obtained:

$$v_0 > v_{\rm th} = \frac{c}{n_*}. \quad (7)$$

Thus, the relativistic generalization of the Landau criterion in (6) and (7) is more common than in condition (4), because (6) and (7) determine the possibility of an energetically favorable realization of the VCR phenomenon for all the angles θ within the interval $0 \leq \theta \leq \theta_m$ [where θ_m may be obtained from (6)], which is in agreement with experimental data [2,26].

Condition (4) coincides with (6) and (7), when $\mathbf{v}_1 = \mathbf{v}_0$ in (5) with $\varepsilon_p > 0$, because, in this case, the left-hand side of (5) is equal to zero.

In the nonrelativistic limit, when $v_0 \ll c$ and $v_p \ll c$ from (5) for $\varepsilon_p > 0$, the Landau criterion [13] may be obtained:

$$\tilde{\varepsilon} - (\mathbf{p}\mathbf{v}_0) < 0, \quad (8)$$

where $\tilde{\varepsilon} = \varepsilon_p(1-\sqrt{1-\frac{v_p^2}{c^2}}) \approx \frac{\varepsilon_p v_p^2}{2c^2}$ is the only kinetic energy of excitation. Then $\tilde{\varepsilon} = \frac{v_p p}{2}$ and the group velocity coincides with the corresponding phase threshold velocity $v_{\rm th} = \frac{v_p}{2}$, as in the Landau criterion [13] [see also Frank (1959) [4]].

In the next part, we consider the threshold conditions (6) and (7) in comparison with some experimental data [2] [Cherenkov (1959)] and with the VCR classic theory [1,3,4,6–8].

3. *Comparison of new theory with classic VCR theory and experiment.*—According to [2], the VCR effect is observed in the whole region of the angles $0 \leq \theta \leq \theta_m^{A,B}$ with the maximum of radiation intensity $I(\theta)$ at the angle $\theta = \theta_0^{A,B} < \theta_m^{A,B}$. Here, the index A corresponds to γ rays of ThC$''$, and the index B corresponds to the VCR induced by Ra. Thus, $I(\theta) = 0$, when $\theta > \theta_m^{A,B}$ and $I(\theta) \neq 0$ for $\theta = 0$. In [26], the same result was also obtained for the VCR realization through the direct use of high-energy electron beams. Let us determine the threshold velocity parameter $\beta_*^{A,B} = \frac{1}{n_* \cos\theta_m^{A,B}}$ from (6) [or from (4) if $\varepsilon \to 0$] and also the same parameter $\beta^{A,B} = \frac{1}{n \cos\theta_m^{A,B}}$ from the classic VCR theory [1,3,4,6–8]. The comparison of these parameters $\beta_*^{A,B}$ and $\beta^{A,B}$ for the values of $\theta_m^{A,B}$, which are obtained from the experimental data [2], is illustrated in Table I for the VCR effect in different mediums.

It follows from Table I that the VCR classic theory yields the unreal values of $\beta^{A,B} > 1$, while $\beta_*^{A,B}$ is smaller than the unit for all the sources of γ rays and for all the

TABLE I. New VCR theory (β_*^A, β_*^B) and classic one (β^A, β^B) for threshold angles (θ_m^A, θ_m^B) of Cherenkov radiation realization in [2].

	n	n_*	$\cos\theta_m^A$	$\cos\theta_m^B$	$\beta_*^A = \frac{v_0}{c}$	$\beta_*^B = \frac{v_0}{c}$	$\beta^A = \frac{v_0}{c}$	$\beta^B = \frac{v_0}{c}$
H_2O	1.3371	2.2247	0.6691	0.7431	0.6718	0.6049	1.1177	1.0064
C_6H_{12}	1.4367	2.4683	0.5	0.6428	0.8103	0.6303	1.392	1.083
C_6H_6	1.5133	2.6491	0.454	0.5736	0.8315	0.6581	1.4556	1.152
$C_{11}H_{12}O_2$	1.5804	2.8049	0.3584	0.5	0.9519	0.6217	1.689	1.103

mediums, where the VCR effect is detected [2]. Of course, if we replace $\theta_m^{A,B}$ by $\theta_0^{A,B}$ in the determination of $\beta^{A,B}$, the physical real result $\beta^{A,B} < 1$ is also obtained for the VCR classic theory [1,3,4,6–8].

The present relativistic generalization of the Landau criterion gives the favorable energetic base for the VCR phenomenon threshold realization in all angles $0 \leq \theta \leq \theta_m$, which are detected in experiment [2,26]. Thus, the integral VCR intensity for all angles $0 \leq \theta \leq \theta_m$ in the present VCR theory and in experiment must be identical. From the other side, the coherent nature of VCR must give an interference distribution of VCR intensity $I(\theta)$, when the interference maximum of intensity $I(\theta)$ takes place at $\theta = \theta_0$ with $\theta_0 < \theta_m$. The traditional VCR theory tied only with this interference maximum of VCR at $\theta = \theta_0$ and does not consider at all the energetic base for a threshold arising of this coherent VCR. Actually, this is clearer for the case of plasma with $n < 1$, where the traditional VCR theory total excludes the possibility of the Cherenkov radiation in the form of transverse high-frequency electromagnetic waves. From the other side, the present VCR theory gives this possibility due to the transformation of longitudinal Bose-condensed plasmon into the transverse Cherenkov photon during the scattering of a plasmon on the relativistic electron, which has superthreshold velocity (see also [20,24]).

Note also that, in connection with the many problems of plasma physics, the conditions (6), (7), and (4) for $n < 1$ may give the useful possibility for effective emission of transverse electromagnetic waves due to the direct VCR effect, when extremely long wave Bose-condensed elementary plasmons may converse into unbremsstrahlung VCR [20].

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