Universality of Soft and Collinear Factors in Hard-Scattering Factorization

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Universality in QCD factorization of parton densities, fragmentation functions, and soft factors is endangered by the process dependence of the directions of Wilson lines in their definitions. We find a choice of directions that is consistent with factorization and that gives universality between e^+e^- annihilation, semi-inclusive deep-inelastic scattering, and the Drell-Yan process. Universality is only modified by a time-reversal transformation of the soft function and parton densities between Drell-Yan and the other processes, whose only effect is the known reversal of sign for T-odd parton densities such as the Sivers function. The modifications of the definitions needed to remove rapidity divergences with lightlike Wilson lines do not affect the results.

Introduction. Much of the predictive power of QCD is provided by universality of the nonperturbative functions in factorization theorems for hard processes. These functions are parton densities, fragmentation functions, etc., Whereas the perturbative parts of factorization formulae can be usefully estimated from first principles by weak coupling methods, the nonperturbative functions cannot. Universality is the process independence of these functions. It allows them to be measured from a limited set of reactions, and then used to predict other reactions, with the aid of factorization and perturbative calculations.

However, recent developments [1–5] show that universality is endangered. For example, the Sivers function [6]—the transverse single-spin asymmetry of a parton density—changes sign [1,3] between the Drell-Yan process and deep-inelastic scattering. This is because the directions of the Wilson lines necessary for a gauge-invariant definition of a parton density depend on whether collinear and soft interactions are before or after the hard scattering. Important current experimental work [7] addresses the associated physics issues.

Although for parton densities time reversal relates the two definitions [1], the situation is not so clear in general. For example, a fragmentation function involves a semi-inclusive sum over out-states:

$$\sum_X |A, X, \text{out}\rangle\langle A, X, \text{out}|. \tag{1}$$

Time reversal converts these to in-states, and therefore does not prove universality with the obvious, process-dependent directions for the Wilson lines [4]—although the nonuniversality did not occur in a one-loop model calculation [8]. Moreover, for hadron production in hadron-hadron collisions, Bomhof, Mulders, and Pijlman [5] found a jungle of Wilson lines whose universality

properties are far from clear; see also the comments of Brodsky, Hwang, and Schmidt [3].

Therefore in this Letter we carefully reexamine the arguments about Wilson lines to discover the true limits of universality, if any. The issues particularly concern processes that need transverse-momentum-dependent (TMD) partonic functions, and we consider three such processes: (a) e^+e^- annihilation with detected almost back-to-back hadrons [9], (b) semi-inclusive deep-inelastic scattering (SIDIS) with measured transverse-momentum for an outgoing hadron [10,11], and (c) the Drell-Yan process with measured transverse momentum [12]. For the first process, Collins and Soper showed a factorization theorem more than two decades ago [9]. But the paper [12] extending the *statement* of factorization to the Drell-Yan process claimed no proof.

Our new methods show, in addition to the time-reversal-modified universality [1] of parton densities, that (1) TMD fragmentation functions are universal between e^+e^- annihilation and SIDIS. (2) The soft factor is universal between all three processes. (3) Universality arguments hold even with a redefinition of the nonperturbative functions needed to remove the divergences due to lightlike Wilson lines. Our arguments delimit the process-dependent choices of direction that are compatible with factorization, and for individual processes the choice is wider than previously used [4,11,13].

The central technical issue is that a proof of factorization requires an appropriate deformation [14] of momentum contours out of the "Glauber region". Allowed directions of the Wilson lines are those compatible with the contour deformation. The possible directions of contour deformation are determined by the space-time location of soft and collinear interactions relative to the hard scattering. Hence a careful analysis of one-gluon corrections, which forms

the bulk of our work in this Letter, should be sufficient to determine the directions.

Factorization and Wilson lines. Factorization into hard, collinear, and soft factors results essentially from a corresponding set of momentum regions. Individual graphs for a cross section have a range of possible leading regions of loop-momentum space, and we will follow the approach of Collins and Hautmann [13] by using a subtractive method that implements a decomposition of graphs by possible regions. Full details of the subtractive method to all orders have not been worked out explicitly, but it should give factorization after a sum over graphs and regions. A

Ward identity argument is needed to disentangle otherwise coupled factors, and it results in simple Wilson lines in the gauge-invariant operator definitions of the factors.

Electron-positron annihilation. We first consider e^+e^- annihilation at large Q with two detected hadrons h_1 and h_2 in almost back-to-back directions, for which some simple graphs with a virtual gluon are shown in Fig. 1. (The contour deformation issues arise only for virtual gluons.)

For the first graph, we make, as in [13], a decomposition into four terms: hard, collinear to h_1, collinear to h_2, and soft. Critical for determining the directions of the Wilson lines is the soft term:

$$S_{\text{epem}} = \int \frac{d^n l}{(2\pi)^n} F_1^+(P_1, -(0, l^-, 0_T)) F_2^-(P_2, (l^+, 0, 0_T)) \frac{i l^- l^+}{l^2 + i\epsilon} \left[\frac{1}{(-l^- + i\epsilon)(l^+ + i\epsilon)} - \frac{1}{(-l^- + i\epsilon)(l^+ - C_A l^- + i\epsilon)} \right.$$
$$\left. - \frac{1}{(-l^- + C_B l^+ + i\epsilon)(l^+ + i\epsilon)} \right] + \overline{\text{MS}} \text{ counterterm}, \tag{2}$$

where $\overline{\text{MS}}$ denotes the modified minimal subtraction scheme. Here we use light-front coordinates in the center of mass with the dominant components of the momenta of h_1 and h_2 being P_1^+ and P_2^-.

In the h_1 and h_2 parts of the graph we have retained only the $-$ and $+$ components of the gluon momentum l, and we have picked out the dominant components of the current to which the gluon couples. We have inserted factors of l^- and l^+ to allow Ward identities to be used and then compensated this in the first term in brackets by dividing by l^- and l^+. With this first term we obtain a good approximation to the original graph in the soft region, provided that we make a suitable deformation out of the Glauber subregion of the soft region. The Glauber region is where $|l^+ l^-| \ll l_T^2$. The second and third terms are counterterms, to cancel the divergences at large positive and negative gluon rapidity that would arise if only the first term were used.

The counterterms have arbitrary positive parameters C_A and C_B of order Q^2/m^2 (with m being a soft scale), they are power suppressed in the soft region, and they allow a Wilson-line definition of the soft factor. The arbitrariness

of C_A and C_B is exploited by the use of the Collins-Soper [9] equation, which controls the C_A and C_B dependence of the soft and collinear factors, and so enables predictions to be made.

The contour deformation must not cross the final-state quark poles in the original graph. We choose to deform symmetrically out of the Glauber region: $\Delta l^+ = iw$, $\Delta l^- = -iw$, where w is a suitable positive function of the real parts of l^μ. Compatibility with this deformation determines uniquely that the lightlike Wilson-line denominators are $-l^- + i\epsilon$ and $l^+ + i\epsilon$, and that the signs of the C_A and C_B terms relative to their $i\epsilon$'s are as shown. But it does not determine whether the counterterm lines are spacelike or timelike. A legitimate possibility not noticed in [13] is that one or both could be timelike: $l^+ + C_A l^- - i\epsilon$, $-l^- - C_B l^+ - i\epsilon$.

There is also an ultraviolet divergence at large l_T which we remove by ordinary renormalization. Neither the rapidity divergences nor the UV divergence affect the validity of the soft approximation in the soft region.

In coordinate space, both lightlike Wilson lines are future pointing, as is intuitively natural. They approximate a fast-moving quark and a fast-moving antiquark as seen by a slow gluon. (However, the nonlightlike lines in the counterterms are past pointing, not so intuitively.)

We choose spacelike lines in the counterterms because they are compatible with one of the possibilities for the counterterms in SIDIS, and so they allow a proof of universality. Furthermore, exact properties of matrix elements of Wilson lines are simpler when the gluon fields are at spacelike separation and hence all commute.

Another possible change is to use an asymmetric contour deformation, such as we choose in SIDIS, i.e., primarily on l^+ only or on l^-. But it can be shown that the only extra resulting cases for the $i\epsilon$ prescriptions violate the charge-conjugation relationships between fragmentation for
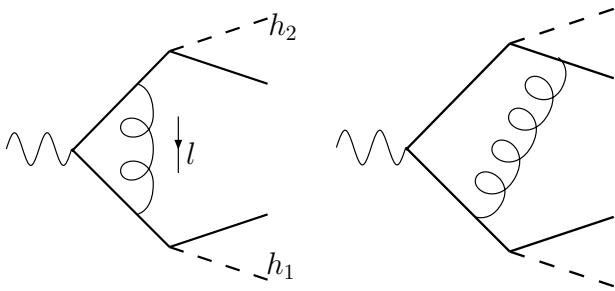

FIG. 1. Sample virtual one-loop diagrams for e^+e^- annihilation, with fragmentation considered in a spectator model.

quarks and antiquarks. They would therefore remove predictive power from factorization.

In the method of Ji, Ma, and Yuan [11] there are no counterterms; instead they use slightly nonlightlike lines to cutoff the rapidity divergences. Their square-bracket factor would be

$$\frac{1}{(l^+/C_A - l^- + i\epsilon)(-l^-/C_B + l^+ + i\epsilon)}. \quad (3)$$

The Wilson lines are actually future pointing, since the denominators are on opposite sides of the graph compared with the corresponding past-pointing spacelike lines in Eq. (2). Some differences with our formulation are inessential power-law corrections. But there are also different leading-power contributions to the soft and collinear factors. We remark without proof that these only occur at large transverse momentum for the gluon and therefore amount to a legitimate scheme change. The subtractive method provides simpler calculations and a cleaner proof of universality.

Once the soft term is fixed, definite prescriptions for the collinear and hard terms follow, just as in [13], so we will not present them here.

There are many other graphs for the process, both with different connections of the gluon, as in Fig. 1, and with arbitrarily many other lines. In all leading regions, the collinear parts are in the final state, so we can continue applying the same prescription for the contour deformations and for the counterterms. Of course, to use Ward identities we must use the same prescription everywhere. So we have determined all the Wilson lines.

Semi-inclusive DIS. We now consider similar graphs for SIDIS, as in Fig. 2. As concerns the contour deformation from the Glauber region, the primary difference is that in the h_1 part of the graph the flow of l relative to collinear momenta is reversed. So we have denominators like $(k_1 + l)^2 - m^2 + i\epsilon$ instead of $(k_1 - l)^2 - m^2 + i\epsilon$. This suggests reversing the contour deformation on l^-, to give
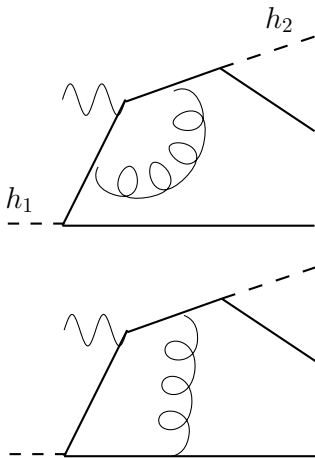

FIG. 2. Sample one-loop diagrams for SIDIS.

$\Delta l^+ = iw$, $\Delta l^- = iw$. Then, following [11], we might reverse the relative signs of l^- and $i\epsilon$ compared with Eq. (2), to get a square-bracket factor

$$\frac{1}{(-l^- - i\epsilon)(l^+ + i\epsilon)} - \frac{1}{(-l^- - i\epsilon)(l^+ + C_A l^- + i\epsilon)}$$
$$- \frac{1}{(-l^- - C_B l^+ - i\epsilon)(l^+ + i\epsilon)}. \quad (4)$$

The counterterm lines are now timelike.

Corresponding timelike lines must also appear in the fragmentation function, so that compared with $e^+ e^-$ annihilation, the definitions of both the fragmentation function and the soft factor are different, and we lose manifest universality. Now both C_A and C_B are large, so that we can in fact use spacelike lines with our symmetric contour deformation (and similarly in the Ji, Ma and Yuan version):

$$\frac{1}{(-l^- - i\epsilon)(l^+ + i\epsilon)} - \frac{1}{(-l^- - i\epsilon)(l^+ - C_A l^- - i\epsilon)}$$
$$- \frac{1}{(-l^- + C_B l^+ + i\epsilon)(l^+ + i\epsilon)}. \quad (5)$$

But even this still differs from the version for $e^+ e^-$ annihilation, so we cannot directly deduce universality.

Moreover, in the second graph of Fig. 2 the symmetric contour deformation is blocked by a trap between initial-state and final-state poles in the target part of the graph. When transverse momenta are of order m, the deformation on l^- is limited to m^2/Q, not enough to get out of the Glauber region. So, as in the proof of factorization for diffractive DIS [15], we should deform primarily on l^+, away from the pole(s) in the outgoing struck quark and its jet; this typically makes l collinear to the target, rather than merely soft. Only small deformations on l^- are necessary to avoid the target side poles both in the original graph and in the Wilson-line approximations.

We can now use exactly the same square-bracket factor as in $e^+ e^-$ annihilation. Since the same direction of contour deformation can be applied generally, for the Glauber regions for all graphs, the Wilson lines in the soft and fragmentation factors are the same as in $e^+ e^-$ annihilation. Thus the soft and fragmentation factors are universal between SIDIS and $e^+ e^-$ annihilation. Spacelike counterterm denominators, like $l^+ - C_A l^- + i\epsilon$, are preferred here, since the large positive imaginary part of l^+ assists rather than hinders the deformation of l^- from an unphysical pole.

Drell-Yan process. In Fig. 3, we show some graphs for the Drell-Yan process. In the first graph, the gluon attaches to two initial-state lines, so we use a contour deformation opposite to that in $e^+ e^-$ annihilation.

But other graphs trap the contour against final-state poles in the target parts of the graphs. Now to prove factorization [16] one can deform away from initial-state poles. Crossing target-related final-state poles produces extra nonfactorizing terms, but these cancel by unitarity, after a sum over all hadronic states in the inclusive cross
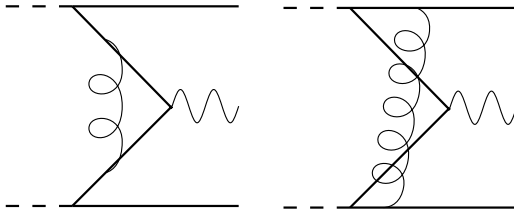
FIG. 3.   Sample one-loop diagrams for the Drell-Yan process.

section. The argument does not depend on the transverse momentum of the lepton pair.

Therefore we find that we can use initial-state Wilson lines for the parton densities and for the soft factor. The time-reversal argument of [1] applies to all these objects, since they all involve only matrix elements of the form $\langle \psi | A_1 A_2 | \psi \rangle$, where A_1 and A_2 are operators and ψ labels a vacuum or a one-particle state. Such states are the same no matter whether they are in-states or out-states, so the transformation by time reversal leaves them unaffected.

This extends the exact universality of parton densities and soft factors to the Drell-Yan process, with the exception of "T-odd" parton densities, which reverse sign [1], as is already known. Our proof now includes Wilson-line factors that implement [17] the cancellation of rapidity divergences.

Conclusions and discussion.   We have shown universality of fragmentation functions, soft factors, and parton densities between $e^+ e^-$ annihilation, semi-inclusive deep-inelastic scattering, and the Drell-Yan process. This applies both to the basic definitions with lightlike Wilson lines and to the correct definitions with removal of rapidity divergences. Regulator lines should be spacelike. In the Drell-Yan process the lines are reversed compared with the other processes we considered, but time reversal relates them to the functions for other processes, with the usual reversal of sign for the Sivers function and other T-odd parton densities.

This eliminates missing elements in the Collins-Soper-Sterman formalism [12] for the Drell-Yan process.

The method of Ji, Ma, and Yuan [11] uses nonlightlike Wilson lines instead of counterterms for removing rapidity divergences. Within this method we find universality if the fragmentation function and the soft factor in SIDIS have future-pointing spacelike Wilson lines, contrary to the choice made by these authors.

Since we need definitions of TMD parton densities that differ from the most obvious ones that use light-front quantization in light-front gauge, we agree with the conclusion of Brodsky *et al.* [18] that parton densities are not literally probability densities. However, our reasoning is different, and builds on the much earlier work of Collins and Soper [9,19].

Our work needs extension to hard hadron-production processes in hadron-hadron collisions. It is not obvious that the extension will succeed. Factorization for these processes is at present an unproved conjecture, at least for cases where transverse-momentum-dependent parton densities and fragmentation functions are needed.

Even for conventional hadron-hadron-to-hadron factorization with ordinary, integrated parton densities, there is no proof in the literature which correctly treats the Glauber region, as far as we know. The factorization proofs of Collins, Soper, and Sterman [16] and Bodwin [20] are only for the Drell-Yan process. A critical reexamination is needed here.

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