Neutrino Superfluidity

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It is shown that Dirac-type neutrinos display BCS superfluidity for any nonzero mass. The Cooper pairs are formed by attractive scalar Higgs boson exchange between left- and right-handed neutrinos; in the standard $SU(2) \times U(1)$ theory, right-handed neutrinos do not couple to any other boson. The value of the gap, the critical temperature, and the Pippard coherence length are calculated for arbitrary values of the neutrino mass and chemical potential. Although such a superfluid could conceivably exist, detecting it would be a major challenge.

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Superconductivity is ubiquitous in nature. It occurs in metals, organic compounds, atomic and molecular gases, nuclear matter, and quark matter. It will be demonstrated here that massive Dirac-type neutrinos can display superfluidity, the analogue of superconductivity for electrically neutral particles, when they are embedded in the standard model of particle physics. What allows neutrino superfluidity is the attractive scalar interaction between left- and right-handed neutrinos (assuming that right-handed neutrinos exist) due to their coupling to the Higgs field from which they obtain their common mass. As was shown by Caldi and Chodos [1], pairing of left-handed neutrinos with left-handed neutrinos cannot occur due to the repulsive vector interaction provided by the Z^0 meson. For the purpose of demonstration, in this Letter only one flavor of lepton is considered. More flavors open the window to neutrino oscillations and an even richer structure of superfluid states. The word neutrino is used in a generic sense to denote any electrically neutral spin-$\frac{1}{2}$ pointlike fermion.

In a convenient gauge the Higgs field can be written as

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_0 + \sigma \end{pmatrix}, \tag{1}$$

where $v_0 = 1/\sqrt{\sqrt{2}G_F} = 246$ GeV is the Higgs condensate and σ is its fluctuation. A Yukawa coupling between the neutrino and the Higgs field gives the neutrino its mass of $m_\nu = h_\nu v_0/\sqrt{2}$, namely,

$$\mathcal{L}_Y = h_\nu \bar{l}_L \Phi_c \nu_R + \text{h.c.} = \left(m_\nu + \frac{h_\nu}{\sqrt{2}}\sigma\right)\bar{\nu}\nu. \tag{2}$$

Standard notation is used, with l_L representing a left-handed lepton doublet, ν_R a right-handed neutrino singlet, and Φ_c, the charge-conjugated Higgs field. For small energy and momentum transfers, the Higgs boson exchange between neutrinos can be replaced by the contact interaction

$$H_I = -\frac{h_\nu^2}{4m_\sigma^2}(\bar{\nu}\nu)(\bar{\nu}\nu), \tag{3}$$

with m_σ the Higgs mass. This interaction is attractive. It can be derived by solving the field equation for σ in terms of the neutrino field and substituting back into the Lagrangian. In Dirac representation we express the neutrino field as

$$\begin{aligned} \nu_L &= \frac{1}{2}(1-\gamma_5)\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_L \\ -\psi_L \end{pmatrix} \\ \nu_R &= \frac{1}{2}(1+\gamma_5)\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_R \\ \psi_R \end{pmatrix}, \end{aligned} \tag{4}$$

where ψ_L and ψ_R are two-component spinors. Then the interaction can be written as

$$\begin{aligned} H_I = -\frac{h_\nu^2}{4m_\sigma^2}\{&2\psi_{La}^\dagger\psi_{Rb}^\dagger\psi_{Lb}\psi_{Ra} + \psi_{La}^\dagger\psi_{Lb}^\dagger\psi_{Rb}\psi_{Ra} \\ &+ \psi_{Ra}^\dagger\psi_{Rb}^\dagger\psi_{Lb}\psi_{La}\}, \end{aligned} \tag{5}$$

where the summation over the spinor indices a, b runs from 1 to 2. Now the familiar path to Cooper pairing in the BCS theory follows naturally [2].

Allowing for condensation of the form

$$\langle \psi_L^a \psi_R^b \rangle = \varepsilon^{ab} D, \tag{6}$$

where ε^{ab} is the Levi-Civita symbol and D is related to the gap, corresponds to spin-0 pairing of left and right-handed neutrinos [3]. As mentioned earlier, pairing of left-handed neutrinos with themselves is not considered because of the repulsive Z^0 exchange. Making the mean field approximation results in the interaction

$$H_I^{MF} = \frac{h_\nu^2}{2m_\sigma^2}(D\psi_{La}^\dagger\psi_{Rb}^\dagger + D^*\psi_{Lb}\psi_{Ra})\varepsilon^{ab}. \tag{7}$$

In terms of particle creation and annihilation operators this is [4]

$$H_{\mathrm{I}}^{\mathrm{MF}} = -\frac{h_\nu^2}{4m_\sigma^2}\sum_{\mathbf{p}}\frac{m_\nu}{\epsilon}\{De^{2i\epsilon t}[b^\dagger(p,+)b^\dagger(-p,-) - b^\dagger(p,-)b^\dagger(-p,+)] + D^* e^{-2i\epsilon t}[b(-p,-)b(p,+) - b(-p,+)b(p,-)]\}, \quad (8)$$

where $\epsilon = \sqrt{p^2 + m_\nu^2}$. The operator $b^\dagger(p,+)$ creates a particle with momentum $\mathbf{p}$ and spin projection $s_z = \frac{1}{2}$, and so on. To this must be added the free particle Hamiltonian

$$H_{\text{free}} = \sum_{\mathbf{p}} \epsilon[b^\dagger(p,+)b(p,+) + b^\dagger(p,-)b(p,-)]. \quad (9)$$

The full Hamiltonian, including the chemical potential μ, can be put in the standard form

$$H = \sum_{\mathbf{p}} E[c^\dagger(p,+)c(p,+) + c^\dagger(p,-)c(p,-)] \quad (10)$$

$$E = \sqrt{(\epsilon - \mu)^2 + (Km_\nu/\epsilon)^2}, \quad (11)$$

where $K = h_\nu^2|D|/2m_\sigma^2$, which has units of energy. This is accomplished by the time-dependent canonical transformation

$$c(p,+) = \cos\theta e^{-i(\alpha+\epsilon t)}b(p,+) - \sin\theta e^{i(\alpha+\epsilon t)}b^\dagger(-p,-)$$
$$c(p,-) = \cos\theta e^{-i(\alpha+\epsilon t)}b(p,-) + \sin\theta e^{i(\alpha+\epsilon t)}b^\dagger(-p,+), \quad (12)$$

with

$$\tan(2\theta) = \frac{Km_\nu}{\epsilon(\epsilon - \mu)}. \quad (13)$$

Here $D = |D|e^{2i\alpha}$.

The gap equation may be derived by demanding self-consistency between the assumed value of the condensate and the value obtained from the canonical transformation of the creation and annihilation operators. One finds that either $D = 0$ or $\alpha = \pi/2$ with the magnitude of the gap determined by

$$\frac{h_\nu^2}{8m_\sigma^2}\int \frac{d^3p}{(2\pi)^3}\frac{m_\nu^2}{\epsilon^2}\frac{1}{\sqrt{(\epsilon-\mu)^2 + (Km_\nu/\epsilon)^2}} = 1. \quad (14)$$

The integral is divergent. It could be cut off with an upper limit Λ of order m_σ or with the form factor $m_\sigma^2/(m_\sigma^2 + 4\mathbf{p}^2)$ befitting the Higgs boson exchange interaction. The gap equation has a nontrivial solution with $K \neq 0$, no matter how small the coupling of the neutrino to the Higgs (expressed as h_ν or m_ν), on account of the divergence at the energy $\epsilon = \mu$. The gap can immediately be inferred to be

$$\Delta = Km_\nu/\mu \quad (15)$$

in the weak coupling limit, $m_\nu^2 h_\nu^2/8m_\sigma^2 < 1$. Notice that there is no pairing and, as a consequence, no superfluidity when the neutrino mass is zero. The reason is simple and was also noticed in the context of color superconductivity [5]: Chirality and helicity coincide when the neutrino is massless, hence, a left-handed neutrino and a right-handed neutrino with equal but opposite momenta give a spin-1 projection along the relative momentum axis and so cannot contribute to a spin-0 condensate.

The gap equation can be written in a form similar to that of ordinary superconductivity

$$\frac{1}{2}gN(0)\int_{\xi_{\min}}^{\xi_{\max}}\frac{d\xi}{\sqrt{\xi^2 + \Delta^2}} = 1, \quad (16)$$

where $\xi = \epsilon - \mu$ and $g = h_\nu^2/4m_\sigma^2$ is the four-point coupling from Eq. (3). The phase space density at the Fermi surface $N(0)$ includes the relativistic factor m_ν^2/ϵ^2 from Eq. (14):

$$N(0) = \frac{1}{2\pi^2}p^2\left(\frac{dp}{d\epsilon}\right)\frac{m_\nu^2}{\epsilon^2}\bigg|_{\epsilon=\mu} = \frac{m_\nu^2 v_F}{2\pi^2}, \quad (17)$$

where v_F is the Fermi velocity. In ordinary superconductivity the integral is cut off by the Debye frequency ω_D, but here the lower limit is minus the Fermi kinetic energy, $\xi_{\min} = -(\mu - m_\nu)$, and the upper limit is essentially the Higgs boson mass, $\xi_{\max} = \Lambda = \sqrt{m_\sigma^2 + m_\nu^2} - \mu$. The solution to the gap equation is

$$\Delta = 2\sqrt{|\xi_{\min}|\xi_{\max}}\exp[-1/gN(0)] = 2\sqrt{(\mu - m_\nu)\Lambda}\exp[-8\pi^2 m_\sigma^2/h_\nu^2 m_\nu^2 v_F]. \quad (18)$$

Since this is a typical BCS-like theory, the critical temperature takes the standard form [2,6]

$$T_c = \frac{e^\gamma}{\pi}\Delta \approx 0.57\Delta. \quad (19)$$

When the coupling is not weak the gap equation can be solved numerically. Up to this point, all formulas have been expressed in terms of h_ν and m_ν without making the connection $m_\nu = h_\nu v_0/\sqrt{2}$.

Unfortunately, for the observed set of three flavors of neutrinos the numbers are uninterestingly small. For example, for a neutrino of mass 1 eV and a Higgs boson of mass 110 GeV the argument of the exponential in the relativistic limit of Eq. (18) is about -1×10^{46}. Therefore, let us consider a very heavy neutrino and the corresponding nonrelativistic limit. The number of antineutrinos is assumed to be negligible in comparison to the number of neutrinos (or vice versa), just as is the case for baryon number. In terms of the neutrino mass density $\rho_\nu = m_\nu n_\nu$, with number density appropriate to a cold Fermi gas, $n_\nu = p_F^3/3\pi^2$, the gap is

$$\Delta = \sqrt{\frac{2\Lambda}{m_\nu}}\left(\frac{3\pi^2\rho_\nu}{m_\nu}\right)^{1/3} e^{-x}, \tag{20}$$

and the Pippard coherence length is

$$\xi = \frac{v_F}{\pi\Delta} = \frac{1}{\pi\sqrt{2\Lambda m_\nu}} e^{x}, \tag{21}$$

where

$$x = \frac{4\pi^2 m_\sigma^2 v_0^2}{m_\nu^2 (3\pi^2 \rho_\nu m_\nu^2)^{1/3}}. \tag{22}$$

First let us apply these results to cosmology. Given that ρ_ν should not exceed the present energy density of the Universe, which is about 5 $\mathrm{keV/cm^3}$, means that the critical temperature T_c is less than 1 K by many orders of magnitude no matter what the neutrino mass is. Hence, neutrino superfluidity does not seem to be relevant to the cosmological expansion of the Universe.

Next let us apply these results to neutron stars, where heavy neutrinos may accumulate due to gravitational attraction. Choose a reference mass scale of 10 TeV and a reference energy density of 10 $\mathrm{MeV/fm^3}$, which is about 1% central density of a neutron star. Such a modest energy density will not significantly alter the structure of the neutron star. Then

$$\begin{aligned} \Delta &= 67.2\left(\frac{\rho_\nu}{10\ \mathrm{MeV/fm^3}}\right)^{1/3}\left(\frac{10\ \mathrm{TeV}}{m_\nu}\right)^{4/3} e^{-x}\ \mathrm{keV} \\ \xi &= 5.71 \times 10^{-4} e^{x}\ \mathrm{fm} \\ x &= 4.73\left(\frac{10\ \mathrm{MeV/fm^3}}{\rho_\nu}\right)^{1/3}\left(\frac{10\ \mathrm{TeV}}{m_\nu}\right)^{8/3}. \end{aligned} \tag{23}$$

To be of interest, the coherence length should be much less than the radius of a neutron star, or about 10 km. In addition, the critical temperature must be less than the interior temperature of the star. After one million years its interior temperature has dropped to about 10^6 K [7]. The quantities Δ, ξ, and T_c are strongly dependent on the neutrino mass. For example, if $m_\nu = 8$, 10, 12 TeV then $\xi = 3.0$, 0.065 and 0.01 fm, while $T_c = 0.11 \times 10^6$, 3.9×10^6, and 19×10^6 K, respectively. So neutrino superfluidity in neutron stars could be interesting if there exist Dirac neutrinos with a mass on the order of, or exceeding, 10 TeV.

In conclusion, it has been demonstrated that neutrino superfluidity is a possibility if Dirac neutrinos exist with nonzero mass. If the neutrino coupled to a much lighter scalar boson than the Higgs, or if a much heavier neutrino exists, then neutrino superfluidity could conceivably be realized in Nature.

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[2] See, for example, the following texts: A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, NJ, 1963); J. R. Schrieffer, *Theory of Superconductivity* (Benjamin, New York, 1964); A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[3] A more general ansatz for pairing is expressed in terms of the Pauli matrices as $\langle \psi_L^a \psi_R^b \rangle = i[\sigma_2(D_0 I + \mathbf{D}_1 \cdot \sigma)]_{ab}$. Here D_0 is the same as the D defined above, corresponding to spin-0 Cooper pairs, $\mathbf{D}_1$ corresponds to spin-1 Cooper pairs. Lengthy calculations show that the latter quantity is zero, at least when the Higgs boson exchange is reduced to a contact interaction.

[4] We can safely neglect the contribution of antiparticles because pairing is dominated by the attractive interaction between particles very near the Fermi surface.

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