Realistic Clocks, Universal Decoherence, and the Black Hole Information Paradox

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(Received 28 June 2004; published 6 December 2004)

Ordinary quantum mechanics is formulated on the basis of the existence of an ideal classical clock external to the system under study. This is clearly an idealization. As emphasized originally by Salecker and Wigner and more recently by others, there exist limits in nature to how "classical" even the best possible clock can be. With realistic clocks, quantum mechanics ceases to be unitary and a fundamental mechanism of decoherence of quantum states arises. We estimate the rate of the universal loss of unitarity using optimal realistic clocks. In particular, we observe that the rate is rapid enough to eliminate the black hole information puzzle: all information is lost through the fundamental decoherence before the black hole can evaporate. This improves on a previous calculation we presented with a suboptimal clock in which only part of the information was lost by the time of evaporation.

DOI: 10.1103/PhysRevLett.93.240401 PACS numbers: 03.65.Yz, 03.67.–a, 04.60.–m, 04.70.Dy

The question of how classical an optimal clock can be was first considered by Salecker and Wigner [1], who noted that in order to measure time with increasing accuracy one needs increasing quantities of energy. Specifically, they construct an elementary clock consisting of a photon bouncing between two mirrors separated by a distance ℓ. The clock "ticks" every time the photon bounces on one of the mirrors [1]. By the time the photon returns after one bounce, the wave packet of the mirror has spread, leading to a bound in the accuracy of the time measurement of $(\delta T)^2 \gtrsim \hbar T_{\max}/(mc^2)$ with m the mass of the clock, c the speed of light, and $T_{\max}$ the maximum interval of time one attempts to measure. The measurement becomes more inaccurate the longer the time to be measured, and the smaller the mass of the clock. Amelino-Camelia and Ng and van Dam further elaborated on this idea [2] by noticing that a fundamental limit exists on how accurate a clock can be: if one needs more accuracy, the energetic demands are so high that the clock collapses into a black hole (the size of the clock cannot be increased to prevent the collapse, since it would imply losing accuracy). In fact, a black hole is the most accurate clock available for a given mass. A simple way of viewing the black hole as a clock is to recall that when excited, black holes behave like a (damped) oscillator. The fundamental frequency is inversely proportional to the mass of the hole, and therefore the resolution of the black hole as a clock is proportional to its mass. Moreover, since Hawking [3] showed that black holes evaporate due to particle production, one has a maximum possible time that can be measured by a black hole clock. If we take this time to be the black hole evaporation time, the inequality listed above is satisfied as an equality. Therefore if one wishes to measure time intervals smaller than a certain value $T_{\max}$, the optimal clock is a black hole with lifetime (at least) $T_{\max}$; a bigger black hole will be less accurate, and a smaller one will evaporate too fast to operate as a clock. The fundamental accuracy with which one can measure a time $T_{\max}$ is therefore determined by the lifetime of the black hole and is given by

$$\delta T \sim t_P \sqrt[3]{T_{\max}/t_P}, \tag{1}$$

where t_P is Planck's time, and from now on we choose units where $\hbar = c = 1$.

In order to do quantum mechanics with realistic clocks, one has to include the clock as part of the system under study. A suitable construction has been proposed by Page and Wootters [4], and a recent reanalysis is present in the paper by Dolby [5]. It consists of computing the probabilities for quantities of the system under study conditional on the quantities describing the clock taking given values. If the clock behaves semiclassically, the resulting probabilities satisfy approximately a Schrödinger equation. However, since the clock can never behave entirely classically, there will be corrections, at least if one wishes to recover Schrödinger's equation at a leading order [6]. We have estimated the type of corrections in Ref. [7] in the context of a discrete theory , but the construction can also be applied to the continuum case. In particular, the corrections imply that the quantum states do not evolve unitarily. Notice that the argument is based on ordinary (unitary) quantum mechanics, we are just recasting the theory in terms of realistic clocks, and this is the root of the loss of unitarity. The magnitude of the loss of unitarity is characterized by a function with units of time that is associated with how accurate the clock one considers is with respect to an ideal classical clock.

We briefly recount the derivation of the decoherence formula from Ref. [7]. We consider a system described by a variable X and a clock described by a variable T. Both variables are treated quantum mechanically and evolve according to Schrödinger's theory with respect to an ideal time t. We can start the system in an optimal quantum state for the clock, in which the probability density for the variable T has the shape of a Dirac delta centered at $T = t_0$. However, upon evolution, the probability distribution spreads and there are several likely values of T for a given instant of the ideal clock t. We do quantum mechanics by computing the conditional probability asking what the probability is of the variable X taking a given value X_0, when the clock variable T takes the value T_0 (if T and X have continuous spectra, we should recast the question in terms of small intervals). Since for later times there are several values of the external time t that correspond to the value of the clock T_0, the resulting probability is a superposition of ordinary Schrödinger probabilities. The latter are evolved unitarily; the former is therefore not. Detailed calculations in Ref. [7] show that one can approximate the evolution (provided the clock is reasonably classical) by a Lindblad type evolution,

$$\frac{\partial \rho}{\partial T} = -i[H, \rho] - \sigma(T)[H, (H, \rho)], \tag{2}$$

where ρ is the density matrix describing the system under study (without the clock) and $\sigma(T)$ is a measure of the rate of spread of the probability distribution of the clock time as a function of the ideal time. Specifically, if we assume the probability distribution is a Gaussian of spread δT, $\sigma(T) = \partial(\delta T)^2/\partial T$.

Since we have argued what an optimal clock is, we can now estimate the minimum rate of nonunitarity that one can expect from quantum mechanics in the real world by providing a concrete model for the spread $\sigma(T)$. Notice that this effect is fundamental; it affects all physical systems and cannot be eliminated. In particular, it does not depend on any interaction of the clock with the system. Quantum systems can decohere due to other effects, and in many practical applications these operate much faster than the fundamental effect we discuss here [7]. The latter is nevertheless ever present. The formula we get starting from (1) for $\sigma(T)$ for an optimal clock is given by $\sigma(T) = (\frac{t_P}{T_{\max} - T})^{1/3} t_P$, where $T_{\max}$ is the length of time we wish to measure, and we take it to coincide with the evaporation time of the black hole.

We now turn our attention to the black hole information paradox. Simply stated (for a review see [9]) the paradox is as follows: take a pure quantum state and collapse it into a black hole. Let the black hole evaporate. The end state is the outgoing thermal radiation, that is, a mixed state. In ordinary quantum mechanics, since evolution has to be unitary, a pure state cannot evolve into a mixed state, hence the puzzle. As we argued above, if one uses realistic clocks in ordinary quantum mechanics, pure states can evolve into mixed states. There is therefore the possibility that the collapse into a black hole and subsequent evaporation of a pure quantum state may not constitute a puzzle. The requirement is that the fundamental decoherence, which would turn the pure state into a mixed one anyway, operate fast enough to occur before the black hole evaporates entirely. We now show that this is the case. In a previous paper we analyzed this problem using a suboptimal clock [10]. The current calculation yields a better picture in the sense that it implies that *all* information is lost by the time the black hole evaporates, which was not the case with the suboptimal clock.

We need to make a quantum model of the black hole in order to study its decoherence. Here we make a very primitive model. We assume the black hole horizon's area (or equivalently its energy) is quantized. This is usually assumed in quantum black hole studies, and, in particular, it is predicted by loop quantum gravity. We choose a basis of states for the black hole labeled by the energy (area). The problem has some resemblance to the problem of an atom that is in an excited state and emits radiation to reach its fundamental state. If one considers the physical system under study to be the atom plus the radiation field, its evolution is unitary. One would expect a similar situation to hold for the black hole interacting with the gravitational and matter fields surrounding it. Here is where the paradox lies, since the evaporation process leads to loss of unitarity for the total system. Our model includes information about the black hole and the surrounding fields such that it starts its evolution in a pure state, and we study its evolution according to Eq. (2). We consider the system as described by a density matrix,

$$\rho = \sum_{ab} \rho_{ab} |E(T) + \epsilon_a, E_0 - E(T)\rangle\langle E(T) + \epsilon_b, E_0 - E(T)|, \tag{3}$$

where the first entry in the bra (ket) represents the energy of the black hole at instant T, which changes with time in an adiabatic fashion, the constant E_0 represents the mean value of the total energy of the system (which is conserved), and $E_0 - E(T)$ is the energy of the field at instant T. We consider the state to be a superposition of states of the black hole that differ in energy from $E(T)$ by ϵ_a. To simplify the analysis we consider only a pair of levels of energy that are separated by an energy proportional to the temperature, as one would expect for an evaporating hole. Concretely, the characteristic frequency for this energy is given by

$$\omega_{12}(T) = \frac{1}{(8\pi)^2 t_P} \left(\frac{t_P}{T_{\max} - T} \right)^{1/3} \tag{4}$$

with $T_{\max}$ the lifetime of the black hole (how long it takes to evaporate) and the subscript 12 denotes that it is the transition frequency between the two states of the system. Although this model sounds simple, it just underlies the robustness of the calculation: it shows that the black hole has discrete energy levels characterized by a separation determined by the temperature of the black hole. It is general enough to be implemented either assuming the Bekenstein spectrum of area or the spectrum stemming from loop quantum gravity [11]. We assume that we start with the black hole in a pure state, which is a superposition of different energy eigenstates (there is no reason to assume that the black hole is exactly in an energy eigenstate, which would imply a stationary state with no radiation being emitted; as soon as one takes into account the broadening of lines due to interaction, one has to consider a superposition of states within the same broadened level with a time dependent separation with a similar behavior). Therefore the density matrix has off-diagonal elements. One now needs to write Eq. (2) in the simplified energy basis we chose, and one can immediately integrate it to yield

$$\log\left(\frac{\rho_{12}(T)}{\rho_{12}(0)}\right) = i\int_0^T \omega_{12}(T')dT' + \frac{1}{(8\pi)^2}\log\left(\frac{T_{\max}-T}{T_{\max}}\right). \tag{5}$$

We therefore see that when time reaches the evaporation time $T = T_{\max}$, the density matrix element vanishes; i.e., the state has decohered completely. Therefore, there is no information puzzle to contend with.

The result presented above is remarkable for being able to erase completely the information before evaporation. On the other hand, it is clear that we have taken a very crude model for the black hole, and a more detailed calculation is needed before one can completely write off the black hole information puzzle; however, the present calculation provides hope that the problem can, indeed, be solved. A realistic calculation seems somewhat beyond the state of the art. For instance, it is clear that the calculation should model quantum mechanically the black hole but also the fields it interacts with in a detailed way in the context of a theory of quantum gravity.

Returning to other physical systems, a similar calculation with a two level system yields that the level of fundamental decoherence is

$$\log\left(\frac{\rho_{12}(T)}{\rho_{12}(0)}\right) = -\frac{3}{2}t_P^{(4/3)}T^{(2/3)}\omega_{12}^2. \tag{6}$$

The effect is too small to be observed in the laboratory, unless one can construct a system with a significant energy difference between the two levels. The most promising candidate systems are given by systems of ''Schrödinger cat'' type. Bose-Einstein condensates could in the future provide a system where the effect would be close to observability [7].

In summary, we have shown that unitarity in quantum mechanics holds only when describing the theory in terms of perfect idealized clocks. If one uses realistic clocks, loss of unitarity is introduced. We have estimated a minimum level of loss of unitarity based on constructing the most accurate clocks possible. The loss of unitarity is universal, affecting all physical phenomena. We have shown that although the effect is very small, it may be important enough to avoid the black hole information puzzle.

This work was supported by Grant No. NSF-PHY0244335 and funds from the Horace Hearne Jr. Institute for Theoretical Physics and the International Centre for Theoretical Physics (Trieste, Italy).

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