Comment on "Nature of the Isotope Effect on Transport in Tokamaks"

In a recent Letter, Tokar, Kalupin, and Unterberg (TKU) [1] suggested that the scaling of the dissipative trapped electron (DTE) mode turbulence could account for the favorable isotope (or mass) scaling observed in tokamaks, when the core plasma is not dominated by the ion temperature gradient (ITG) mode turbulence. The authors make their arguments within the broad outline of standard diffusivelike drift wave gyro-Bohm scaling theories (see Horton, Ref. [10] of Ref. [1], for a review) and generally discount the possible proximity to a critical temperature gradient which appears to be required to account for profile stiffness. The precise parameter dependence (or scaling) of such theories is clearly dependent on the assumed form of the turbulence mixing length approximation used to evaluate the saturated level of density fluctuations in the quasilinear transport theory. [Equation (4) of Ref. [1] is an example of the weak turbulence rule: $\chi = C(\gamma_{\max}/k_{\max}^2)[\gamma_{\max}^2/(\omega_{\max}^2 + \gamma_{\max}^2)]$.] Such theories (usually with the strong turbulence mixing rule $[\chi = C(\gamma_{\max}/k_{\max}^2)]$ involving the DTE and ITG mode scaling [as well as the collisionless trapped electron (CTE) and edge collisional drift wave (CDW)] have long been used to explain empirical confinement time scaling with power (P), density (n), and magnetic field (B) [2], but they fail to account for the favorable mass (or atomic weight A) scaling. It is often argued that the edge CDW can account for some of the unfavorable q (safety factor) scaling and perhaps some of the favorable A scaling. The particular form of the TKU theory for the DTE apparently has the long sought favorable A scaling (Figs. 2 and 3 of Ref. [1]), but the Letter fails to demonstrate that the form is quantitatively consistent with the empirical scaling for the other parameters, particularly n and P. Furthermore, the Letter apparently demonstrates quantitative A scaling at fixed temperature (T), whereas empirical A scaling of confinement time $\tau \propto a^2/\chi$ is usually expressed at fixed P. When combined with unfavorable T (or P) scaling, A scaling at fixed T is considerably weakened at fixed P.

In more detail, it can be argued that any collisionless gyro-Bohm model will be generally described by $\chi = (c_s/a)\rho^2 A^{1/2} F$, where following the notation of the Letter norms to the proton (hydrogen) mass $(c_s/a)\rho_s^2 \propto T^{3/2}/aB^2$. F can depend on q, $s = d\ln q/dr$, a/L_n, a/L_T, a/R, r/a, and β. These do not depend on A and, restricting ourselves to electrostatic theories, can be ignored. Ignoring passing electrons as a source of electron nonadiabaticity [ratio of electron transit frequency to mode frequency (ω_{te}/ω)] and ion-ion collisionality as a source A dependence, we are left only with electron-ion collisionality appropriate to the trapped electron physics, which we can express as electron collision frequency over mode frequency (ν_{ei}/ω). It can be assumed (as do the authors) that wave number spectrum scaled to the ion gyroradius will remain invariant $(k\rho_s A^{1/2} = \text{const})$, so that taking $\omega \approx \omega_*$, we have $F(\nu_{ei}/\omega) = F(A^{1/2}an/T^2) = C/(A^{1/2}an/T^2)^\alpha$. F is not really a simple power law (with fixed α) as indicated, but it is monotonically decreasing with electron collisionality. $\alpha = 0$ in the CTE regime; in the DTE regime $\alpha = 1$ for strong turbulence mixing and close to $\alpha = 2$ or possibly 3 for the weak mixing assumed by the authors. ($\alpha = -1$ for the strong mixing CDW, but then we also must include the ω_{te}/ω dependence.) We are left with the scaling law form $\chi = C(T^{3/2}/aB^2)A^{1/2}/(A^{1/2}an/T^2) \propto A^{-1/2}$ for $\alpha = 2$ (consistent with Fig. 2 of Ref. [1]). Converting to the fixed P scaling form, we have $\tau = (a^3nP^{-1})[C^{-1}B^2a^\alpha n^{\alpha-1}A^{1/2(\alpha-1)}P]^{1/(5/2+2\alpha)}$. For $\alpha = 0$ we get the CTE (and ITG) scaling $\tau = A^{-1/5}B^{4/5}C^{-2/5}n^{3/5}P^{-3/5}a^3$, which is rather close to empirical beam heated scaling apart from weakly unfavorable A scaling, which is empirically closer to $A^{1/5\to1/2}$. (It is actually quite close to TEXTOR-RI mode scaling for n and P [3].) For $\alpha = 2$, we get $\tau = A^{+1/13}B^{4/13}C^{-2/13}n^{15/13}P^{-11/13}a^{42/13}$ where the very weakly favorable A scaling buys a quite wrong P scaling. $\alpha = 1$ has null A scaling, but it very well describes neo-Alcator Ohmic (low-n but also low-T) scaling [2].

Isotope or A scaling remains poorly understood. It may be the result of some correlation with impurity content (hydrogen plasmas have higher Z_{eff} then deuterium), or, more likely, it is correlated with the diamagnetic (rho-star) component of $E \times B$ shear stabilization [4], not included in standard drift models before the mid 1990s.

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