The Song of Dunes as a Wave-Particle Mode Locking

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Singing dunes, which emit a loud sound as they avalanche, constitute a striking and poorly understood natural phenomenon. We show that, on the one hand, avalanches excite elastic waves at the surface of the dune, whose vibration produces the coherent acoustic emission in the air. The amplitude of the sound ($\simeq$ 105 dB) saturates exactly when the vibration makes the grains take off the flowing layer. On the other hand, we show that the sound frequency ($\simeq$100 Hz) is controlled by the shear rate inside the sand avalanche, which for granular matter is equivalent to the mean rate at which grains make collisions. This proves the existence of a feedback of elastic waves on particle motion, leading to a partial synchronization of the avalanching sand grains. It suggests that the song of dunes results from a wave-particle mode locking.

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Since Marco Polo [1], many travelers have given accounts of puzzling sounds heard in the desert. Throughout the 19th century, these phenomena have been related to the harmonious acoustic emission produced by some sand dunes as they avalanche [2–5]. This loud sound can be heard up to 10 km away and resembles the sound of a makhnovist drum or of a low-flying twin-engined jet. Sounds recorded in different locations have exhibited a well-defined frequency ranging from 65 to 100 Hz. Not all the dunes sing but all the singing dunes are composed by well sorted and very dry sand [2–8]. Infrared spectroscopy [9] has suggested that sound-producing grains could be covered by a characteristic silica gel layer. Still, a complete and satisfactory understanding of the dynamical mechanism [10] at the origin of the phenomenon has not been reached yet.

We performed our measurements in the Atlantic Sahara (Morocco), a coastal desert where all the dunes—more than 10 000 barchans—produce sound, whenever the Sun is able to dry out the humidity of the night. The wind erodes the back of the dune and accumulates sand at the top of the slip face at the front. When the slope is too large, a spontaneous avalanche nucleates and propagates down the dune, and sound is produced (Fig. 1). Artificial avalanches induced by sliding down the slip face have exactly the same acoustic emission: this proves that the sound is not due to the wind but directly to the grains motion. Whatever the size of the dune and the localization of the avalanche, we measured a frequency f of 100 ± 5 Hz (Figs. 1 and 4), meaning that it is not controlled by a resonance involving the geometry of the whole slip face but by intrinsic properties of the grain dynamics within an avalanche.

The sound and the motion of the ground were both recorded on the same digital audio tape [Fig. 1(a)]. The pressure wave in the air was measured with a cardioid condenser microphone, the signal of which was shifted in phase after calibration in the lab. A piezoelectric accelerometer was used to measure one component of the ground acceleration (usually along the normal to the slip face). Its size (typically 1 cm) was large compared to the grain size (180 μm) but small compared to the acoustic wavelengths. To measure the normal acceleration of the sand surface (Fig. 2 left), we mounted it on a small plastic floater of diameter 10 cm. Alternatively, the acceleration in the bulk of the avalanche was measured by fixing the transducer in approximately the same position—initially at the surface—by thin pieces of string (Fig. 2 right). In both cases, the accelerometer is let free to vibrate, but in the first case (surface), it can move along

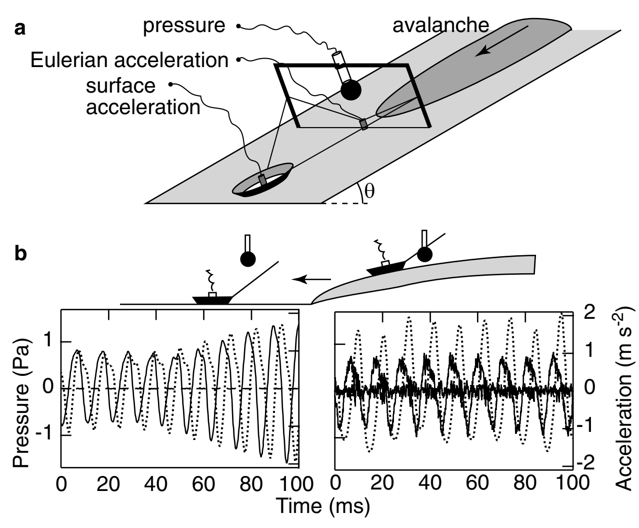

FIG. 1. (a) Measurement setup. (b) Samples of the sound (dotted line) and surface acceleration (solid line) signals, just before the avalanche front reaches the transducers (left) and just after (right). The dashed line is the same acceleration signal from which the Fourier components smaller than 500 Hz have been removed. It evidences the high frequency noise induced by avalanching grains (right), not present before the avalanche (left). The sound may be listened to at Ref. [1].

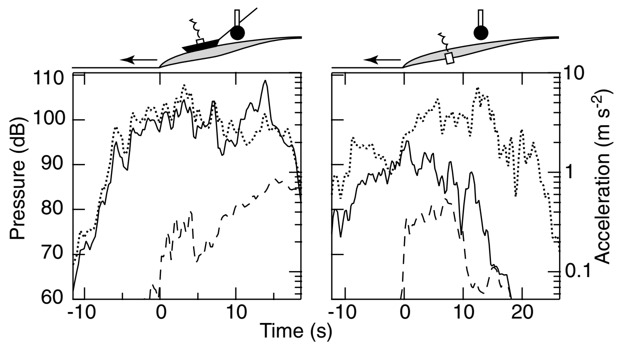

FIG. 2. Root mean square amplitude of the pressure (dotted line) and acceleration (solid line) signals, measured on a 1 s moving window. The avalanche propagates at approximately $0.5\ \mathrm{m\,s^{-1}}$ and reaches the transducers at $t = 0$, as can be observed on the noise amplitude (dashed line, see Fig. 1). The scales have been chosen in order to make the curves collapse if the surface behaves as a loud speaker [Eq. (1)]. When the accelerometer is fixed in approximately the same place (right), it is engulfed inside the avalanche so that the signal gets reduced, compared to the measurements at the surface (left).

the direction normal to the slip face to follow the evolution of the free surface, while in the second, its position is fixed (Eulerian measurement in the bulk).

The normal acceleration of the free surface shows that the sound is associated with a vibration of the sand surface in the avalanche zone, but also in the surrounding region where there is no surface flow (Fig. 1). The surface of the sand bed acts as the membrane of a loud speaker and its vibration is directly responsible for the acoustic emission in the air. The acceleration is approximately in phase with the pressure time derivative and we verified in the lab that it is the same for a loud speaker whose membrane is covered by sand. Furthermore, the power I emitted per unit surface in the air, measured with the microphone, is related to the mean square acceleration of the free surface $\langle a^2 \rangle$ by

$$I = \frac{\rho_{\mathrm{air}} c_{\mathrm{air}} \langle a^2 \rangle}{(2\pi f)^2}, \tag{1}$$

where ρ_{air} is the density of air and c_{air} the sound velocity in air. The sound is incredibly loud as the amplitude usually reaches 100 to 105 dB in the core of the avalanche (Fig. 2) which is close to the threshold of pain, 120 dB. At 106 dB, the peak acceleration of the surface grains is equal to $8.5\ \mathrm{m\,s^{-2}}$ and just balances the component of gravity perpendicular to the slip face $g\cos\theta$, where $\theta = 30°$ is the repose angle. The amplitude thus saturates when the grains at the surface start taking off. When the avalanche front passes the transducers, the acceleration signal becomes noisy but the sound signal remains smooth (Fig. 1). The oscillation of the sand bed, and thus the sound in air, do not present a discontinuity at the transition between rest and flowing. This indicates that the global oscillation of the sand bed (solid line) is responsible for the sound emission in air (dotted line), whereas the noise (dashed line) is due to the motion of the grains inside the avalanche. So the grains are both subjected to elastic deformations and relative displacements, meaning that the phenomenon results from a combination of solidlike and liquidlike behaviors.

The sand bed oscillations correspond to surface elastic waves. We will limit ourself to present an overview of their characteristics, sufficient to get a reasonable view of the phenomenon, and we postpone the detailed study to a forthcoming paper. Simultaneous measurements with two accelerometers, one in surface and the other buried in the sand below the first one, have shown that the amplitude of vibration abruptly decreases with depth, over typically 10 cm. The elastic waves are thus localized close to the surface, just like Rayleigh waves [11] (see Fig. 2). Measuring now two orthogonal components of the acceleration at the same place, we found that these waves were elliptically polarized. Finally, we performed measurements with two accelerometers mounted on floaters aligned along the steepest slope (schematic at the bottom of Fig. 3). The phase between these two signals, determined by cross correlation, is almost constant and of an opposite sign before and after the avalanche. So, the source of the surface elastic waves is inside the avalanche. The distance between the transducers was tuned to 21 cm, value for which the two signals remain almost in quadrature during the whole experiment: the wavelength $\lambda \simeq 42$ cm (star on Fig. 3) is almost the same when the sand is at rest or avalanching. In order to determine the dispersion relation, we used an external excitation: sinusoidal signals, recorded on a tape, were played through an amplified loud speaker that was kept at the surface of the avalanche slip face of a singing dune (schematic at the top of Fig. 3). As the mechanical transmission with the

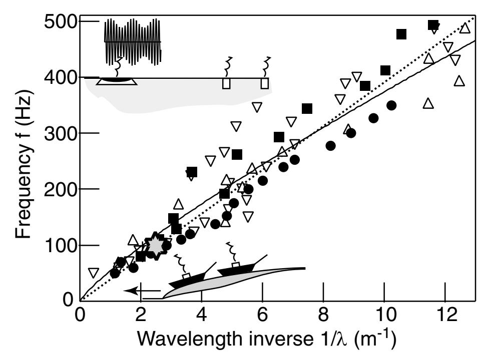

FIG. 3. Dispersion relation of surface elastic waves, the sand being at rest [transducers distant by 5 cm (down triangle), 15 cm (up triangle), 25 cm (square), and 42 cm (circle)] or avalanching (star). The dotted line corresponds to a nondispersive relation $f = c\lambda^{-1}$, with $c = 40\ \mathrm{m\,s^{-1}}$ and the solid line to the Rayleigh-Hertz relation $f = c_0^{2/3} g^{1/6} \lambda^{-5/6}$, with $c_0 = 230\ \mathrm{m\,s^{-1}}$.

sand bed is nonlinear, the signal is distorted and presents harmonics. We used this imperfection to derive several points of the dispersion relation from each recording. From the phase between the Fourier components of the signals of two accelerometers aligned with the loud speaker, we measured the wavelength λ associated with each frequency f. In order to unwrap the phase, the amplitude of the signals was slightly modulated at low frequency. The measurements reveal an incredibly low phase speed $c = 40 \pm 10\ \mathrm{m\,s^{-1}}$ (dotted line), 2 orders of magnitude lower than the speed c_b of elastic waves in the bulk of the material (quartz). In a pile composed by elastic grains in Hertz contact [11], the propagation speed should increase with the pressure P as $c_b^{2/3}\rho^{-1/6}P^{1/6}$ [12]. In the present case, gravity makes P increase linearly with depth, so that elastic waves inside the sand bed should be refracted toward the surface. This does not preclude the existence of propagative modes localized over a typical depth λ below the surface. The typical pressure P is then $\rho g \lambda$ and these Rayleigh-Hertz modes have thus a dispersion relation of the form

$$f = c_0^{2/3} g^{1/6} \lambda^{-5/6}, \tag{2}$$

where c_0 should be of the order of c_b. Our measurements are compatible with this relation but with a value of c_0 ($230\ \mathrm{m\,s^{-1}}$) at least 1 order of magnitude lower than c_b. This remains to be explained—in particular its possible relation with the silica gel layer on the surface of the grains [9]—and compared in the future to the case of glass beads and silent sand.

In silent granular avalanches, the grain motion results from the balance between gravity and dissipation due to inelastic collisions [13,14]. As in a Sisyphus-like nightmare, to make a relative displacement d, the grain has to overcome an energy barrier and then, precisely when its kinetic energy is maximal, to collide with the neighbor beneath. During the shock, a part of its translation energy is transferred into elastic deformations. As this could constitute a simple mechanism explaining the excitation of elastic waves, we have measured in a laboratory experiment the average velocity profile inside a small-scale avalanche of sound-producing grains. A homogeneous and steady flow [15] is generated between two glass plates spaced by 30 mm, at a controlled flow rate. The flow is visualized at the boundary with a fast video camera (1 kHz) inclined to be parallel to the free surface. From the time correlation of one image line, we extract the mean velocity $U(z)$ at the corresponding position z. The velocity profiles, shown on Fig. 4, closely resemble those obtained with glass beads [15]: they exhibit a strong shear layer near the free surface followed by a zone of intermittent motion known as the "creep tail." The velocity gradient Γ, which in granular flows is the typical rate at which grains jump over their neighbors and make collisions, is independent of the flowing depth H and is equal

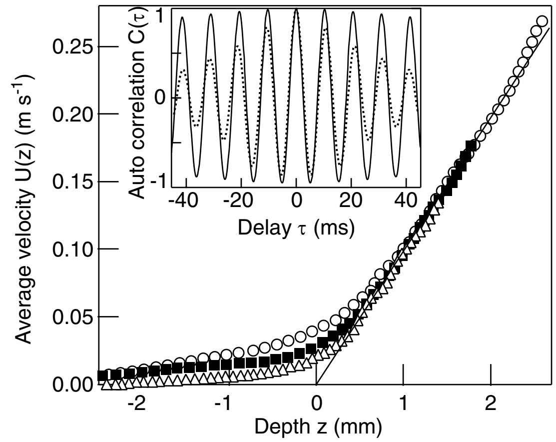

FIG. 4. Average velocity profile in a laboratory avalanche for different flow heights: $H = 8d$ (triangle), $H = 10d$ (square), $H = 15d$ (circle). The solid line corresponds to the velocity profile $U(z) = fz$ with the sound frequency $f = 100$ Hz. Inset: Autocorrelation of the microphone signal measured in the core of a steady and homogeneous avalanche (solid line) and a very intermittent one (dotted line).

to $100 \pm 5\ \mathrm{s^{-1}}$. This is precisely the spontaneous frequency of acoustic emission f, which strongly suggests that they are intimately related. The value is the same as for silent grains of the same diameter d [15] (here $d = 180\ \mu\mathrm{m}$): $\Gamma \simeq 0.4\sqrt{g/d}$. The scaling of f as $d^{-1/2}$ is approximately verified [6,7] in the narrow range of diameters d of sounding grains across the world: from 180 μm ($f = 100$ Hz) to 380 μm ($f = 66$ Hz) [7].

This scaling is valid as long as the flow is homogeneous. As soon as the flow is inhomogeneous, inertial effects add to gravity and modify Γ. Correspondingly, it has been remarked [6–8] that the emission frequency f increases significantly when one quickly pushes a small avalanche by hand, or releases a large quantity of sand on a flat surface. The avalanche inhomogeneity is the essential limit for the temporal coherence of the emission. Using the first maximum of autocorrelation C_m (inset of Fig. 4), we define the period τ_m and the coherence time $\tau_m/(1 - C_m)$. When one slides steadily down the slip face, a very homogeneous avalanche can be maintained and the coherence time is of the order of 10 periods ($\simeq$100 ms). When, on the contrary, one slides down intermittently, the avalanche is inhomogeneous and the coherence time drops to few periods ($\simeq$20 ms). The spatial coherence of the acoustic source can be ascribed to that of the surface elastic waves so the coherence length in a homogeneous avalanche is around 4 m which is the size of the avalanche itself.

On the basis of these results, we are able to conclude that the sound emission is induced by surface elastic waves whose frequency f is slaved to the collision rate Γ inside the avalanche. This strongly suggests the existence of an interaction between the two. A grain inside

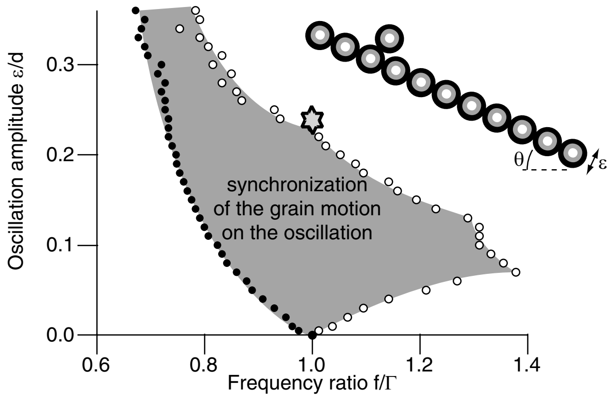

FIG. 5. Dynamics of a single grain rolling down an inclined array of grains of same diameter that oscillates at the frequency f with an amplitude ϵ. Γ is the frequency of collisions at $f = 0$. The diagram shows the region of mode-locking, i.e., the region of parameters space—the rescaled amplitude ϵ/d and the rescaled frequency f/Γ—where the grain collisions synchronize on the oscillation. The star indicates the parameters selected in booming avalanches ($\epsilon \simeq 0.24d$ and $f = \Gamma$).

the avalanche moves at a mean velocity Γd with respect to the grains beneath it. So the collisions, which happen at a mean frequency Γ, excite elastic waves. This is why the characteristics of the grain surface and particularly its softness are so important. If the whole sand layer oscillates at a frequency f close to Γ, it induces a further force that oscillates in time at frequency f. As demonstrated on Fig. 5, this feedback mechanism, which is formally similar to a phase-locked loop [16], tends to synchronize the motion of the grain with the oscillation. Obviously, the synchronization is the most efficient for moderate amplitudes of oscillations: as soon as the acceleration becomes larger than gravity—when the peak-peak amplitude reaches $d/4$—the grain takes off and its trajectory becomes chaotic. In summary, the vibration of the bed tends to synchronize the collisions, which themselves excite this vibration.

Our results show that the acoustic emission of a harmonious sound—and not a random noise—by booming avalanches is induced by coherent elastic waves, localized in surface, elliptically polarized, and of amplitude one fourth of grain diameter. The very low propagation speed of these waves—and more generally the behavior of granular media at low frequencies and large wavelength—remains unexplained. These waves are themselves excited by the collisions of grains inside the avalanche. To produce a sound, this excitation has to be—at least partially—synchronized with the elastic waves, which suggests a mechanism of wave-particle mode locking. For a typical avalanche (4 m long, 1 m wide), the power extracted to gravity is about 3 kW, the power radiated in the elastic waves is about 10 W, and that radiated in the acoustic emission is about 150 mW. This means that actually, a very small part of the moving grains are synchronized. The song of dunes turns out to transform an external incoherent mechanical work into coherent acoustic radiation, but with a very low output and with a poor coherence due to the size of the acoustic source (approximately 10 wavelengths). Future investigations, with the help of molecular dynamics simulations, are needed to determine the conditions on the softness of the grains and the restitution coefficient under which this feedback loop produces a coherent vibration of the surface.

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