Nonlinear Evolution of Surface Gravity Waves over Highly Variable Depth

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New nonlinear evolution equations are derived that generalize those presented in a Letter by Matsuno [Phys. Rev. Lett. **69**, 609 (1992)] and a terrain-following Boussinesq system recently deduced by Nachbin [SIAM J Appl. Math. **63**, 905 (2003)]. The regime considers finite-amplitude surface gravity waves on a two-dimensional incompressible and inviscid fluid of, highly variable, finite depth. A Fourier-type operator is expanded in a wave steepness parameter. The novelty is that the topography can vary on a broad range of scales. It can also have a complex profile including that of a multiply valued function. The resulting evolution equations are variable coefficient Boussinesq-type equations. The formulation is over a periodically extended domain so that, as an application, it produces efficient Fourier (fast-Fourier-transform algorithm) solvers.

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The study of weakly nonlinear (finite-amplitude) water waves has been of great research interest for quite some time. Many fundamental models (namely partial differential equations) have been derived according to the physical situation and carefully analyzed for their mathematical properties. In a recent Letter Matsuno [1] derived a Boussinesq-type model that arises from an expansion in a steepness parameter and not in the dispersion parameter corresponding to a long wave regime. Matsuno develops a method based on the theory of complex functions and a systematic perturbation theory with respect to the steepness parameter $\varepsilon \equiv a/\ell$ (a is the amplitude scale and ℓ is a wavelength scale). We start by using dimensionless parameters such as in Whitham's book [2] and in [3]: let $\alpha = a/h_o$ be the nonlinearity parameter (with the typical, say, average depth denoted by h_o) and $\beta = h_o^2/\ell^2$ be the dispersion parameter. As pointed out in [1] the formulation naturally suggests combining these parameters as $\alpha\sqrt{\beta} \equiv \varepsilon$. Hence the characteristic depth h_o cancels out and the free surface perturbations are controlled through ε. The starting point for the asymptotic analysis are the dimensionless potential theory equations.

The novelty in this Letter is that we accommodate the asymptotic modeling to consider very general topographic profiles, as, for example, the one presented in Fig. 1. We also start from the nonlinear, dimensionless potential theory equations but follow an analytical route that is somewhat different from Matsuno's. Naturally some of the transforms used are similar, but they are "brought into the picture" by different means. In particular, the Fourier analysis presented naturally suggests the use of fast-Fourier-transform algorithm (FFT) based methods to generate efficient numerical schemes. The formulation presented here also extends a weakly dispersive terrain-following Boussinesq system, recently published in [3]. A regime of interest is pulse shaped surface gravity waves interacting with rapidly varying, disordered, topographies.

We use an orthogonal curvilinear coordinate system for the potential theory equations. Within this frame we are able to write a Dirichlet-to-Neumann (DTN) operator which automatically reduces the entire dynamics to the free surface. This formulation is possible in the presence of complex multivalued profiles, or even rapidly varying topographies. Our transforms not only resemble, but are clearly related to, those in very recent work [1,4–6]. Some of the differences are that we work in Fourier space and Matsuno [1] uses complex functions, while Craig and Sulem [4] an iterative DTN operator inversion procedure. We define our curvilinear coordinate system through the conformal mapping of the *undisturbed corrugated channel.* Byatt-Smith [5] maps the (finite-amplitude) time dependent flat channel (i.e. without a topography) while Zakharov *et al.* [6] map a channel of infinite depth. As mentioned by Matsuno [1] the equations presented in [5] are quite complicated. In our formulation we work in the physical domain, through the curvilinear coordinate system and we arrive at Fourier-type transforms which are easily incorporated into a numerical method. This was also efficiently used in [4,6] but in the absence of topographies. All schemes deal with (singular) Hilbert-

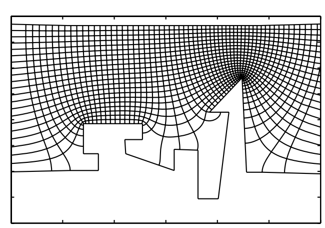

FIG. 1. Topography profile together with the level curves of the $\xi\zeta$ coordinate system.

differential equations. Our final result is a, variable coefficient, weakly nonlinear evolution equation of the Boussinesq-type. Nonlinear waves in a highly heterogeneous medium is of great interest in physics and mathematics. Boussinesq-type models are of great interest in geophysical applications. Recently, using a power series expansion for the velocity potential, Nachbin [3] derived a terrain-following Boussinesq system valid for weakly dispersive, weakly nonlinear waves in the presence of a rapidly varying topography. This model was used to study long waves interacting with disordered (randomly varying) topographies [7–10], in particular for the time-reversal refocusing of solitary waves [8–10]. Our generalization of Matsuno's Boussinesq-type system has, as its (weakly dispersive) leading order approximation the terrain-following system presented in Nachbin [3]. This is a nice consistency check since here we do not use a power series expansion for the potential, but rather we do asymptotics with the (Fourier) symbol of integral operators.

Having defined the dimensionless parameters α and β, we are also led to defining a parameter controlling the depth irregularities: $\gamma \equiv \ell_b/\ell$, where ℓ_b is the (horizontal) characteristic length scale for the bottom variations, described through $y = -H(x/\gamma)$ [3]. The topography's amplitude does not need to be small. If $\gamma \ll 1$ the topography is rapidly varying. Using the Schwarz-Christoffel transformation [11] we define a mapping from a uniform strip in the $\xi\zeta$ plane onto the undisturbed corrugated strip in the physical xy plane. This is possible even when $H(x/\gamma)$ represents a polygonal, multivalued profile as in Fig. 1. This formulation defines $x(\xi, \zeta)$ and $y(\xi, \zeta)$ as harmonic functions together with (ξ, ζ) level curves of an orthogonal curvilinear coordinate system in the physical domain (c.f. Fig. 1). In the curvilinear coordinate system we have that $(\partial_{\xi\xi} + \partial_{\zeta\zeta}) = |J|^2(\partial_{xx} + \partial_{yy})$, where, by the Cauchy-Riemann equations [3], the Jacobian of the $(\xi, \zeta) \mapsto (x, y)$ transformation is $|J| \equiv y_\xi^2 + y_\zeta^2$. Initially, for convenience, we will keep track of the free surface (FS) elevation in both coordinate systems. In the mapped domain we have $\zeta \equiv S(\xi, t) = \varepsilon N(\xi, t)$, while in the physical domain it is defined by $y[\xi, S(\xi, t)] = \alpha\sqrt{\beta}\eta \times (x[\xi, S(\xi, t)], t)$, with η denoting the surface gravity wave. When the FS has a small steepness (as indicated by εN above) the Jacobian can be approximated as $|J| \approx y_\zeta^2(\xi, 0) + O(\varepsilon^2)$ [3] and the relevant FS metric coefficient is $M(\xi; \sqrt{\beta}, \gamma) \equiv y_\zeta(\xi, 0)$, where

$$M(\xi; \sqrt{\beta}, \gamma) = \frac{\pi}{4\sqrt{\beta}} \int_{-\infty}^{\infty} \frac{H[x(\xi_o, -\sqrt{\beta})/\gamma]}{\cosh^2 \frac{\pi}{2\sqrt{\beta}}(\xi_o - \xi)} d\xi_o.$$

The metric term carries vertical and horizontal bottom variation scales through the parameters $\sqrt{\beta}$ and γ.

Laplace's equation, for the velocity potencial ϕ, is invariant through our change of coordinates. Thus we start directly with the nonlinear potential theory equations in curvilinear $\xi\zeta$ coordinates, as presented in Nachbin [3]:

$$\phi_{\xi\xi} + \phi_{\zeta\zeta} = 0, \qquad -\sqrt{\beta} < \zeta < S(\xi, t). \tag{1}$$

At the free surface the free boundary conditions are

$$N_t + \frac{\alpha}{|J|}\phi_\xi N_\xi - \frac{1}{|J|\sqrt{\beta}}\phi_\zeta = 0, \tag{2}$$

$$\phi_t + \frac{\alpha}{2|J|}(\phi_\xi^2 + \phi_\zeta^2) + \eta = 0. \tag{3}$$

We have a trivial Neumann condition along the impermeable topography given as $\zeta \equiv -\sqrt{\beta}$: $\phi_\zeta = 0$. No approximation has been made up to this point. In the sequel we will calculate the source distribution for the integral representation of the FS's Dirichlet data and its impact on representing the DTN operator. For a nonlinear FS the source distribution is found implicitly and through a mild steepness hypothesis ($\varepsilon \ll 1$) we find an efficient approximation for the DTN operator.

The Dirichlet-to-Neumann (DTN) operator.—We formulate a DTN operator for the finite depth wave problem based on Guidotti's [12] work for a time independent harmonic function on a corrugated half plane. To simplify the presentation we analyze the harmonic problem at a frozen instant of time:

$$\phi_{\xi\xi} + \phi_{\zeta\zeta} = 0 \text{ in } -\sqrt{\beta} < \zeta < S(\xi), \tag{4}$$

$$\phi[\xi, S(\xi)] = \varphi(\xi) \quad \text{at the FS } \zeta = S(\xi), \tag{5}$$

$$\phi_\zeta(\xi, -\sqrt{\beta}) = 0 \quad \text{at the topography } \zeta = -\sqrt{\beta}. \tag{6}$$

Our goal is to express the Neumann data directly from its corresponding Dirichlet data $\varphi(\xi)$. By reflecting our domain about the topographic level curve $\zeta \equiv -\sqrt{\beta}$ we convert the problem above into the Dirichlet problem

$$\phi_{\xi\xi} + \phi_{\zeta\zeta} = 0 \text{ in } -2\sqrt{\beta} - S(\xi) < \zeta < S(\xi), \tag{7}$$

$$\phi = \varphi(\xi) \quad \text{at } \zeta = S(\xi), \tag{8}$$

$$\phi = \varphi(\xi) \quad \text{at } \zeta = -S(\xi) - 2\sqrt{\beta}. \tag{9}$$

Moreover, due to the symmetry about $\zeta \equiv -\sqrt{\beta}$ we have that $\phi_\zeta(\xi, -\sqrt{\beta}) = 0$ is automatically satisfied. We proceed in obtaining an integral representation for the solution of problem (7)–(9). Aiming at producing Fourier-type transforms, that can be easily converted into an efficient numerical (FFT) scheme, we extend our domain periodically and normalize it to having period one. The time independent potential is cast into the integral representation

$$\phi(\xi, \zeta) = \int_0^1 K(\xi, \tilde{\xi}, \zeta, S(\tilde{\xi}))f(\tilde{\xi})|\Gamma|d\tilde{\xi}, \tag{10}$$

$K(\xi, \tilde{\xi}, \zeta, S(\tilde{\xi})) = G(s, -\zeta + \tilde{S}) + G(s, \zeta + \tilde{S} + 2\sqrt{\beta})$, where $f(\tilde{\xi})$ is the symmetric source distribution over the boundary curves, $|\Gamma|^2 \equiv 1 + S_\xi^2(\xi)$, $s = \xi - \tilde{\xi}$, and $\tilde{S} = S(\tilde{\xi})$. The kernel proposed by Guidotti [12], $G(\xi, \zeta) = 1/2\pi \ln[1 + e^{-4\pi\zeta} - 2e^{-2\pi\zeta}\cos(2\pi\xi)]$, has a logarithmic singularity and can also be written as

$$G(\xi,\zeta) = -\sum_{\kappa\neq 0} \frac{1}{2\pi|\kappa|} e^{-2\pi|\kappa|\zeta} e^{2\pi i\kappa\xi} \quad \text{if } \zeta \geq 0. \qquad (11)$$

It also has the property that $\lim_{\zeta\to 0^+} \partial_\zeta G(\xi,\zeta) = \delta(\xi) - 1$. One can then verify that the integral representation satisfies $\phi_\zeta(\xi, -\sqrt{\beta}) = 0$ and $\phi(\xi,\zeta) = \phi(\xi, -\zeta - 2\sqrt{\beta})$ as expected. Thus we have formally solved the potential problem (4)–(6) once we find the source distribution $f(\xi)$.

But first we outline the calculation of the DTN operator which maps the FS Dirichlet data directly onto its corresponding Neumann data. The integral representation in (10) can be written as

$$\phi(\xi,\zeta) = \int_0^1 K(\xi,\tilde{\xi},0,0) f(\tilde{\xi})|\Gamma| d\tilde{\xi} + \int_0^1 \{K(\xi,\tilde{\xi},\zeta,S(\tilde{\xi})) - K(\xi,\tilde{\xi},0,0)\} f(\tilde{\xi})|\Gamma| d\tilde{\xi}. \qquad (12)$$

The first term, containing $K(\xi,\tilde{\xi},0,0)$, is the linear (singular) term corresponding to infinitesimal perturbations about the undisturbed water surface. In this way the nonlinear term arises as a *desingularized correction term*. Taking the normal derivative of the potential in (12) and letting the interior point approach the top boundary we obtain, in the limit, the Dirichlet-to-Neumann operator $\mathrm{DTN}(\varphi)(\xi) \equiv \partial\phi/\partial n(\xi, S(\xi))$, which gives the Neumann data along the FS. A more explicit description will be given below in the gravity wave context.

Now we go back to calculating the source distribution $f(\xi)$ along the FS. Motivated by (12) we decompose the Dirichlet data into its linear and nonlinear parts: $\varphi(\xi) \equiv \varphi_{\mathrm{L}}(\xi) + \varphi_{\mathrm{NL}}(\xi)$. Using the Fourier representation of the Green's function G in the linear term of (12), and carefully computing the limit as we approach the top boundary, we have that $\varphi_{\mathrm{L}}(\xi) = \mathbf{P}[f|\Gamma|]$ where

$$\mathbf{P}[f|\Gamma|] \equiv -\sum_{\kappa\neq 0} \frac{1 + e^{-2\pi|\kappa|2\sqrt{\beta}}}{2\pi|\kappa|} \mathbf{F}_\kappa[f|\Gamma|] e^{2\pi i\kappa\xi}. \qquad (13)$$

By $\mathbf{F}_\kappa[g]$ we denote the Fourier series of $g(\xi)$. The contribution from the linear part is easily computed through $f|\Gamma| = \mathbf{P}^{-1}[\varphi_{\mathrm{L}}]$. This is equivalent to the flat FS case, but recall that $S(\xi) = O(\varepsilon)$. The main difficulty resides on inverting (12), for computing the source distribution $f(\xi)$ along the nonlinear FS, in the presence of a kernel depending on S. Hence we perform an $O(\varepsilon)$ approximation in (12) so that $\phi(\xi, S) = \varphi(\xi) \approx \mathbf{P}[f|\Gamma|] + \mathbf{R}_S[f|\Gamma|]$. The operator $\mathbf{R}_S$, which depends on the FS profile S, is an $O(\varepsilon)$ approximation to the full nonlinear term $\varphi_{\mathrm{NL}}(\xi)$ and will appear combined with $\mathbf{P}^{-1}$ as indicated below. By applying the operator $\mathbf{P}^{-1}(\mathbf{I} - \mathbf{R}_S\mathbf{P}^{-1})$ to the approximation above, to leading order, we obtain

$$f(\xi)|\Gamma| = \mathbf{P}^{-1}[\varphi] - \mathbf{P}^{-1}\mathbf{R}_S\mathbf{P}^{-1}[\varphi] + O(\varepsilon^2). \qquad (14)$$

Using the Fourier representation of G, as we did above for the linear analysis, we arrive at

$$\mathbf{R}_S\mathbf{P}^{-1}[\varphi] = S\sum_{\kappa\neq 0} 2\pi\kappa \tanh(2\pi\kappa\sqrt{\beta}) \mathbf{F}_\kappa[\varphi] e^{2\pi i\kappa\xi} + \sum_{\kappa\neq 0} \{1 + e^{-2\pi|\kappa|2\sqrt{\beta}}\} \mathbf{F}_\kappa[S\mathbf{P}^{-1}[\varphi]] e^{2\pi i\kappa\xi}$$

where, through the approximate inversion procedure (14), we have that

$$f(\xi)|\Gamma| = \mathbf{P}^{-1}[\varphi] - \mathbf{P}^{-1}[S\mathrm{DTN}_0[\varphi]] + \sum_{\kappa\neq 0} 2\pi|\kappa| \mathbf{F}_\kappa[S\mathbf{P}^{-1}[\varphi]] e^{2\pi i\kappa\xi} + O(\varepsilon^2). \qquad (15)$$

This approximation for the nonlinear source distribution $f(\xi)$ is expressed by a linear contribution [inverting (13)] corrected by [$S(\xi)$-dependent] iterates of linear objects: namely, the linear ($S \equiv 0$) Dirichlet-to-Neumann operator DTN_0 and again $\mathbf{P}^{-1}[\varphi]$. In the sequel we will confirm this interpretation for the DTN_0 operator, which is defined by the first sum in the expression for $\mathbf{R}_S\mathbf{P}^{-1}$. Thus we have approximated the calculation of the source distribution by straightforward Fourier transforms (namely FFTs) of smooth functions. We will show how the nonlinear DTN operator can be approximated by straightforward compositions of the linear DTN_0 operator.

Evolution of weakly nonlinear surface gravity waves.— In the context of nonlinear surface gravity waves, the Neumann data is needed at the second FS condition. Writing the norm (squared) of the velocity in terms of its tangential and normal components, implies that we need to find the ξ and ζ derivatives as in

$$\mathrm{D\,TN}[\varphi](\xi) \equiv \phi_n(\xi, S) = \left[-\frac{S_\xi}{|\Gamma|}\phi_\xi + \frac{1}{|\Gamma|}\phi_\zeta \right]_{(\xi,S)}. \qquad (16)$$

Differentiating (12) and using (16) we have that at $\zeta = S$

$$\begin{aligned} \phi_\zeta &= \mathrm{DTN}_0[\varphi - S\mathrm{DTN}_0[\varphi]] - S\varphi_{\xi\xi} + O(\varepsilon^2), \\ \phi_\xi &= \varphi_\xi + O(\varepsilon). \end{aligned} \qquad (17)$$

When $S \equiv 0$, ζ is the normal direction and (17) is exact, confirming the interpretation given to DTN_0. There are other ways of representing the $\mathrm{DTN}_0[\varphi] \equiv \mathbf{T}[\varphi_\xi]$:

$$\mathbf{T}[\varphi_\xi] = -i\sum_{\kappa\neq 0} \tanh[2\pi\kappa\sqrt{\beta}] \mathbf{F}_\kappa[\varphi_\xi] e^{2\pi i\kappa\xi}. \qquad (18)$$

The dispersion relation appears as the symbol of the linear DTN_0 operator (c.f. also [4]). For example, in Berger and Milewski [13] a Fourier-type integral transform, also having the dispersion relation as its symbol, is used in studying surface gravity wave interaction and wave turbulence. We point out that $\mathbf{T}$ is the periodic counterpart of the (singular) integral operator used by Matsuno [1]:

$$\tilde{\mathbf{T}}[\varphi_x] = \frac{1}{2\sqrt{\beta}} - \int_{-\infty}^{\infty} \frac{\varphi_x(x')}{\sinh[\frac{\pi}{2\sqrt{\beta}}(x - x')]} dx'. \qquad (19)$$

Moreover it is important to notice that the operator acts on slightly different velocity components. Here we have a terrain-following (ϕ_ξ) velocity component along the FS, which clearly is tangent to neither the undisturbed, nor the perturbed FS. On the other hand Matsuno uses the horizontal (ϕ_x) velocity component at the FS (see (18) in [1] and also (27) in Craig and Sulem [4]).

Furthermore, recall that the surface wave profile is denoted by $S(\xi, t) \equiv \varepsilon N(\xi, t)$ which, together with the operator $\mathbf{T}$ and (16) and (17), leads to the asymptotic expression $|\Gamma|\mathrm{DTN}[\varphi](\xi) = \mathbf{T}[\varphi_\xi] - \varepsilon\{\mathbf{T}[(N\mathbf{T}[\varphi_\xi])_\xi] - (N\varphi_\xi)_\xi\} + O(\varepsilon^2)$. Define $U \equiv \phi_\xi$ as the "horizontal" terrain-following velocity component. Then $\varphi_\xi = U + \varepsilon N_\xi \mathbf{T}[U] + O(\varepsilon^2)$ and the *normal derivative approximation* reads as

$$|\Gamma|\phi_n = \mathbf{T}[U] - \varepsilon\mathbf{T}[N\mathbf{T}[U]_\xi] - \varepsilon[NU]_\xi + O(\varepsilon^2). \quad (20)$$

The kinematic condition, in terms of the normal derivative to $\zeta = \varepsilon N$, is written as $N_t - |\Gamma|\phi_n/[M(\xi)^2 \times \sqrt{\beta}] = 0$. Substituting (20), using the fact that $\eta(x, t) = M(\xi)N(\xi, t) + O(\varepsilon^2)$ [3] and dropping the $O(\varepsilon^2)$ term, we obtain a fully dispersive evolution equation for the wave elevation:

$$\eta_t - \frac{1}{M\sqrt{\beta}}\left\{\mathbf{T}[U] - \varepsilon\left(\left(\frac{\eta U}{M}\right)_\xi - \mathbf{T}\left[\frac{\eta}{M}\mathbf{T}(U)_\xi\right]\right)\right\} = 0. \quad (21)$$

For the dynamic FS condition we start by taking its ξ-derivative. We should keep in mind that $\phi[\xi, S(\xi, t), t] = \varphi(\xi, t)$ is such that $d\phi/dt = \varphi_t = \phi_t + \phi_\zeta S_t$ and $\phi_t = \varphi_t - \phi_\zeta S_t$. Moreover $d(\phi_t)/d\xi = \varphi_{t\xi} - \phi_{\zeta\xi}S_t - \phi_\zeta S_{t\xi}$. Using approximations given above, together with (17), we have

$$\begin{aligned}\frac{d}{d\xi}\phi_t &= U_t + \varepsilon N_{\xi t}\mathbf{T}[U] + \varepsilon N_\xi \mathbf{T}[U]_t \\ &\quad - \phi_{\zeta\xi}S_t - \phi_\zeta S_{t\xi} + O(\varepsilon^2) \\ &= U_t + \varepsilon N_\xi \mathbf{T}[U]_t - \varepsilon\mathbf{T}[U]_\xi N_t + O(\varepsilon^2).\end{aligned}$$

Going back to the dynamic condition and using the fact that we are working with the velocity components $\phi_\xi \equiv U$ and $\phi_\zeta = \mathbf{T}[U] + O(\varepsilon)$, after some computation, we arrive at

$$U_t + \eta_\xi + \varepsilon\left\{\frac{1}{2\sqrt{\beta}}\left[\left(\frac{U^2}{M^2}\right)_\xi + \left(\frac{1}{2M^2}\right)_\xi \mathbf{T}(U)^2\right] - \left(\frac{\eta}{M}\right)_\xi \mathbf{T}[\eta_\xi]\right\} = 0, \quad (22)$$

where we have again dropped the $O(\varepsilon^2)$ term. Eqs. (21) and (22) represent a *fully dispersive terrain-following Boussinesq system.* For a flat bottom [i.e. $M(\xi) \equiv 1$] this system reduces to system (19)-(20) in Matsuno [1]. Also by expanding in β, for the weakly dispersive regime, we can recover the terrain-following Boussinesq system as in [3], when using the vertically averaged terrain-following velocity component (equation (5.12) in [3]). Moreover for $\beta > 0.25$ it is known [14], from the linear potential theory, that the surface wave does not feel the bottom anymore. Therefore in this regime there is no point in generalizing Matsuno's formulation.

Following Matsuno [1] system (21) and (22) can be transformed into a variable coefficient, second order Boussinesq equation:

$$\sqrt{\beta}M\eta_{tt} + \mathbf{T}[\eta_\xi] + \varepsilon\Big(-\frac{1}{2}\Big(\frac{(\eta^2)_\xi}{M}\Big)_\xi + \sqrt{\beta}\Big(\frac{\eta_t}{M}\mathbf{T}^{-1}[M\eta_t]\Big)_\xi + \frac{\sqrt{\beta}}{2}\mathbf{T}\Big[\eta_t^2 + \mathbf{T}^{-1}(M\eta_t)^2\frac{1}{M^2}\Big]_\xi - \mathbf{T}\Big[\frac{\eta}{M}\mathbf{T}(\eta_\xi)\Big]_\xi\Big) = 0.$$

For the flat bottom case ($M \equiv 1$), using condition $\mathbf{T}[\mathbf{T}^{-1}(f)^2] = \mathbf{T}[f^2] + 2\mathbf{T}[f]f$ given in [1], this equation reduces to Matsuno's main result (namely Eq. (22); c.f. definition of δ and κ in [1] page 609). In the presence of the metric term the simplification of the equation, through the identity above, does not apply.

In this Letter we have generalized Matsuno's [1] equation for a general class of topography profiles as well as the terrain-following system [3] to being fully dispersive. The formulation is such that an efficient (FFT based) numerical scheme is readily available.

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