Bound on Lorentz and *CPT* **Violating Boost Effects for the Neutron**

F. Canè, D. Bear, D. F. Phillips, M. S. Rosen, C. L. Smallwood, R. E. Stoner, and R. L. Walsworth *Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA*

V. Alan Kostelecky´

Physics Department, Indiana University, Bloomington, Indiana 47405, USA (Received 29 August 2003; published 30 November 2004)

A search for an annual variation of a daily sidereal modulation of the frequency difference between colocated 129Xe and 3He Zeeman masers sets a stringent limit on boost-dependent Lorentz and *CPT* violation involving the neutron, consistent with no effect at the level of 150 nHz. In the framework of the general standard-model extension, the present result provides the first clean test for the fermion sector of the symmetry of spacetime under boost transformations at a level of 10^{-27} GeV.

DOI: 10.1103/PhysRevLett.93.230801 PACS numbers: 06.30.Ft, 11.30.Cp, 11.30.Er, 32.60.+i

The invariance of the laws of nature under transformations between inertial reference frames that differ in relative velocity (''boosts'') is an essential feature of Lorentz symmetry, along with invariance under rotations. Experimental tests of boost invariance, such as the Kennedy-Thorndike experiment [1], have been performed for many years with increasing precision [2]. These experiments typically search for a variation of the velocity of light with the laboratory velocity and test boost invariance in the context of classical electrodynamics. However, the fundamental role of Lorentz symmetry in physics makes it desirable to test boost invariance for other systems, such as massive particles with spin.

In this Letter, we present a high-sensitivity experimental test of boost invariance for the neutron, which we interpret in the context of the standard-model extension (SME) [3], a general theoretical framework that allows a comprehensive and systematic study of the implications of Lorentz-symmetry violation at observable energies. The SME provides a widely accepted formalism for the interpretation and comparison of experimental searches for violations of Lorentz symmetry and associated violations of *CPT* symmetry (the product of charge conjugation, parity inversion, and time reversal). The SME has been applied to many systems, including mesons [4], photons [2,5], and leptons [6–8], as well as the neutron [9,10] and proton [11]. An observable Lorentz violation could be a remnant of Planck-scale physics. One attractive origin is spontaneous Lorentz breaking in a fundamental theory [12], but other sources are possible [13].

Our experiment consists of a long-term monitoring of the frequencies of colocated 3 He and 129 Xe Zeeman masers as the Earth rotates and revolves around the Sun. We search for a specific signature of a violation of boost invariance: an annual variation of the nuclear Zeeman splitting, modulated at the frequency of the Earth's daily sidereal rotation. Such an effect could arise from couplings of the 3 He and 129 Xe nuclear spins (each largely determined by a valence neutron) to background tensor fields, including a dependence of the Zeeman frequencies on the instantaneous velocity (magnitude and direction) of the laboratory. The appeal of the noble-gas maser experiment is the excellent absolute frequency stability [10,14,15], and thus the sensitivity to small, slow variations in the magnitude of Lorentz-violating spin couplings.

Using the two-species noble-gas maser, we recently constrained the possible rotation-symmetry-violating couplings of the neutron spin with respect to an inertial reference frame based on the Earth [10]. Here, we choose a Sun-based inertial reference frame, which allows us to study cleanly—for the first time in the fermion sector the symmetry of spacetime with respect to *boost* transformations. (The pioneering work of Berglund *et al.* [16] does not distinguish between the boost and rotation effects to which it is sensitive.) Our experiment's rest frame moves with the Earth around the Sun at a velocity of magnitude $v_{\phi}/c = \beta_{\phi} \approx 9.9 \times 10^{-5}$, and the Lorentz transformation that describes the change of coordinates from the laboratory frame to the Sun-based frame includes both a rotation, **R**, and a boost along the velocity β .

The most general, coordinate-independent Hamiltonian, *H*, containing the Zeeman effect (from an applied magnetic field \vec{B}) and Lorentz-symmetry-violating couplings of the noble-gas nuclear spins, \vec{I} , including leading terms to first order in β , takes the simple form

$$
H = \vec{I} \cdot (\gamma \vec{B} + \mathbf{R}(t)\vec{\lambda}_0 + \vec{\beta}(t)\mathbf{R}(t)\Lambda_0).
$$
 (1)

Here, the vectors \vec{l} and \vec{B} are expressed in the lab frame, whereas the explicit Lorentz-symmetry-violating vector $\vec{\lambda}_o$ and 3 \times 3 matrix Λ_o have elements that are combinations of SME coefficients, which may be determined in terms of fundamental Lorentz-violating interactions [17,18], and are assumed constant in the Sun frame. The second term of Eq. (1) leads to a rotation-dependent modulation of the maser frequency. The third term contains cross couplings, in which the rotation induces daily sidereal modulations of the maser frequencies, while the boost transformation induces a sinusoidal variation of the daily modulation amplitude over the course of the sidereal year [19] as the direction of the velocity of the Earth varies with respect to the Sun. Terms from higher rank tensors (such as a yearly modulation in the maser frequency—for which our maser does not have the stability to set strong limits) have been neglected.

We refer the reader to previous publications [10,14,15] for details on the design and operation of our two-species noble-gas Zeeman maser. Here, we provide a brief review. Colocated ensembles of ^{129}Xe and ^{3}He atoms at pressures of hundreds of millibar are held in a double-chamber glass cell placed in a homogeneous magnetic field of \sim 1.5 G. Both species have spin-1/2 nuclei and the same sign nuclear magnetic dipole moment, but no higherorder electric or magnetic nuclear multipole moments. In one chamber of the glass cell, the noble-gas atoms are nuclear-spin polarized by spin-exchange collisions with optically pumped Rb vapor [20]. The noble-gas atoms diffuse into the second chamber, which is surrounded by an inductive circuit resonant both at the ³He and 129Xe Zeeman frequencies (4.9 and 1.7 kHz, respectively). For a sufficiently high flux of population-inverted nuclear magnetization, active maser oscillation of both species can be maintained indefinitely.

Because of the generally weak interactions of noblegas atoms with the walls and during atomic collisions, the 3 He and 129 Xe ensembles can have long Zeeman coherence (T_2) times of hundreds of seconds. It is thus possible to achieve excellent absolute frequency stability with one of the noble-gas masers by using the second maser as a comagnetometer. For example, Zeeman frequency measurements with a sensitivity of \sim 100 nHz are possible with averaging intervals of about an hour [15]. This twospecies noble-gas maser can also serve as a sensitive NMR gyroscope [21]: the above quoted frequency stability implies a rotation sensitivity of 0.13 deg/h.

For the boost-symmetry test, we choose a set of laboratory coordinates (t, x, y, z) , such that the \hat{x} axis points south, the \hat{y} axis points east, and the \hat{z} axis points vertically upwards in the laboratory [22]. With the reasonable approximation that the orbit of the Earth is circular, the rotation, **R**, from the Sun-centered celestial equatorial frame to the standard laboratory frame is given by

$$
R^{jJ} = \begin{pmatrix} \cos\chi\cos\omega_{\oplus}T_{\oplus} & \cos\chi\sin\omega_{\oplus}T_{\oplus} & -\sin\chi \\ -\sin\omega_{\oplus}T_{\oplus} & \cos\omega_{\oplus}T_{\oplus} & 0 \\ \sin\chi\cos\omega_{\oplus}T_{\oplus} & \sin\chi\sin\omega_{\oplus}T_{\oplus} & \cos\chi \end{pmatrix}.
$$
 (2)

In this equation, $j = x, y, z$ denotes the spatial index in the laboratory frame, while $J = X$, Y , Z denotes the spatial index in the Sun-centered frame using celestial equatorial coordinates. The Earth's sidereal angular rotation frequency is $\omega_{\phi} \simeq 2\pi/(23 \text{ h } 56 \text{ min})$, and $\chi \simeq 47.6^{\circ}$ is the colatitude of the laboratory, located in Cambridge, Massachusetts. The time T_{\oplus} is measured in the Suncentered frame from the beginning of the sidereal day, which begins when the \hat{y} and \hat{Y} axes align.

The velocity 3-vector of the laboratory in the Suncentered frame is

$$
\vec{\beta} = \beta_{\oplus} (\sin \Omega_{\oplus} T, -\cos \eta \cos \Omega_{\oplus} T, -\sin \eta \cos \Omega_{\oplus} T). \tag{3}
$$

Here, Ω_{Θ} is the angular frequency of the Earth's orbital motion. The time *T* is measured by a clock at rest at the origin, with $T = 0$ taken at 2:35 AM (U.S. Eastern Standard Time), March 20, 2000 [23]. The angle between the *XY* celestial equatorial plane and the Earth's orbital plane is $\eta = 23.4^{\circ}$. We have ignored the laboratory's velocity due to the rotation of the Earth, whose magnitude, $\beta_L = r_{\theta} \omega_{\theta} \sin \chi/c \approx 1.1 \times 10^{-6}$ (where r_{θ} is the radius of the Earth), is 2 orders of magnitude smaller than the orbital velocity.

We assume that the Lorentz-violating coefficients of $\vec{\lambda}_\mathrm{o}$ and Λ_{\circ} are static and spatially uniform in the Sun frame, at least over the course of a solar year. The corresponding coefficients in the laboratory frame thus acquire a time dependence due to both the Earth's rotation and its revolution around the Sun. We also assume observer Lorentz covariance; hence direct Lorentz transformations yield the coefficients in the laboratory frame.

In the boost-symmetry test, we used the 129 Xe maser as a comagnetometer to stabilize the magnetic field, which was oriented along the *y* axis (i.e., west to east). Thus the leading Lorentz-violating frequency variation of the freerunning 3 He maser was given by

$$
\delta \nu_{\text{He}} = \delta \nu_X \sin \omega_{\oplus} T_{\oplus} + \delta \nu_Y \cos \omega_{\oplus} T_{\oplus}, \tag{4}
$$

where

$$
\delta \nu_X = k[\lambda_s + \beta_{\mathbf{\Theta}} (\Lambda_{ss} \sin \Omega_{\mathbf{\Theta}} T + \Lambda_{sc} \cos \Omega_{\mathbf{\Theta}} T)].
$$
 (5)

$$
\delta \nu_Y = k[\lambda_c + \beta_{\Phi} (\Lambda_{cs} \sin \Omega_{\Phi} T + \Lambda_{cc} \cos \Omega_{\Phi} T)].
$$

Here λ_c , λ_s , Λ_{ss} , Λ_{sc} , ..., are combinations of Sun-frame Lorentz-violating coefficients of λ_{o} and Λ_{o} , and $k =$ -8.46×10^{32} nHz/GeV [10].

We note that Eqs. (4) and (5) cleanly distinguish the effects of rotation alone (terms proportional to λ_c and λ_s) from the effects of boosts due to the Earth's motion (terms proportional to Λ_{cc} , Λ_{cs} , Λ_{sc} , Λ_{ss}). In addition, these equations indicate that the sensitivity of our experiment to violations of boost-symmetry is reduced by a factor of $\beta_{\Phi} \simeq 10^{-4}$ with respect to the sensitivity to rotation-symmetry violation. However, for models of Lorentz violation that are isotropic in the frame of the cosmic microwave background [24], our experiment has greater sensitivity to boost-symmetry violation than to rotation-symmetry violation.

As discussed in [10], we acquired noble-gas maser data in four different runs spread over about 13 months (see Fig. 1). Each run lasted about 20 d, and we reversed the direction of the magnetic field after the first \sim 10 d in each run to help distinguish possible Lorentz-violating effects from diurnal systematic variations. We fit this data to Eq. (4). Table I lists, for each run, the mean values we determined for $\delta \nu_X$ and $\delta \nu_Y$, the amplitudes of sidereal day modulations of the ³He-maser frequency due to Lorentz-violating coefficients in the \hat{X} and \hat{Y} directions (Sun-centered frame). For each run, $\delta \nu_X$ and $\delta \nu_Y$ correspond to a very good approximation to a single highprecision measurement of the *X* and *Y* components of $\delta \nu_{\text{He}}$ performed at the mean time T_j .

Next, we fit the values of $\delta \nu_X$, $\delta \nu_Y$, and T_i in Table I to Eq. (5), thus obtaining the fit parameters reported in Table II and shown graphically in Fig. 1. We treated all fit parameters as independent and we extracted energy bounds for Lorentz-violating coefficients disregarding the possibility of accidental mutual cancellations. This analysis yielded no significant violation of boost invariance, with a limit of about 150 nHz on the magnitude of an annual modulation of the daily sidereal variation in the ³He-maser frequency.

To confirm that our result is consistent with the null hypothesis (i.e., no Lorentz-violating effect), we performed two checks. First, we generated 10 000 faux ³He-maser data sets including sidereal day frequency variations drawn from a normal distribution of zero mean but with standard errors for $\delta \nu_X$ and $\delta \nu_Y$ at each time T_i equal to the corresponding values found in the experiment. For each faux data set, we calculated the χ^2 of the fit to Eq. (5) and found that the value $\chi^2 = 0.30$ from the real experimental data is highly probable for a system in which there is no daily sidereal modulation of the 3He-maser frequency at the experiment's level of sensitivity. In the second check, we performed a series of F tests to estimate the probabilities that the values of the fit parameters, determined from the maser data, arise

FIG. 1. Time course of the mean values of $\delta \nu_X$ and $\delta \nu_Y$. For each plot the dashed line is the best fit obtained from Eq. (5), using the fit parameters λ_c , λ_s , Λ_{cc} , Λ_{cs} , Λ_{sc} , and Λ_{ss} (see also the fit results in Table II). Dotted lines indicate the 1σ confidence bands for the fit model.

entirely from statistical fluctuations. For all fit parameters, the F tests yielded probabilities greater than 30%, whereas it is customary to consider that a fit parameter is significantly different from zero only if the F test probability is smaller than 5% or 1%.

We also performed a series of checks for systematic effects, including sidereal day and year variations in maser temperature and signal amplitude (e.g., driven by variations in the optical-pumping laser). Temperature fluctuations in the 3 He and the 129 Xe detection circuit can induce small maser-frequency shifts. Accurate temperature monitoring over the course of the 13-month experiment showed a maximum 1.6 mK sidereal day variation of maser temperature, corresponding to a maximum sidereal day ³He-maser-frequency modulation of about 4 nHz, which is an order of magnitude smaller than our statistical sensitivity. A careful analysis of the maser amplitude showed a lack of phase coherence in sidereal day modulations over the 13-month data set, and hence an insignificant systematic sidereal-year variation in the 3 He-maser frequency.

To interpret this test of boost invariance, we follow the conventions of Ref. [25], Appendix C, which allows us to relate the maser frequencies to the various SME coefficients for Lorentz and *CPT* violations. In particular, the neutron—and hence the frequency of each noble-gas maser—is sensitive to Lorentz and *CPT* violations controlled by the SME coefficients b_{Λ} , $d_{\Lambda\Sigma}$, $H_{\Lambda\Sigma}$, and $g_{\Lambda\Sigma\Gamma}$ [17]. Table II shows the corresponding bounds provided by our experiment to combinations of Sun-frame SME coefficients, including the clean limit of $\sim 10^{-27}$ GeV on boost violation.

In conclusion, we used colocated 3 He and 129 Xe Zeeman masers to perform a high-sensitivity search for a violation of boost invariance of the neutron. We found no significant sidereal annual variation in the freerunning 3He-maser frequency at a level of approximately 150 nHz. This result provides the first clean test of boost symmetry for a fermion, and, in the context of the general standard-model extension, places a bound of about 10^{-27} GeV on 11 previously unexplored coefficients among the 44 coefficients describing possible leadingorder Lorentz- and *CPT*-violating couplings of the neutron. Significant improvements may be possible with a

TABLE I. Mean and standard error of the two quadratures $\delta \nu_X$ and $\delta \nu_Y$ of the sidereal day ³He-maser-frequency modulations, for each of four runs. T_i indicates the mean date of the *j*th run.

T_i	$\delta v_{\rm X}$ (nHz)	$\delta \nu_Y$ (nHz)
4/15/99	156 ± 90	37 ± 90
9/15/99	-100 ± 112	-162 ± 148
3/10/00	42 ± 86	$25 + 76$
4/22/00	$125 + 80$	-25 ± 99

TABLE II. Limits from the present work on Lorentz violation of the neutron, expressed in terms of (i) the fit parameters of Eqs. (4) and (5), i.e., coefficients for the general Lorentz-symmetry-violating vector $\vec{\lambda}_0$ and Λ_0 (both in the Sun frame); and (ii) combinations of Sun-frame SME coefficients for Lorentz and *CPT* violation (defined in Appendix B of Ref. [18]). Bounds on rotation-symmetry violation are set by the limits on λ_c and λ_s , whereas bounds on boost-symmetry violation are determined from Λ_{cc} , Λ_{cs} , Λ_{ss} , and Λ_{sc} .

Measurable combinations of SME coefficients	Fit parameters	Fit results (GeV)
$\tilde{b}_Y = 0.0034 \tilde{d}_Y + 0.0034 \tilde{g}_{DY}$	Λ_c	$(8.0 \pm 9.5) \times 10^{-32}$
$-\tilde{b}_x + 0.0034 \tilde{d}_x - 0.0034 \tilde{g}_{DX}$	$\Lambda_{\rm s}$	$(2.2 \pm 7.9) \times 10^{-32}$
$-\cos\eta[(\frac{1}{2}\tilde{b}_T + \frac{1}{2}\tilde{d}_- - \tilde{g}_c - \frac{1}{2}\tilde{g}_T) + (\tilde{g}_T - 2\tilde{d}_+ + \frac{1}{2}\tilde{d}_Q)] + \sin\eta(\tilde{d}_{YZ} - \tilde{H}_{XT})$	Λ_{cc}	$(-1.1 \pm 1.0) \times 10^{-27}$
$-\tilde{H}_{ZT}$	Λ_{cs}	$(0.2 \pm 1.8) \times 10^{-27}$
	Λ_{ss}	$(-1.8 \pm 1.9) \times 10^{-27}$
$[(\frac{1}{2}\tilde{b}_T + \frac{1}{2}\tilde{d}_- - \tilde{g}_c - \frac{1}{2}\tilde{g}_T) - (\tilde{g}_T - 2\tilde{d}_+ + \frac{1}{2}\tilde{d}_Q)]$ $\cos \eta (\tilde{H}_{ZT} - \tilde{d}_{XY}) - \sin \eta \tilde{H}_{YT}$	Λ_{sc}	$(-1.1 \pm 0.8) \times 10^{-27}$

 21 Ne 3 He Zeeman maser [26], with masers located on a rotating table [27], or with space-based clocks [28].

We acknowledge support from NASA Grants No. NAG8-1784, No. NAG3-2878, No. NAG3-2194, and No. NAG8-1770, DOE Grant No. DE-FG02-91ER40661, and the Smithsonian Institution.

- [1] R. J. Kennedy and E. M. Thorndike, Phys. Rev. **42**, 400 (1932).
- [2] D. Hils and J. L. Hall, Phys. Rev. Lett. **64**, 1697 (1990); J. A. Lipa *et al.*, Phys. Rev. Lett. **90**, 060403 (2003); H. Müller et al., Phys. Rev. Lett. 91, 020401 (2003).
- [3] D. Colladay and V.A. Kostelecký, Phys. Rev. D 55, 6760 (1997); **58**, 116002 (1998); V. A. Kostelecky´ and R. Lehnert, Phys. Rev. D **63**, 065008 (2001); V. A. Kostelecky´, Phys. Rev. D **69**, 105009 (2004).
- [4] KTeV Collaboration, H. Nguyen, in *CPT and Lorentz* Symmetry II, edited by V.A. Kostelecký (World Scientific, Singapore, 2002); OPAL Collaboration, R. Ackerstaff *et al.*, Z. Phys. C **76**, 401 (1997); BELLE Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **86**, 3228 (2001); *BABAR* Collaboration, B. Aubert *et al.*, hep-ex/ 0303043; FOCUS Collaboration, J. M. Link *et al.*, Phys. Lett. B **556**, 7 (2003); V. A. Kostelecký, Phys. Rev. Lett. **80**, 1818 (1998); Phys. Rev. D **61**, 016002 (2000); **64**, 076001 (2001).
- [5] S. M. Carroll *et al.*, Phys. Rev. D **41**, 1231 (1990); V. A. Kostelecky´ and M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).
- [6] H. Dehmelt *et al.*, Phys. Rev. Lett. **83**, 4694 (1999); R. Mittleman *et al.*, Phys. Rev. Lett. **83**, 2116 (1999); G. Gabrielse *et al.*, Phys. Rev. Lett. **82**, 3198 (1999); R. Bluhm *et al.*, Phys. Rev. Lett. **82**, 2254 (1999); **79**, 1432 (1997); Phys. Rev. D **57**, 3932 (1998).
- [7] B. Heckel, in *CPT and Lorentz Symmetry II*, edited by V. A. Kostelecky´ (World Scientific, Singapore, 2002); L.-S. Hou *et al.*, Phys. Rev. Lett. **90**, 201101 (2003); R. Bluhm and V. A. Kostelecky´, Phys. Rev. Lett. **84**, 1381 (2000).
- [8] V.W. Hughes *et al.*, Phys. Rev. Lett. **87**, 111804 (2001); R. Bluhm *et al.*, Phys. Rev. Lett. **84**, 1098 (2000).
- [9] V.W. Hughes *et al.*, Phys. Rev. Lett. **4**, 342 (1960); T. E. Chupp *et al.*, Phys. Rev. Lett. **63**, 1541 (1989).
- [10] D. Bear *et al.*, Phys. Rev. Lett. **85**, 5038 (2000); **89**, 209902 (2002).
- [11] D. F. Phillips *et al.*, Phys. Rev. D **63**, 111101 (2001).
- [12] V. A. Kostelecky´ and S. Samuel, Phys. Rev. D **39**, 683 (1989); Phys. Rev. Lett. **63**, 224 (1989); Phys. Rev. D **40**, 1886 (1989); V. A. Kostelecký and R. Potting, Nucl. Phys. **B359**, 545 (1991); Phys. Rev. D **51**, 3923 (1995).
- [13] For reviews of approaches to Lorentz and *CPT* violation, see, for example, *CPTand Lorentz Symmetry I*, edited by V. A. Kostelecky´ (World Scientific, Singapore, 1999); *CPT and Lorentz Symmetry II*, edited by V. A. Kostelecky´ (World Scientific, Singapore, 2002).
- [14] R. E. Stoner *et al.*, Phys. Rev. Lett. **77**, 3971 (1996).
- [15] D. Bear *et al.*, Phys. Rev. A **57**, 5006 (1998).
- [16] C. J. Berglund *et al.*, Phys. Rev. Lett. **75**, 1879 (1995); L. R. Hunter *et al.*, in *CPT and Lorentz Symmetry I*, edited by V. A. Kostelecky´ (World Scientific, Singapore, 1999).
- [17] V. A. Kostelecky´ and C. D. Lane, Phys. Rev. D **60**, 116010 (1999).
- [18] R. Bluhm *et al.*, Phys. Rev. D **68**, 125008 (2003).
- [19] The sidereal-year lasts 365 d, 6 h, 9 min, and 9.5 s.
- [20] T. E. Chupp *et al.*, Phys. Rev. A **38**, 3998 (1988); G. D. Cates *et al.*, Phys. Rev. A **45**, 4631 (1992).
- [21] K. F. Woodman *et al.*, J. Navig. **40**, 366 (1987).
- [22] The coordinate system adopted here differs from that used in our earlier work, Ref. [10].
- [23] At this instant, Earth crossed the equatorial plane during the vernal equinox. *T* and T_{\oplus} are equivalent, modulo the sidereal day period.
- [24] An example of such a model expressed in terms of standard-model extension coefficients consists of two background tensor fields (b_Λ and $d_{\Lambda\Sigma}$) with the following simple relationships: $d_{XX} = d_{YY} = d_{ZZ}$, $d_{TT} = 3d_{XX}$, $b_T = 4md_{XX}$, where *m* is the mass of the neutron and all other terms are zero. For this model, our experiment has an order of magnitude greater sensitivity to boostsymmetry violation than to rotation-symmetry violation, despite the factor of $\beta_{\varphi} \simeq 10^{-4}$.
- [25] V. A. Kostelecky´ and M. Mewes, Phys. Rev. D **66**, 056005 (2002).
- [26] R. E. Stoner and R. L. Walsworth, Phys. Rev. A **66**, 032704 (2002).
- [27] Successful tests of Lorentz symmetry have been performed using rotating tables. See, e.g., Ref. [7].
- [28] R. Bluhm *et al.*, Phys. Rev. Lett. **88**, 090801 (2002).