Spin Current and Polarization in Impure Two-Dimensional Electron Systems with Spin-Orbit Coupling

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We derive the transport equations for two-dimensional electron systems with Rashba spin-orbit interaction and short-range spin-independent disorder. In the limit of slow spatial variations, we obtain coupled diffusion equations for the electron density and spin. Using these equations we calculate electric-field induced spin accumulation and spin current in a finite-size sample for an arbitrary ratio between spin-orbit energy splitting Δ and elastic scattering rate τ^{-1}. We demonstrate that the spin-Hall conductivity vanishes in an infinite system independent of this ratio.

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Introduction.—The subject of the novel and quickly developing field of spintronics is the transport of electronic spins in low-dimensional and nanoscale systems. A possibility of coherent spin manipulation represents an ultimate goal of this field. Typically, spin transport is strongly affected by a coupling of spin and orbital degrees of freedom. The influence of the spin-orbit interaction is twofold. The momentum relaxation due to diffusive scattering of carriers, e.g., by disorder, inevitably leads to spin relaxation and destroys spin coherence. On the other hand, the controlled orbital motion of carriers can result in a coherent motion of their spins. Thus, spin-orbit coupling is envisaged as a possible tool for spin control in electronic devices. In particular, it is possible to generate spin polarization and spin currents by applying electric field, the phenomenon known as the spin-Hall effect.

Although the study of the spin-Hall effect recently evolved into a subject of intense research [1–11], the issue remains highly controversial. Sinova *et al.* [2] have predicted that in a clean, infinite, homogeneous two-dimensional electron system (2DES) the spin current $\hat{j}_k^i = \frac{1}{4}\{\hat{\sigma}_i, \hat{v}_k\}$ develops a nonzero expectation value under an external electric field $\mathbf{E}$. (Here $\frac{1}{2}\hat{\boldsymbol{\sigma}}$ and $\hat{\mathbf{v}}$ are the operators of the electron spin and velocity, respectively.) The spin-Hall conductivity, defined as the ratio $\sigma_{sH} = -j_y^z/E_x$, was predicted to have a universal value $\sigma_{sH} = \frac{e}{8\pi}$, independent of the magnitude of the spin-orbit energy splitting Δ. The effect of impurity scattering on a spin current has been discussed in Refs. [4,9,10]. References [4,10] show that the spin-Hall conductivity disappears in the dirty limit $\Delta \ll \tau^{-1}$, reaching the universal value only for a sufficiently clean regime, $\Delta \gg \tau^{-1}$. The clean regime has been analyzed by Inoue *et al.* [9], who argued that the spin current completely disappears due to vertex corrections. Recently, Dimitrova [11] obtained the universal value independent of the relation between the spin-orbit splitting Δ and the impurity scattering rate.

Because the spin current is not measurable directly, its physical meaning is obscure. In the presence of spin-orbit interaction, electron spin is not a conserved quantity, and a spin current is not directly related to the transport of spins. In particular, Rashba [8] demonstrated that spin current can be nonzero even in equilibrium, as the symmetry of an isotropic spin-orbit Hamiltonian allows nonzero in-plane currents $j_y^x = -j_x^y \neq 0$. A more meaningful quantity is spin polarization (spin accumulation) rather than a spin current. Equilibrium currents do not lead to spin accumulation. It remains unclear whether the predicted *nonequilibrium* spin-Hall currents j_y^z accumulate near sample boundaries. Bulk polarization has been studied in both the three-dimensional [12] and two-dimensional [13] electron systems in the electric field.

In this Letter, we develop a consistent microscopic approach to spin transport in impure 2DES. We derive a quantum kinetic equation which describes the evolution of a density matrix of a noninteracting 2DES. For length scales exceeding the mean free path, this equation reduces to a modified diffusion equation. We then compute spin polarization and spin current in a general situation when the finite-size system is driven out of equilibrium by an external electric field as well as by the density gradient. We find that the spin current actually *vanishes* in an infinite system for arbitrary $\Delta\tau$.

However, in a mesoscopic conductor connected to two massive metallic contacts, nonequilibrium spin currents j_y^z flow in the vicinity of the contacts (as shown in Fig. 1). A nonzero spin-Hall effect can also be achieved in an infinite system by applying a finite frequency electric field. We evaluate the ac spin-Hall conductivity, which is maximal for a frequency of order of the spin-relaxation rate. This result is instructive for making a connection with previous works and clarifying the "universality" issue of the spin-Hall conductivity.

Kinetic equation.—Noninteracting electrons in an asymmetric quantum well can be described by a single particle Hamiltonian

$$H = [\mathbf{p} - e\mathbf{A}(t)]^2/2m + \alpha\hat{\boldsymbol{\eta}} \cdot [\mathbf{p} - e\mathbf{A}(t)] + U_i, \quad (1)$$

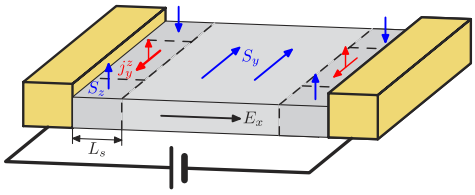

FIG. 1 (color online). In a spin-Hall bar setup, the electric current is driven through 2DES contacted by the metallic leads connected to a voltage source. The electric field E_x creates an in-plane spin polarization S_y in the bulk. Spin currents j_y^z are running in the vicinity of the contacts while vanishing in the bulk. Out-of-plane polarization S_z is accumulated at the sample corners.

where $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$ is electron momentum, m is the effective mass, $\mathbf{A}(t)$ is a vector potential of the uniform electric-field $\mathbf{E} = -\dot{\mathbf{A}}$, and $\hat{\boldsymbol{\eta}}$ is proportional to the electron spin operator. (We neglect terms cubic in $\mathbf{p}$.) The disorder potential U_i is assumed to be random, short range, and spin independent. For the isotropic ("Rashba") spin-orbit interaction [14], $\hat{\boldsymbol{\eta}} = \mathbf{z} \times \hat{\boldsymbol{\sigma}}$, where $\hat{\boldsymbol{\sigma}}$ are the Pauli matrices. To describe a nonequilibrium state of the system, we use the Keldysh approach [15]. We introduce the retarded and advanced Green's functions G^R and G^A, and Keldysh function G^K satisfying Dyson's equation

$$(\hat{G}_0^{-1} - \underline{\hat{\Sigma}})\underline{\hat{G}} = 1, \qquad \underline{\hat{G}} = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{pmatrix}. \tag{2}$$

Here the lower bar denotes a matrix in Keldysh space, $\hat{G}_0^{-1} = i\partial_t - \hat{H}$, and $\nu = \frac{m}{2\pi}$ is a density of states per spin direction. Neglecting weak-localization effects, one can relate the self-energy $\underline{\hat{\Sigma}}$ to the Green's function $\underline{\hat{G}}$ by a standard disorder averaging technique [16], $\underline{\hat{\Sigma}} = \delta_{\mathbf{x}\mathbf{x}'} \times \underline{\hat{G}}(\mathbf{x}, \mathbf{x})/m\tau$. We consider only the limit where τ^{-1} and Δ are small compared to the Fermi energy $p_F^2/2m$. In the absence of electron-electron interactions, functions $\hat{G}^R$ and $\hat{G}^A$ are independent of the nonequilibrium state of the system. In the Fourier representation, they are given by

$$\hat{G}^{R,A}_{\mathbf{p}\varepsilon} = \frac{1}{\epsilon - \xi_p - \Delta_p \hat{\eta}_{\mathbf{p}} \pm \frac{i}{2\tau}}. \tag{3}$$

Here $\xi_p = (p^2 - p_F^2)/2m$ is the kinetic energy counted from the equilibrium chemical potential, $\Delta_p = \alpha p$ is the energy of the spin-orbit splitting, and $\hat{\eta}_{\mathbf{p}} = \hat{\boldsymbol{\eta}} \cdot \mathbf{p}/p$ is the projection of the spin operator $\hat{\boldsymbol{\eta}}$ onto the direction of the electron momentum. The Keldysh function $\hat{G}^K$ satisfies the equation

$$[\hat{G}^R]^{-1}\hat{G}^K - \hat{G}^K[\hat{G}^A]^{-1} = \hat{\Sigma}^K\hat{G}^A - \hat{G}^R\hat{\Sigma}^K. \tag{4}$$

It is now customary to apply the Wigner transformation to Eq. (4), i.e., the Fourier transform with respect to the relative time and space arguments,

$$\hat{G}^K(t_+\mathbf{x}_+; t_-\mathbf{x}_-) = \frac{i}{\pi}\int \frac{d\varepsilon d^2p}{(2\pi)^2}\hat{g}_{\mathbf{p}\varepsilon}(t, \mathbf{x})e^{i[\mathbf{p}+e\mathbf{A}(t)]\delta\mathbf{x} - i\varepsilon\delta t}, \tag{5}$$

where $t_\pm = t \pm \delta t$ and $\mathbf{x}_\pm = \mathbf{x} \pm \delta\mathbf{x}/2$. In the semiclassical approximation, the Wigner transform of the right-hand side of Eq. (4) can be replaced by a product of the Wigner transforms of Σ and G:

$$\frac{\partial\hat{g}_{\mathbf{p}\varepsilon}}{\partial t} + \frac{1}{2}\left\{\frac{\mathbf{p}}{m} + \alpha\boldsymbol{\eta}, \tilde{\nabla}\hat{g}_{\mathbf{p}\varepsilon}\right\} + i\alpha[\hat{\boldsymbol{\eta}} \cdot \mathbf{p}, \hat{g}_{\mathbf{p}\varepsilon}] = -\frac{\hat{g}_{\mathbf{p}\varepsilon}}{\tau} + \frac{i}{\tau}(\hat{G}^R_{\mathbf{p}\varepsilon}\hat{\rho}_\epsilon - \hat{\rho}_\epsilon\hat{G}^A_{\mathbf{p}\varepsilon}), \tag{6}$$

where $\tilde{\nabla} = \nabla + e\mathbf{E}\partial_\varepsilon$, and

$$\hat{\rho}_\varepsilon = \frac{1}{2\pi\nu}\int \frac{d^2p}{(2\pi)^2}\hat{g}_{\mathbf{p}\varepsilon} \tag{7}$$

is the density matrix of electrons with the energy ε. The total number of particles and their total spin can be expressed via $\hat{\rho}_\varepsilon$ as follows:

$$N = \mathrm{Tr}\,\nu\int d\varepsilon\hat{\rho}_\varepsilon, \qquad \mathbf{S} = \frac{1}{2}\mathrm{Tr}\,\nu\int d\varepsilon\hat{\boldsymbol{\sigma}}\hat{\rho}_\varepsilon. \tag{8}$$

In the limit $\tau \to \infty$ Eq. (6) reduces to the ballistic equation of Ref. [17]. Note, however, that the function $\hat{g}_{\mathbf{p}\epsilon}$ is not a distribution function in the conventional sense, since it depends on both energy and momentum.

A stationary solution to the quantum kinetic Eq. (6) is of the form $\hat{g}_{\mathbf{p}\varepsilon} = A_\varepsilon(\hat{G}^R_{\mathbf{p}\varepsilon} - \hat{G}^A_{\mathbf{p}\varepsilon})$, where A_ε is an arbitrary scalar function of the electron energy ε. This solution represents the state in which the charge density is uniform, and spin density is zero. In a nonequilibrium state with the characteristic length scales of the spin and charge densities exceeding the electron mean free path $l = v_F\tau$, the distribution $\hat{g}_{\mathbf{p}\varepsilon}$ relaxes slowly to equilibrium. To describe this relaxation, we derive the equation for the density matrix $\hat{\rho}_\varepsilon(\mathbf{r}, t)$. It is useful to move small gradient terms to the right-hand side of the kinetic Eq. (6), so that its left-hand side describes fast relaxation to the local equilibrium distribution:

$$(\partial_t + \tau^{-1})\hat{g}_{\mathbf{p}\varepsilon} + i\Delta_p[\hat{\eta}_{\mathbf{p}}, \hat{g}_{\mathbf{p}\varepsilon}] = \hat{\mathcal{K}}_{\mathbf{p}\varepsilon} \equiv \hat{\mathcal{K}}^{(0)}_{\mathbf{p}\varepsilon} + \hat{\mathcal{K}}^{(1)}_{\mathbf{p}\varepsilon}, \tag{9}$$

where

$$\hat{\mathcal{K}}^{(0)}_{\mathbf{p}\varepsilon}[\hat{\rho}_\varepsilon] = i\tau^{-1}[\hat{G}^R_{\mathbf{p}\varepsilon}\hat{\rho}_\varepsilon - \hat{\rho}_\varepsilon G^A_{\mathbf{p}\varepsilon}],$$
$$\hat{\mathcal{K}}^{(1)}_{\mathbf{p}\varepsilon}[\hat{g}_{\mathbf{p}\varepsilon}] = -\frac{1}{2}\left\{\frac{\mathbf{p}}{m} + \alpha\hat{\boldsymbol{\eta}}, \tilde{\nabla}\hat{g}_{\mathbf{p}\varepsilon}\right\}. \tag{10}$$

Small anisotropic deviations from local equilibrium are due to the gradient term $\hat{\mathcal{K}}^{(1)}_{\mathbf{p}\varepsilon}$ in the kinetic equation which can be treated perturbatively. The solution to Eq. (9) can formally be written (in the Fourier represen-

tation with respect to time) as

$$\hat{g}_{\mathbf{p}\varepsilon} = i\frac{(2\Delta_p^2 - \Omega^2)\hat{\mathcal{K}}_{\mathbf{p}\varepsilon} + 2\Delta_p^2\hat{\eta}_{\mathbf{p}}\hat{\mathcal{K}}_{\mathbf{p}\varepsilon}\hat{\eta}_{\mathbf{p}} - \Omega\Delta_p[\hat{\eta}_{\mathbf{p}}, \hat{\mathcal{K}}_{\mathbf{p}\varepsilon}]}{\Omega(4\Delta_p^2 - \Omega^2)} \equiv \mathcal{L}[\hat{\mathcal{K}}_{\mathbf{p}\varepsilon}], \tag{11}$$

where $\Omega = \omega + i/\tau$. In a zeroth order, one can neglect the gradient term $\hat{\mathcal{K}}^{(1)}_{\mathbf{p}\varepsilon}$ altogether, so that Eq. (11) gives the distribution $\hat{g}^{(0)}_{\mathbf{p}\varepsilon}$ in terms of the density matrix $\hat{\rho}_\varepsilon$. In the first order, we substitute the obtained expression for $\hat{g}^{(0)}_{\mathbf{p}\varepsilon}$ in the gradient term $\hat{\mathcal{K}}^{(1)}_{\mathbf{p}\varepsilon}$ to obtain an improved expression for the distribution function, $\hat{g}^{(1)}_{\mathbf{p}\varepsilon}$. This procedure is then to be repeated to the necessary order,

$$\begin{aligned} \hat{g}^{(0)}_{\mathbf{p}\varepsilon} &= \mathcal{L}[\hat{\mathcal{K}}^{(0)}_{\mathbf{p}\varepsilon}(\hat{\rho}_\varepsilon)], \\ \hat{g}^{(i)}_{\mathbf{p}\varepsilon} &= \hat{g}^{(i-1)}_{\mathbf{p}\varepsilon} + \mathcal{L}[\hat{\mathcal{K}}^{(1)}_{\mathbf{p}\varepsilon}(\hat{g}^{(i-1)}_{\mathbf{p}\varepsilon})], \qquad i \geq 1. \end{aligned} \tag{12}$$

Integrating the second-order approximation over the momentum $\mathbf{p}$, one arrives at the diffusion equation for the density matrix $\hat{\rho}_\varepsilon$. In a quasistationary regime ($\omega\tau \ll 1$) the equation takes the following form:

$$\frac{\partial\rho_\varepsilon}{\partial t} + D\tilde{\nabla}^2\hat{\rho}_\varepsilon + iC[\boldsymbol{\eta}, \tilde{\nabla}\hat{\rho}_\varepsilon] + B\{\boldsymbol{\eta}, \tilde{\nabla}\hat{\rho}_\varepsilon\} = \frac{\hat{\rho}_\varepsilon}{\tau_s} - \frac{\boldsymbol{\eta}\cdot\hat{\rho}_\varepsilon\boldsymbol{\eta}}{2\tau_s}. \tag{13}$$

The first two terms in this equation describe spin and charge diffusion with $D = v_F^2\tau/2$ being the conventional diffusion constant, and $v_F = p_F/m$ the Fermi velocity. The third term describes a spin precession due to the drift velocity, and the fourth term describes the coupling between charge and spin. The right-hand side of Eq. (13) describes spin relaxation due to the Dyakonov-Perel mechanism [18]. The coefficients of the spin relaxation, spin-density coupling, and spin precession are

$$\frac{1}{\tau_s} = \frac{2\Delta\zeta}{1+4\zeta^2}, \qquad B = \frac{\alpha\zeta^2}{1+4\zeta^2}, \qquad C = \frac{v_F\zeta}{(1+4\zeta^2)^2},$$

where $\Delta = \Delta_{p_F}$, and the dimensionless parameter $\zeta = \Delta\tau$ describes the relative strength of spin-orbit coupling and disorder scattering. In deriving Eq. (13), we assumed that the spin-orbit splitting is small compared to the Fermi energy ($\Delta \ll E_F$), while the parameter $\zeta = \Delta\tau$ is arbitrary. (Physically, ζ represents the angle of spin precession between two consecutive collisions.) In the case of weak spin-orbit coupling or a very clean sample ($\zeta \ll 1$), the Dyakonov-Perel relaxation time is large compared to the elastic mean free time $\tau_s \sim \tau/\zeta^2 \gg \tau$ and the characteristic spin-relaxation length $\sqrt{D\tau_s}$ is large compared to the mean free path. The spin dynamics is thus slow both in space and in time, and Eq. (13) has a meaning of a real diffusion equation for the coupled density and spin degrees of freedom. The spin-density coupling coefficient B differs, in the limit $\zeta \ll 1$, from the corresponding term that was given in the original version of Ref. [10]. However, Ref. [10] was corrected in proof and now agrees with our result. We see below that the value of B is crucial for the magnitude of the spin-Hall effect.

In the opposite limit, $\zeta \gg 1$, spin relaxation is fast, $\tau_s \sim \tau$, and occurs on a length scale of the mean free path l, i.e., locally as compared to the system size $L \gg l$. Spin-relaxation dynamics (e.g., propagation of a spin-polarized injected beam) is therefore beyond the reach of the diffusion equation and must be studied with the kinetic Eq. (6). However, Eq. (13) can still be used to study a steady state in which spin polarization changes slowly on a scale of l (which is the case for spin-Hall conductivity; see below). One then has to retain the terms describing density diffusion, spin relaxation, and spin-density coupling. In the vector basis,

$$\hat{\rho}_\varepsilon = (n_\varepsilon/2) + \hat{\boldsymbol{\sigma}}\cdot\mathbf{s}_\varepsilon, \tag{14}$$

Eqs. (13) are reduced to

$$\tilde{\nabla}^2 n_\varepsilon = 0, \qquad \mathbf{s}_\varepsilon = -B\tau_s\mathbf{z}\times\tilde{\nabla}n_\varepsilon. \tag{15}$$

Total density and spin polarization are expressed in this basis as $N = \nu\int d\varepsilon n_\varepsilon$, and $\mathbf{S} = \nu\int d\varepsilon \mathbf{s}_\varepsilon$.

Spin accumulation.—We now apply the spin diffusion Eq. (13) to analyze spin accumulation in a finite-size sample of the length L contacted by two ideal unpolarized metallic leads. The sample is infinite in the transverse direction so that $\hat{\rho}_\varepsilon(x)$ depends on the longitudinal coordinate x only. Note that the electric field in the sample enters Eq. (13) only via $\tilde{\nabla} = \nabla + e\mathbf{E}\partial_\varepsilon$ and therefore can be eliminated by shifting the energy as $\varepsilon \to \varepsilon + eEx$. Thus, the electric field may be treated via the boundary conditions, $\hat{\rho}_\varepsilon(0) = F_{\varepsilon - eV}$, $\hat{\rho}_\varepsilon(L) = F_\varepsilon$, where $V = EL$ is the voltage bias between the two leads, and F_ε is the equilibrium Fermi-Dirac distribution. Substituting the expansion (14) into Eq. (13) we observe that $s^x_\varepsilon = s^z_\varepsilon = 0$. The other two equations yield

$$\begin{aligned} n_\varepsilon(x) &= 2(1 - x/L)F_{\varepsilon - eV} + 2xF_\varepsilon/L, \\ \frac{d^2 s^y_\varepsilon}{dx^2} - \frac{s^y_\varepsilon}{L_s^2} &= \frac{B}{D}\frac{dn_\varepsilon}{dx}, \qquad L_s^2 = D\tau_s. \end{aligned} \tag{16}$$

Note that the B term in the equation for n_ε leads to small corrections, $\sim\alpha^2/v_F^2$, which must be neglected in the considered approximation. The solution to the second of Eqs. (16) yields

$$S^y(x) = \frac{eE_{\text{eff}}\zeta}{2\pi v_F}\left(1 - \frac{\cosh[\gamma - x/L_s]}{\cosh\gamma}\right), \tag{17}$$

where $\gamma = L/2L_s$, and $E_{\text{eff}} = V/L$. For $\gamma \to \infty$, this agrees with the previous calculation by Edelstein [13].

Spin current.—The spin current, as defined in the introduction, is found from the Keldysh Green's function,

$$j^i_k = \frac{1}{8m}\mathrm{Tr}\,\hat{\sigma}^i(\nabla'_k - \nabla_k)_{\mathbf{x}'\to\mathbf{x}}\hat{G}^K + \frac{\alpha}{2}\epsilon_{ikz}N. \tag{18}$$

The function $\hat{G}^K$ can be expressed via the density $\hat{\rho}_\varepsilon$ with the help of the equation, $\hat{G}^K = \hat{G}^R\hat{\Sigma}^K\hat{G}^A$, which follows

from Dyson's Eq. (4). After simple transformations,

$$j_k^i = \frac{i}{8\pi m\tau}\mathrm{Tr}\,\hat{\sigma}^i(\nabla'_k - \nabla_k)_{\mathbf{x}'\to\mathbf{x}}\int d\varepsilon d\mathbf{y} \times \hat{G}^R_\varepsilon(\mathbf{x}-\mathbf{y})\hat{\rho}_\varepsilon(\mathbf{y})\hat{G}^A_\varepsilon(\mathbf{y}-\mathbf{x}') + \frac{\alpha}{2}\epsilon_{ikz}N. \quad (19)$$

Keeping now in the integrand only the zero and first-order terms in the expansion of $\hat{\rho}_\varepsilon$ over $\mathbf{y}-\mathbf{x}$, we arrive at the final expression for the nonequilibrium spin current in terms of the density and spin distribution functions,

$$j_k^i = -e\frac{\delta_z^i\zeta^2[\mathbf{z}\times\mathbf{E}_{\mathrm{eff}}]_k}{2\pi(1+4\zeta^2)} + \frac{v_F\zeta(\delta_z^i S^k - \delta_k^i S^z)}{1+4\zeta^2}. \quad (20)$$

Here $\mathbf{E}_{\mathrm{eff}} = \mathbf{E} - \boldsymbol{\nabla}N/2e\nu$ is the gradient of the electrochemical potential including both the electric field and the gradient of electron density. Substituting Eq. (17) into Eq. (20) we observe that the two contributions to j_k^i cancel each other in the bulk of a sample. Therefore, the dc spin current vanishes independent of ζ.

However, near the contacts where the spin polarization deviates from its bulk value, the spin current is nonzero. Using the expression (17) in Eq. (20), we find that the spin current near the contacts decays as ($\zeta \ll 1$)

$$j_y^z(x) = -\frac{eE}{2\pi}\zeta^2 e^{-x/L_s}. \quad (21)$$

For a sample of finite width, this spin current should lead to a nonzero spin accumulation S_z within a distance L_s of the corners of the sample, as illustrated in Fig. 1.

Note that for a nonuniform system in thermal equilibrium, where $\mathbf{E}_{\mathrm{eff}} = 0$, the spin density given by Eq. (16) is zero, as well as the spin current. Small equilibrium spin currents [8], proportional to $(\alpha/v_F)^3$, are beyond the approximation used when deriving Eq. (20). Our derivation of the diffusion Eq. (13) and the spin current (20) relies on the approximation (3) that neglects contributions from diagrams with crossed impurity lines (ladder approximation). This is usually justified provided that $E_F\tau \gg 1$. In an infinite system the result (20) is equivalent to a calculation within the Kubo formalism with the first term representing a single-loop contribution and the second term originating from the ladder impurity diagrams.

To reconcile our result for spin current with the predictions of Ref. [2], it is helpful to consider the ac spin-Hall effect [9]. When the applied electric field is time dependent, spin polarization is retarded with respect to the field, due to the finite spin-relaxation time. As a result, the spin polarization contribution in Eq. (20) does not exactly cancel the electric-field contribution, and spin-Hall conductivity becomes nonzero. Solving Eq. (9) for the homogeneous infinite system and generalizing Eq. (19) for a time-dependent state, we find,

$$\sigma_{sH}(\omega) = \frac{e\Delta^2}{2\pi}\frac{\omega\tau}{\omega\tau[4\Delta^2 - (\omega+\frac{i}{\tau})^2] + 2i\Delta^2}. \quad (22)$$

For low frequencies, $\omega\tau_s < 1$, the spin-Hall conductivity remains small, $\sigma_{sH} \sim -i\omega\tau$. When the frequency exceeds the spin-relaxation rate ($\omega\tau_s \geq 1$), σ_{sH} reaches its maximum value $e\tau/(4\pi\tau_s)$. For clean samples, this is the universal value $e/8\pi$ predicted in Ref. [2], while for dirty samples ($\zeta \ll 1$) the maximum value of the spin-Hall conductivity remains strongly suppressed, $\sigma_{sH} \approx e\zeta^2/(2\pi)$.

To conclude, we derived a quantum kinetic equation for 2D electrons in the presence of spin-orbit coupling and short-range potential scattering. We proved that the dc spin-Hall effect disappears in a bulk sample, and we computed the spin accumulation in a finite-size system for a wide range of parameters.

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