# Spontaneous Pattern Formation with Incoherent White Light

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We present the first experimental observation of modulation instability and spontaneous pattern formation with incoherent white light emitted from an incandescent light bulb. We show experimentally that modulation instability of white light propagating in a noninstantaneous self-focusing medium is a collective effect, where the entire temporal spectrum of the light beam becomes unstable at the same threshold value and collectively forms a pattern with a single periodicity. We experimentally demonstrate that the temporal spectrum of the evolving perturbation self-adjusts to match the collective pattern formation phenomenon.

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Self-organization and spontaneous ordering into periodic structures can be found perhaps in all fields of science. This fundamental phenomenon, known as spontaneous pattern formation, was extensively studied during the last century in the context of fluid dynamics [1], biology [2], chemistry [3], matter-wave physics [4], and other areas. Essentially, spontaneous pattern formation arises from the interplay between self-action (nonlinearity) and long-range interaction (e.g., diffusion or diffraction). When these tendencies act in an opposite fashion on a local perturbation, they result in the breakup of a homogeneous state, initiated by small random fluctuations (“noise”); perturbations with a particular periodicity are enhanced, forming regular patterns, such as stripes or hexagons. Although the mechanisms driving this “modulational instability” (MI) process and pattern formation may significantly differ from one system to another, this behavior is universal, exhibiting similar features in all systems supporting spontaneous pattern formation [5]. In optics, this phenomenon manifests itself as the disintegration of a light beam of uniform intensity into an ordered structure. This behavior was shown to occur in many materials with diverse nonlinearities [6–9]. Similar effects were observed also in the temporal domain [10]. Traditionally, pattern formation in nonlinear optical systems was studied with coherent light, relying on the intuition that interference effects, and therefore the coherence of the interacting waves, are necessary for nonlinear wave amplification and for the spontaneous creation of long-range ordering. Recently, however, several studies have shown theoretically [11,12] and experimentally [13,14] that a partially spatially incoherent yet quasimonochromatic light beam can also undergo MI, resulting in a periodic array of spatially incoherent 1D or 2D solitonlike filaments [15]. This incoherent MI is distinguished from coherent MI by the presence of a distinct instability threshold, which can be understood as the balance point between two opposing tendencies: the nonlinear self-focusing of small perturbations, and their linear diffusive washout resulting from the incoherence [11]. Below this threshold value, a uniform-intensity beam is stable and no pattern will emerge [11–14]. The instability threshold directly depends on the correlation statistics [11,12,16]. The essential feature underlying pattern formation with incoherent light is that the response time of the nonlinearity should be much longer than the characteristic time of random phase fluctuations. Hence, the nonlinearity responds to the time-averaged intensity rather than to the speckled instantaneous intensity structure [11–14]. On the experimental front, the spatially incoherent light sources used in the experiments studying incoherent MI [13,14] and solitons [17] were constructed by passing a (monochromatic) laser beam through a rotating diffuser. The exception is the experimental observation of incoherent white light solitons made of light emanating from an incandescent light bulb [18]. Such studies of nonlinear phenomena with light from natural sources, such as incandescent bulbs, or the Sun, are especially intriguing. Unfortunately, experimental results with such sources are scarce in nonlinear optics. Sources like light bulbs or the Sun, apart from their everyday appearance, are both spatially and temporally incoherent (not quasimonochromatic), and thus nonlinear interactions among a broad continuum of temporal frequencies become an important ingredient of the underlying nonlinear dynamics. This had motivated us to study MI and pattern formation with (spatially and temporally) incoherent white light originating from an ordinary incandescent light bulb.

In a recent theoretical study, a model was derived to describe the propagation of incoherent white light in a noninstantaneous nonlinear medium, and the MI process of such a light beam was investigated using linear stability analysis [19]. Apart from showing that the white light beam would go unstable only above a threshold value in a similar fashion to the spatially incoherent quasimonochromatic case, this theoretical work predicted two new features, which characterize MI with white light: (i) The whole temporal spectrum is collectively becoming unstable at *the same* threshold value of nonlinearity, while all temporal frequencies obtain an intensity structure with the same spatial periodicity. Both of these indicate that *the process of white light MI is a collective effect.* (ii) Above the threshold value, *the temporal spectrum of the perturbations (growing on top of the underlying uniform beam) self-adjusts* so as to match the collective instability process, and different wavelengths participate with different strengths in the MI process.

Here, we present the first experimental observation of modulation instability and spontaneous pattern formation with spatially and temporally incoherent white light emitted from an incandescent light bulb. We show experimentally that white light modulation instability is indeed a collective effect, where the entire temporal spectrum of the light beam becomes unstable at the same threshold value and collectively forms a pattern with a single periodicity. Furthermore, we experimentally demonstrate that the temporal spectrum of the evolving perturbation self-adjusts to match the collective pattern formation phenomenon. This holds even when the modulation depth of the pattern is high and the system is in a highly nonlinear state, where the linear stability analysis [19] is inapplicable.

A sketch of our experimental setup is presented in Fig. 1: An incandescent light bulb is used as a light source, emitting a white light beam with a wide temporal spectrum. The emitted light is collected using an elliptical reflector and focused onto an aperture, which is used to adjust the spatial coherence of the beam. We set the spatial correlation distance $l_c$ to be $\sim$8 $\mu$m, which is estimated (at the central wavelength) by measuring the diffraction angle $\theta$ of the incoherent beam, when passed through a second aperture (much larger than $l_c$), and by using $\theta \approx \lambda/\pi l_c$. We use a filter to pass only the visible range of the (temporal) spectrum. The beam is then collimated and launched into an 8 mm long SBN:60 crystal, possessing the photorefractive screening nonlinearity [20]. An external bias field, which is applied on the crystal parallel to its *c* axis, is used to control the strength of the nonlinearity. The light beam entering the crystal is nonpolarized. The ordinarily polarized (perpendicular to the crystalline *c* axis) fraction of the light beam is propagating through the medium in a basically linear fashion, thus merely serving a background beam to set the degree of saturation of the screening nonlinearity [20]. The extraordinarily polarized (parallel to the *c* axis) light propagates in a nonlinear fashion and serves as a signal beam [20]. Since the entire light beam is nonpolarized, the signal-to-background intensity ratio is 1. After the crystal, a linear polarizer is placed so as to filter out the ordinarily polarized background beam. We use a CCD camera to monitor the intensity distribution at the output face of the crystal. Finally, a set of narrow-band color filters (10 nm FWHM) is placed before the camera to enable monitoring for each wavelength separately. Figure 1 also shows the temporal spectrum of the beam, measured at the input face of the crystal, and the measured absorption coefficient (which relates to material sensitivity) as a function of wavelength. The photosensitivity of the crystal is higher at shorter wavelengths; hence the power spectrum as “seen“ by the crystal is enhanced at shorter wavelengths. Since the width of the spectrum is approximately half the central wavelength, we estimate the temporal coherence length to be $\sim$0.5 $\mu$m, which is twice the central wavelength in the material (0.25 $\mu$m). Apart from the absorption, the temporal power spectrum of the signal beam is unchanged during propagation. The response time of the medium ($\sim$0.1 s) is too slow and the optical intensity is too low for significant generation of new frequencies [21].

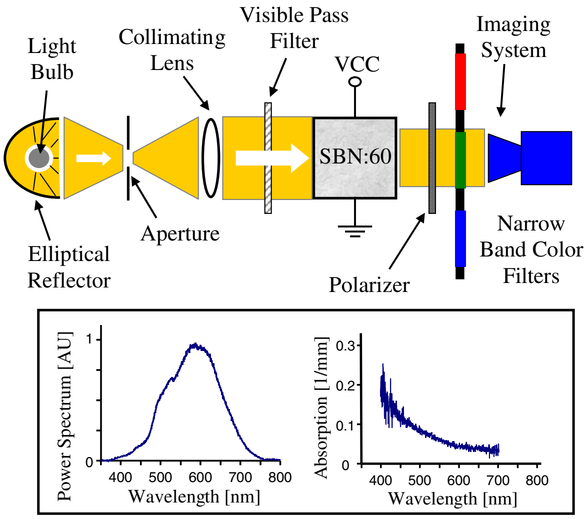

FIG. 1 (color online). A sketch of the experimental setup. The inset shows the temporal spectrum of the light at the input face of the crystal (left) and the crystal absorption (right).

Figure 2 depicts typical experimental results, showing the light intensity distribution (with the entire temporal spectrum present) at the crystal output, for various values of nonlinearity. Figure 2(a) shows the homogeneous output beam with the nonlinearity off. Figure 2(b) shows the output light beam with an applied electric field of 1000 V/cm. The nonlinearity strength is below the MI threshold; hence, the beam is stable and no significant change in the uniform output intensity distribution is apparent. By setting a bias of 1200 V/cm, a low-visibility periodic pattern of stripes starts to form [Fig. 2(c)], with a typical periodicity of 40 $\mu$m. When the applied voltage is further increased [Fig. 2(d)], the modulation depth of the pattern increases to produce an array of one-dimensional white-light filaments with a typical width

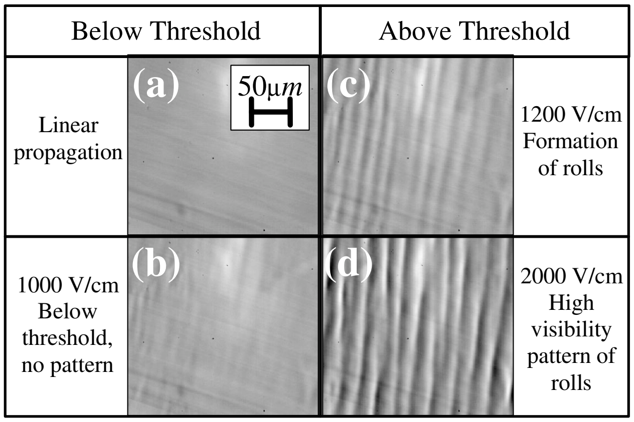

FIG. 2. Total intensity distribution at the output face of the crystal: homogeneous intensity at linear propagation (a) and stable nonlinear propagation below the MI threshold (b); pattern formation above the MI threshold (c),(d).

of 15 $\mu$m. The light trapped in each filament is temporally incoherent (with a wavelength span over the entire visible range), and partially spatially incoherent, as the spatial correlation distance ($\sim$8 $\mu$m) is smaller than the width of each filament ($\sim$15 $\mu$m).

The temporal power spectrum of the light bulb is broad and continuous, and all the temporal frequencies propagate simultaneously. From the study of quasimonochromatic spatially incoherent MI [11], we know that by changing the temporal frequency of the carrier wave, while keeping all other parameters the same, the threshold for MI and the pattern periodicity would change. By simply extending these findings, one might believe that during incoherent white-light MI not all temporal frequencies would become unstable at the same threshold value and that the unstable frequencies could give rise to patterns with different periodicities. However, the theoretical model [19] predicted the opposite: all temporal frequencies should act collectively, by having the same threshold value and pattern periodicity. We are therefore motivated to measure the behavior of different temporal frequencies during white light MI dynamics. We measure the standard deviation (relative to the mean intensity) of the intensity distribution as the nonlinearity is increased, for each wavelength (temporal frequency) separately, by placing narrow-band color filters in front of the camera. We sample the spectrum from 500 to 700 nm, with intervals of 50 nm. The nonlinearity strength is characterized by the maximal index change $\Delta n_0 = 1/2 n_0^3 r_{33} E_0$, where $n_0 = 2.37$ is the linear refractive index, $r_{33} = 210 pm/V$ is the relevant electro-optic coefficient, and $E_0$ is the externally applied electric field. The results, presented in Fig. 3, show that the pattern formation process of incoherent white light is indeed a collective phenomenon. Up to a particular value of the nonlinearity, the visibility is very low for all wavelengths and does not depend on the nonlinearity strength. However, when the nonlinearity strength exceeds the MI threshold value ($\sim 1.5 \times 10^{-4}$, marked by an arrow), the modulation depth of the emerging pattern starts to grow sharply and simultaneously at every temporal frequency. *The threshold value is the same for all temporal frequencies, and so is the periodicity of the emerging pattern, indicating that the process is indeed a collective phenomenon.* Increasing the nonlinearity beyond the threshold results in the enhancement of the modulation depth of the periodic pattern until it saturates. In addition, Fig. 3 shows that *the modulation depth of the pattern is monotonically decreasing with the optical wavelength.* This indicates that the temporal spectrum of the perturbation growing on top of a uniform beam self-adjusts so that shorter wavelengths (higher frequencies) contribute more to the modulation depth of the intensity pattern. For clarity, we restate that the temporal power spectrum of the beam as a whole (growing perturbations plus the underlying uniform beam) is unchanged during propagation. The intuitive explanation for the observed effect is that longer wavelengths have a stronger tendency to diffract. Consequently, their coupling into the collectively induced structure is weaker and the visibility of the pattern at longer wavelengths is lower (Fig. 3). Examining the structure of a single filament reveals that this effect is also manifested as a reddish shade (shown only in the color version online) of the filament edges. Such a characteristic filament is shown in Fig. 4, which is a magnified real-color photograph, taken with a regular stills camera. It should be noted that the equivalent effect was theoretically predicted to appear for incoherent white light solitons [22]; that is, the pattern of colors characteristic of such solitons should correspond to the colors observed in Fig. 4.

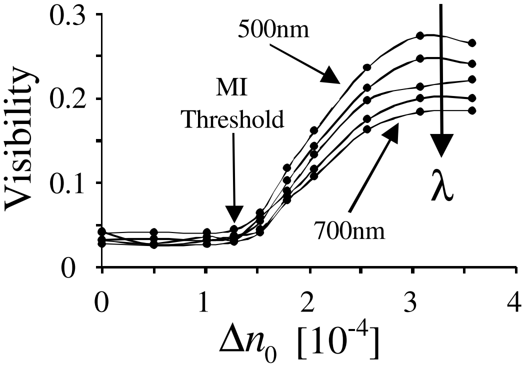

FIG. 3. Visibility as a function of the nonlinearity strength. Different curves correspond to different wavelengths (temporal frequencies).

In conclusion, we have reported the first observation of modulational instability and spontaneous pattern formation with incoherent white light emitted from a bulb. We

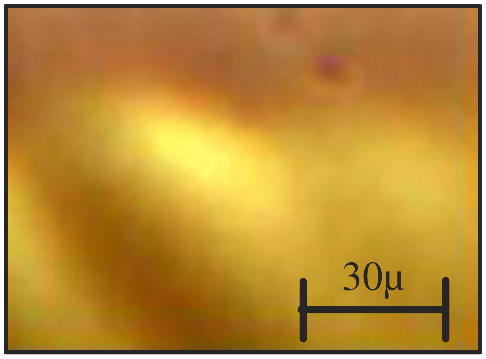

FIG. 4 (color online). True color magnified image of a typical filament in white light MI, showing color separation [reddish shade towards the filament edges (shown only in the color version online)] and self-adjustment of the temporal spectrum.

have demonstrated that pattern formation in this system is a collective phenomenon, where all temporal frequencies undergo instability at the same threshold value and attain the same pattern periodicity. We have also shown that different temporal frequencies have different values of visibility within the collectively forming pattern. The breakup and spontaneous formation of patterns of incoherent white light reported here is an example of such a process in a general statistical wave system, that is, a system with limited temporal and spatial correlation. The results of this work imply that similar behavior may be found in various physical wave systems, where nonlinearity interplays with random statistics and noise. Charge density waves in plasmas, matter waves in Bose-Einstein condensates, and sound waves in superfluids, all at finite temperature, are a few such systems. For example, the behavior of weakly interacting Bose-Einstein condensates at finite temperature is analogous to the behavior of incoherent light in noninstantaneous nonlinear media. This follows from the similarity between the theories describing the systems (e.g., the equations of the static Hartree-Fock mean field approximation [23] are analogous to the equations describing incoherent solitons in noninstantaneous Kerr nonlinear media [24]).

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[1] J.W.S. Rayleigh (Lord), Philos. Mag. **32**, 529 (1916).

[2] A. M. Turing, Philos. Trans. R. Soc. London B **237**, 37 (1952).

[3] A. N. Zaikin and A. M. Zhabotinsky, Nature (London) **225**, 535 (1970).

[4] K. E Strecker *et al.*, Nature (London) **417**, 150 (2002).

[5] M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).

[6] V. I. Bespalov and V. I. Talanov, JETP Lett. **3**, 307 (1966).

[7] M. D. Iturbe-Castillo *et al.*, Opt. Lett. **20**, 1853 (1995).

[8] R. Malendevich *et al.*, Opt. Lett. **26**, 1879 (2001).

[9] L. A. Lugiato, Chaos Solitons Fractals **4**, 1251 (1994); F. T. Arecchi, S. Boccaletti, and P. Ramazza, Phys. Rep. **318**, 1 (1999).

[10] K. Tai, A. Hasegawa, and A. Tomita, Phys. Rev. Lett. **56**, 135 (1986).

[11] M. Soljačić *et al.*, Phys. Rev. Lett. **84**, 467 (2000).

[12] S. M. Sears *et al.*, Phys. Rev. E **65**, 036620 (2002).

[13] D. Kip *et al.*, Science **290**, 495 (2000); D. Kip *et al.*, J. Opt. Soc. Am. B **19**, 502 (2002); J. Klinger, H. Martin, and Z. Chen, Opt. Lett. **26**, 271 (2001).

[14] Z. Chen *et al.*, Proc. Natl. Acad. Sci. U.S.A. **99**, 5223 (2002).

[15] More recently, our group had also studied, theoretically and experimentally, spontaneous pattern formation in cavities with spatially incoherent light. See H. Buljan *et al.*, Phys. Rev. E **68**, 016616 (2003); T. Carmon, H. Buljan, and M. Segev, Opt. Express **12**, 3481 (2004).

[16] D. Anderson *et al.*, Phys. Rev. E **69**, 025601 (2004). This paper analyzes the influence of the shape of the correlation function of the beam on the onset of the instability. For example, it shows that for the case of a rectangular power spectrum there is always an interval of spatial frequencies that are unstable, with no threshold. However, for Gaussian-Schell sources, e.g., a laser beam passing through a rotating diffuser, or a thermal source [see P. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995), Chap. 5 and J. D. Farina *et al.*, Opt. Commun. **32**, 203 (1980)], there is always a threshold for incoherent MI, below which all spatial frequencies are stable [11,12].

[17] M. Mitchell *et al.*, Phys. Rev. Lett. **77**, 490 (1996).

[18] M. Mitchell and M. Segev, Nature (London) **387**, 880 (1997).

[19] H. Buljan *et al.*, Phys. Rev. E **66**, 035601 (2002).

[20] M. Segev *et al.*, Phys. Rev. Lett. **73**, 3211 (1994); M. Segev, M. Shih, and G. C. Valley, J. Opt. Soc. Am. B **13**, 706 (1996).

[21] In instantaneous nonlinearities at sufficiently high intensities, new frequencies are generated and the temporal spectrum broadens considerably. In some cases, this gives rise to spatiotemporal instabilities, which may have an extremely large bandwidth [see D. Salerno *et al.*, physics/0405119 v1].

[22] H. Buljan *et al.*, J. Opt. Soc. Am. B **21**, 397 (2004); H. Buljan *et al.*, Opt. Lett. **28**, 1239 (2003).

[23] V.V. Goldman, I.F. Silvera, and A.J. Leggett, Phys. Rev. B **24**, 2870 (1981).

[24] M. I. Carvalho *et al.*, Phys. Rev. E **59**, 1193 (1999).