Ensemble Quantum Computation with Atoms in Periodic Potentials

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(Received 5 May 2004; published 23 November 2004)

We show how to perform universal quantum computation with atoms confined in optical lattices which works both in the presence of defects and without individual addressing. The method is based on using the defects in the lattice, wherever they are, both to "mark" different copies on which ensemble quantum computation is carried out and to define pointer atoms which perform the quantum gates. We also show how to overcome the problem of scalability in this system.

DOI: 10.1103/PhysRevLett.93.220502 PACS numbers: 03.67.Lx, 03.67.Pp, 42.50.Vk

Neutral atoms confined in (quasi)periodic optical potentials and manipulated by lasers provide us with one of the most promising avenues to implement a quantum computer or to perform quantum simulations [1]. For example, a Bose-Einstein condensate can be loaded in an optical lattice achieving almost unit occupation per lattice site through a superfluid-Mott insulator quantum phase transition [2]. A universal set of quantum gates can then be implemented by individual laser manipulation and inducing cold collisions between the atoms [3]. In several remarkable experiments, all these phenomena have been observed [4–6].

At the moment, quantum computation with atoms in optical lattices is hindered by three major obstacles: (1) Lack of addressability; (2) Presence of defects (empty site); (3) Uncontrolled number of atoms. The first obstacle is due to the fact that the separation between atoms is of the order of an optical wavelength (determined by the laser which creates the confining potential), so that individual addressing with another laser beam would require focusing beyond the diffraction limit. A possible way to circumvent this problem consists of using optical superlattices [7], or other optical microtraps [8], where the separation between atoms is larger. To implement quantum gates in these setups, however, may be harder than in conventional lattices. The second obstacle occurs due to the fact that some sites may have no atom at all. A single defect will unavoidably spoil any quantum computation, and may also have important consequences in quantum simulations. In current experiments, one can estimate that the number of defects is relatively high [9]. The last obstacle can be overcome to a very large extent by a filtering process [10], where lattice sites with more than one atom are emptied until a single atom remains there. Alternatively, one can define collective qubits independent of the number of atoms per site [11]. Both procedures should avoid situations in which defects are present. Finally, the number of atoms forming the quantum computer must be well defined since, otherwise, when performing quantum gates the rest of the atoms will act as an environment.

In this Letter we introduce a novel method of performing quantum computations in optical lattices (or, more generally, periodic potentials) which circumvents the above mentioned obstacles. One of the fundamental ideas of our method is to use defects (which are delocalized in the lattice) in order to mark the atoms that build the quantum computer and to break the translational symmetry in order to obtain addressability. Note that we do not know where the defects are, but only their presence (wherever they are) is sufficient for our purposes. On the other hand, the defects allow us to build "pointer" sites, also delocalized, which will be used to perform a universal set of quantum gates. Note also that since there will be several defects in the atomic sample, we will have several quantum computers running in parallel, randomly distributed all over the optical lattice. This situation resembles ensemble quantum computation of Ref. [12], and in fact some of the ideas developed there can be directly incorporated in our method to make it more efficient. In general, these methods suffer from the scalability problem. Here, we will also present a method to overcome it and to make the present proposal scalable. Note that even though our method is developed for atoms in optical lattices, some of these ideas may also apply to very different implementations where similar obstacles are present.

We consider a three dimensional periodic potential in which atoms are loaded. The atoms have three internal states, $|a\rangle$, $|b\rangle$ and $|p\rangle$. The first two will store the qubit, whereas the third one will be used by the pointer. We will consider each 1D lattice independently, i.e., we assume that tunnelling is switched off for all times along the y and z directions. Thus, we can effectively reduce the system to a set of 1D periodic lattices. We will use second quantization description of the states; that is, for each lattice site k we will write a state $|m_k, m'_k, n_k\rangle_k$, where m_k, m'_k, and n_k are natural numbers that indicate the

occupation number of levels $|a\rangle$, $|b\rangle$, and $|p\rangle$, respectively. Thus, a typical product state of one 1D lattice will be

$$\otimes_k |m_k, m'_k, n_k\rangle_k. \quad (1)$$

We will assume that four kinds of basic operations are available. These operations act in exactly the same way in each lattice site, since we do not assume individual addressing. On the other hand, they are based on physical processes which have been demonstrated (or are feasible) in the current experimental setups:

(i) Particle transfer between internal states. We will consider two kinds: (i.1) An integer number of particles is transferred between states a and p. For example,

$$U_{m,n}^{m+x,n-x}: |m, 0, n\rangle \leftrightarrow |m + x, 0, n - x\rangle, \quad (2)$$

where x is an integer. Note that for the unitary operator describing this process at each site, the relation $U_{m,n}^{m+x,n-x} = U_{m+x,n-x}^{m,n}$ holds. Another example that we will use later is W: $|1, 0, 1\rangle \leftrightarrow |0, 1, 1\rangle$. These operations can be carried out using blockade mechanisms due to atom-atom interactions [10]. (i.2) Transfer of particles that generate superpositions. For example, $V = \exp(-iH_{ab}\pi/8)$, which acts only on the a and b levels, with $H_{ab} = \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}$, where $\hat{a}$ and $\hat{b}$ are the annihilation operators for particles in states $|a\rangle$ and $|b\rangle$, respectively. These operations can be easily realized using laser or rf fields [6].

(ii) Collisional shifts: They are due to interactions between particles in the states a and p. For example, the unitary operation $C(\varphi)$: $|1, 0, 1\rangle \leftrightarrow e^{i\varphi}|1, 0, 1\rangle$ can be obtained by waiting the appropriate time [3].

(iii) Lattice shifts: We denote by S_x the operations which shift the pointer states x steps to the right. For example, S_{-1} transforms the state in Eq. (1) to $\otimes_k |m_k, m'_k, n_{k+1}\rangle_k$. They can be realized by changing the intensity and polarization of the lasers [3,13].

(iv) Emptying sites: All atoms in internal states a or p are thrown away. This can be done, for example, by switching off the lattice potential for the corresponding internal state. We will denote them by E_b or E_p and they transform the state in Eq. (1) into $\otimes_k |m_k, m'_k, 0\rangle_k$, and $\otimes_k |m_k, 0, n_k\rangle_k$, respectively.

Initially, all atoms are in the internal state $|a\rangle$, distributed along the lattice according to some probability distribution; i.e., the state is a mixture of states in the form of Eq. (1) with $m'_k = n_k = 0$. Thus, the goal is to show how with these random states, and without requiring individual addressing, we can perform arbitrary quantum computations. This will be achieved in two steps. First there will be a preparation step, and then a computation step. At the end we will show how to include an additional step to make the system scalable. In the preparation stage of our protocol, only states a and p will be occupied. Thus, we will simplify our notation denoting $|m, n\rangle := |m, 0, n\rangle$. Moreover, the states that we will use now will be product states of the form

$$|m_1, n_1\rangle \otimes |m_2, n_2\rangle \ldots \otimes |m_N, n_N\rangle, \quad (3)$$

where we have not included the subscript k to simplify the notation. This step starts by reducing all occupation numbers larger than 2 to 2 (Fig. 1). This is done by applying the operation $U_{x,0}^{2,x-2}$ first and then E_p, and then repeating those actions for $x > 2$ (up to some value of x in which we are confident that no site with this number of particles is present).

The next step is to "format" the lattice. We produce several areas, randomly distributed across the lattice, with exactly n neighboring sites having a single atom in a and one site at the right edge with two atoms, one in p and the other in a (see Fig. 1). In order to accomplish this, we have to keep only the areas in which initially there are n neighboring two-atom sites and a one-atom site at the edge. The rest of the atoms are thrown away, and then we manipulate the remaining atoms to obtain the desired states. The sites in which initially there was a single atom that has survived will now contain the pointers (the extra atom in level p). This atom will then be used to perform the quantum gates.

First, we change the internal states of the one-atom sites from a to p. These atoms are now called the pointers. They will be essential to create the quantum computers in the lattice. Each of those atoms marks the position where we try to establish one of those quantum computers. We want such a pointer to survive during the following protocol if it has on its left at least n sites with exactly two atoms in each. We thus proceed as follows. We shift the pointer one lattice site to the left. We transfer the pointer atom to the state a if there are two atoms in that site by applying $U_{2,1}^{3,0}$. By emptying the internal states p we delete all pointers which fail to have a two-atom site next to their starting position. Then we raise the pointer again by $U_{2,1}^{3,0}$. By repeating this procedure for the next $n - 1$ sites we delete all pointers that fail to have n two-atom sites on the left of their starting position. Note that this also implies that every pointer in one of the n sites on the right of each surviving pointer is deleted. This means that every pointer can act on its own "reserved" n sites, i.e., there are no overlapping reserved areas. Having the

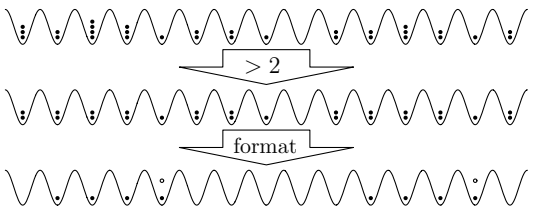

FIG. 1. First, the sites with more than two atoms are depopulated. The format step ($n = 3$) produces sites with two atoms in levels a and p, surrounded by a "reserved area" to their left which contains exactly n sites with a single atom.

pointer and the reserved n two-atom sites, we can effectively address single sites of this reserved space. This enables us to reduce the number of atoms in each site in this reserved area to one and afterwards to empty the remaining sites that are not reserved by any pointer. In terms of the operations described above, the protocol is given by a sequence of the following operators: (1) $U_{1,0}^{0,1}$; (2) S_{-1}, $U_{2,1}^{3,0}$, E_p, $U_{2,1}^{3,0}$, and then repeat this whole step $n-1$ times; (3) $U_{2,1}^{1,2}$; (4) S_1, $U_{2,2}^{3,1}$, E_p, $U_{3,0}^{1,2}$, and then repeat this whole step $n-2$ times; (5) S_1, $U_{2,0}^{0,2}$,E_p, $U_{2,0}^{1,1}$.

The randomly distributed quantum computers consist of n sites, all of them with a single atom in state $|a\rangle$, and the pointer atom in state $|p\rangle$ in the rightmost site (see Fig. 1). The first atoms store a qubit each, with states $|\downarrow\rangle = |1,0,0\rangle$ and $|\uparrow\rangle = |0,1,0\rangle$, whereas the pointer atom in state $|0,0,1\rangle$ carries out the quantum gates.

Now we show how to carry out a universal set of quantum gates using the operations defined above. The idea is to move the pointer atom to the sites which participate in the quantum gate and then apply the appropriate operations. The set is composed of [14] (a) control-π phase-gate on two arbitrary qubits, at the conventional locations "first site" and "second site": we move the pointer to the first site, and apply the operator $U_{1,1}^{0,2}$. This turns the pointer to the level a, if the qubit atom is in level a. After this operation, the pointer can be found in any superposition of levels p and a. In the next step we move the pointer to the second site and we wait until the collisional shift operation $C(\pi)$ is applied. Here, "moving the pointer" means displacing the lattice associated with the pointer level p. Finally, we move the pointer back to the first site and apply again $U_{1,1}^{0,2}$. It is simple to show that this will only add a π phase if both qubits are in the state $|\uparrow\rangle$; (b) Phase gate φ on an arbitrary qubit (see Fig. 2): We bring the pointer to the corresponding site and wait for the appropriate collisional shift operation $C(\varphi)$; (c) Hadamard gate on an arbitrary qubit: We first bring the pointer to the site. Then we apply the following sequence of operations: $C(\pi/2)$, V, $C(\pi)$, $V^\dagger$, $C(\pi/2)$.

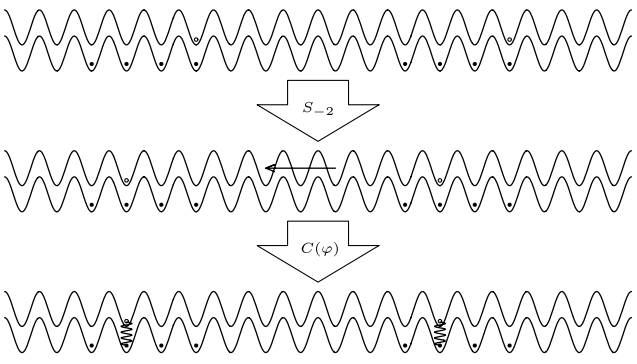

FIG. 2. The levels a, b will store the qubits, and the p level will contain the pointer, which can be moved around allowing us to address single sites, e.g., applying a phase gate φ on an arbitrary qubit.

Measurement on an arbitrary qubit in the $\{|\downarrow\rangle, |\uparrow\rangle\}$ basis can be performed as follows. We promote the corresponding atom to the pointer level provided it is in state $|a\rangle$, i.e., if the qubit is in the state $|\downarrow\rangle$. For the measurement, we count the numbers of atoms in the pointer level (by analyzing the fluorescence coming from that level) and drop them afterwards. Note that this occurs in the same way as in usual ensemble quantum computation [14], where we get the global information about all quantum computers. To save the original pointer from being emptied we need an extra resting site, with one atom in the ground state (for example, the rightmost qubit can be reserved for this purpose). In summary, we (1) move the pointer to the corresponding site and apply $U_{1,1}^{0,2}$; (2) move the pointer to the resting site and apply $U_{1,1}^{2,0}$ and $U_{1,2}^{2,1}$; (3) count atoms in pointer level and apply E_p; (4) apply $U_{2,0}^{1,1}$. The measured qubit site is emptied, if the qubit was found in state $|\downarrow\rangle$, while the pointer and the resting qubit survived unchanged. We can continue by moving the pointer back to the target qubit, applying W and then repeating the above protocol. The number of atoms counted in the pointer level is equal to the number of qubits measured in state $|\uparrow\rangle$. Alternatively, we can leave out this step and relate the number of qubits found in $|\downarrow\rangle$ to the total number of quantum computers in the lattice. This number can be estimated either by the statistic of the initial distribution or by counting the pointers or atoms at the end of the computation.

So far we have shown how to build a quantum register of n qubits, for an arbitrary integer n, and how to perform quantum computations. Note that in order to prepare the initial state it is necessary that lattice areas with no defects exist, i.e., neither empty sites nor one-atom sites. If the number of such defects inside the lattice is larger than the number of 1D lattices, then the probability of ending up with at least one quantum computer will decrease exponentially with n, and thus the method proposed here will not be scalable. In detail, if we assume that every site of the lattice is filled independently with zero, one, or two atoms, according to the probabilities distribution p_0, p_1, p_2, then the expected number of quantum computer of length n in a 1D lattice can be estimated by $Np_1p_2^n = Np_1(1-p_0-p_1)^n$, where the length N of the lattice has to be much larger than n. This quantity decreases exponentially with n which makes the proposed method not scalable.

In the following we show how to boost the probability of creating a quantum computer in the lattice by correcting the defects, i.e., making p_0 and p_1 arbitrarily small. Having this possibility, we change the probabilities to $p_0 = 0$ and $p_1 = 1/n$. The resulting expected number of quantum computers in a lattice of size N is then given by $N/n(1-1/n)^n$, which goes to $N/(ne) \sim 1/n$ for large n; i.e., our method becomes scalable. The procedure of cor-

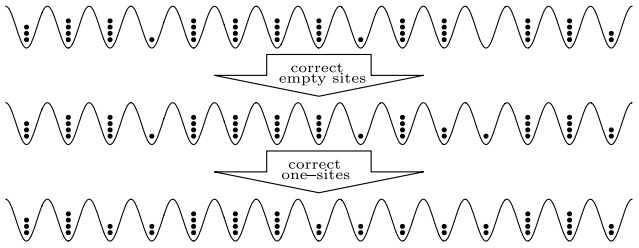

FIG. 3. Overpopulated sites are used to first fill empty sites and then to put two atoms in sites with one atom.

recting the defects will also be useful if one performs quantum simulations with large spin chains.

The main idea of the protocol is to first fill all sites which are empty with one atom coming from a different site which is overpopulated. Then, the sites with one atom are filled with another atom also coming from overpopulated sites (see Fig. 3). Thus, we have to assume that there are as many overpopulated sites as defects, an achievable requirement for sufficiently high densities.

First, we reduce all occupation numbers larger than 4 to 4 [15]. The protocol starts by promoting two atoms to the state p whenever a site has four atoms. Then, we shift the lattice corresponding to level p by a random amount x and try to deposit one of such atoms in an empty site. The remaining atom in the p level is thrown away. Note that for every corrected defect we lose one atom in this protocol. Losing atoms while correcting defects is unavoidable, since it is the only way to reduce the entropy of the state in our setup. We proceed in the same way until we make the probability of having sites with no atoms vanishingly small. In more detail, we apply the following sequence of operations several times: $U_{4,0}^{2,2}$, S_x, $U_{0,2}^{1,1}$, S_{-x}, $U_{4,0}^{2,2}$, E_p to fill the empty sites. Now, we can do the same but replacing $U_{4,0}^{2,2}$ and $U_{0,2}^{1,1}$ by $U_{x,0}^{x-2,2}$ and $U_{1,2}^{2,1}$, so that sites with a single atom get double occupation. For a finite lattice of length N there are only N different possibilities for the x, so the protocol requires at most N repetitions.

We still need some defects to act as pointers. So we either do not fill up all the one-atom defects or we have to create new defects. The latter can be done by first reducing all occupation numbers to two and then applying a unitary operation that changes $|2, 0, 0\rangle$ to the superposition state $\sqrt{\varepsilon}|1, 1, 0\rangle + \sqrt{1-\varepsilon}|2, 0, 0\rangle$, followed by E_b. With probability ε a one-atom-site defect is created out of a two-atom site.

We have shown that it is possible to perform quantum computations in optical lattices in the presence of lattice defects, and without the necessity to address single sites or specify the total number of atoms in the lattice. In practice, a very high degree of control is required, something which is being achieved in current experiments. The ideas presented here not only apply to the field of quantum computation but they can also be used to prepare and manipulate the states in the lattice, and to build some prescribed atomic patterns [10]. Furthermore, all these methods can be generalized in a straightforward way to two-dimensional or three-dimensional lattices. Finally, note that some of the protocols given here require a large number of steps, something which is experimentally demanding. We are currently using the ideas of quantum compression in order to develop new efficient methods for loading the lattices [16].

This work is supported by EU IST projects, the DFG, and the Kompetenznetzwerk Quanteninformationsverarbeitung der Bayerischen Staatsregierung.

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