Bright and Dark Gap Solitons in a Negative Index Fabry-Pérot Etalon

Giuseppe D'Aguanno,[1,2] Nadia Mattiucci,[1,2,3] Michael Scalora,[2] and Mark J. Bloemer[2]

[1]*Time Domain Corporation, Cummings Research Park 7057 Old Madison Pike, Huntsville, Alabama 35806, USA*

[2]*Research Development and Engineering Center, U.S. Army Aviation & Missile Command, Building 7804, Redstone Arsenal, Alabama 35898-5000, USA*

[3]*Università "RomaTre", Dipartimento di Fisica "E. Amaldi", Via Della Vasca Navale 84, I-00146 Rome, Italy*

(Received 2 July 2004; published 16 November 2004)

We predict the existence of bright and dark gap solitons in a single slab of negative index material. The formation of gap solitons is made possible by the exceptional interplay between the linear dispersive properties of the negative index etalon and the effect of a cubic nonlinearity.

DOI: 10.1103/PhysRevLett.93.213902 PACS numbers: 42.65.Tg, 42.65.Pc, 78.20.Ci

The presence of a cubic (Kerr) nonlinearity in structures characterized by a periodic variation of the linear refractive index leads to the formation of localized electromagnetic modes in spectral regions that otherwise would just allow evanescent modes. These localized modes are generally referred to as gap solitons (GS) [1]. GS have attracted the attention of many researchers for almost two decades, beginning with the theoretical predictions of Chen and Mills for one-dimensional (1D) photonic lattices with a Kerr nonlinearity [1]. Subsequently GS have been studied both theoretically [2] and experimentally [3], and their existence has also been predicted in 1D periodic media with shallow gratings and a quadratic nonlinearity [4]. GS in 2D and 3D photonic crystals have also been theoretically studied using different mathematical approaches [5].

The aim of this Letter is to show that the presence of bright and dark GS is supported in a single slab of material. This surprising outcome is born out of the peculiar dispersive properties of a new class of materials called "negative index materials" (NIMs), or left handed materials [6–8]. NIMs' most impressive property is their ability to refract light in the opposite way with respect to what an ordinary material does. Very recently, nonlinear effects in NIMs have been also investigated [9]. While it is not surprising that a single slab of frequency dispersive material together with a cubic nonlinearity can support soliton waves in general, what it is surprising is that in this case the *single slab appears to support both bright and dark GS.*

Before going into detail, it is worth saying a few words to define the terms "bright" and "dark" GS in the case of NIMs. By the term "bright GS" in NIMs we refer to a highly localized electromagnetic mode with approximately decaying tails excited inside the gap of a NIM [see Figs. 3(b) and 3(c)]. These modes have localization properties similar to the classical GS excited in the gap of distributed feedback structures with a cubic nonlinearity [1–3]. We emphasize that the formation of the gap in the NIM is due to the peculiar dispersive properties of the bulk of the material, while the formation of the gap in distributed feedback structures is due to interference effects. On the other hand, the physical mechanism that leads to the formation of bright GS in both cases is the same: a dynamical change in the refractive index of the material occurs due to the presence of a cubic nonlinearity that shifts the position of the band gap, and allows the formation of localized modes in a spectral region that would otherwise support only evanescent modes. While bright GS in NIMs are localized over the structure in way similar to GS in distributed feedback structures, in contrast "dark GS" are excited in the gap of a NIM in the form of delocalized modes, with approximately nondecaying tails. These states display a low intensity at the center of the structure, and a high intensity at the edges [see Fig. 5(c)]. We note that, contrary to bright GS, the intensity inside the structure never exceeds one with respect to a unitary input intensity. Therefore, dark GS have no counterpart in the case of the structures studied in Refs. [1–4]. Dark solitons generated at frequencies outside the gap or in other systems where there is no photonic band gap structure are not uncommon. For example, light waves in the form of dark solitons appear in optical fibers operating in the normal dispersion regime [10], in Raman scattering [11], and atomic, out-of-gap dark soliton waves are supported in a Bose-Einstein condensate interacting with a periodic optical field [12]. In contrast, *here we present numerical evidence that dark solitons can also occur when the incident light frequency is tuned inside the photonic band gap of a NIM.*

Let us begin by describing the effective susceptibility and magnetic permeability of a NIM with a lossy Drude model [13]:

$$\varepsilon(\tilde{\omega}) = 1 - \frac{1}{\tilde{\omega}(\tilde{\omega} + i\tilde{\gamma}_e)}, \qquad \mu(\tilde{\omega}) = 1 - \frac{(\omega_{pe}/\omega_{pe})^2}{\tilde{\omega}(\tilde{\omega} + i\tilde{\gamma}_m)}, \tag{1}$$

where $\tilde{\omega} = \omega/\omega_{pe}$ is the normalized frequency, ω_{pe} and ω_{pm} are the respective electric and plasma frequencies, $\tilde{\gamma}_e = \gamma_e/\omega_{pe}$ and $\tilde{\gamma}_m = \gamma_m/\omega_{pe}$ are the respective elec-

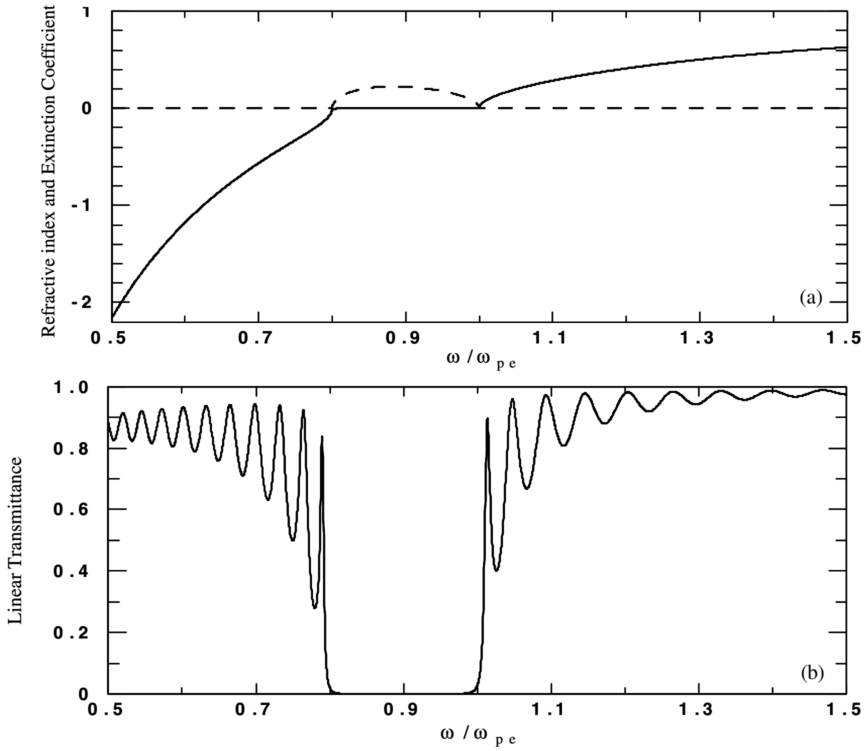

FIG. 1. (a) Refractive index (solid line) and extinction coefficient (dashed line) vs normalized frequency (ω/ω_{pe}) for a NIM with $\omega_{pm}/\omega_{pe} = 0.8$ and $\tilde{\gamma}_e \approx \tilde{\gamma}_m \approx 4.5 \times 10^{-4}$. (b) Linear transmittance vs normalized frequency (ω/ω_{pe}) for a Fabry-Pérot etalon of length $L = 5\lambda_{pe}$ where $\lambda_{pe} = 2\pi c/\omega_{pe}$ is the wavelength corresponding to the electric plasma frequency.

tric and magnetic loss terms normalized with respect to the electric plasma frequency.

In Fig. 1(a) we show the refractive index and the extinction coefficient for a NIM with $\omega_{pm}/\omega_{pe} = 0.8$ and $\tilde{\gamma}_e \approx \tilde{\gamma}_m \approx 4.5 \times 10^{-4}$. The refractive index n and the extinction coefficient β of the material are given by $n + i\beta = \pm\sqrt{\varepsilon\mu}$. The sign in front of the square root must be chosen in a way that ensures the Poynting vector of the light refracted into a semi-infinite slab of NIM will always be directed away from the interface into the refracting material itself. Of course, only one of the two possible solutions of the square root satisfies this requirement. In Fig. 1(b) we show the linear transmission property of a Fabry-Pérot (FP) etalon made by the same NIM. Figure 1(b) shows that the transmission spectrum of the FP etalon is similar to the transmission spectrum that occurs in structures that have a periodic variation of the refractive index. The center-gap frequency $\omega_{c,\text{gap}}$ and the spectral width of the gap $\Delta\omega_{\text{gap}}$ depend on the electric and magnetic plasma frequency as follows: $\omega_{c,\text{gap}} = (\omega_{pe} + \omega_{pm})/2$ and $\Delta\omega_{\text{gap}} = |\omega_{pe} - \omega_{pm}|$. The only gap that forms disappears when $\omega_{pe} = \omega_{pm}$. Moreover the gap appears in the region where values of the refractive index n are near zero.

Let us now suppose the FP possesses a Kerr nonlinearity. The Helmholtz equation that governs the nonlinear dynamic at normal incidence is given by

$$\frac{d^2E}{dz^2} + \frac{\omega^2}{c^2}\varepsilon\mu E = -\frac{\omega^2}{c^2}\mu\chi^{(3)}|E|^2E, \qquad (2)$$

where ε and μ are the effective electric susceptibility and magnetic permeability given by Eq. (1), and $\chi^{(3)}$ is the coefficient of the cubic nonlinearity. The boundary con-

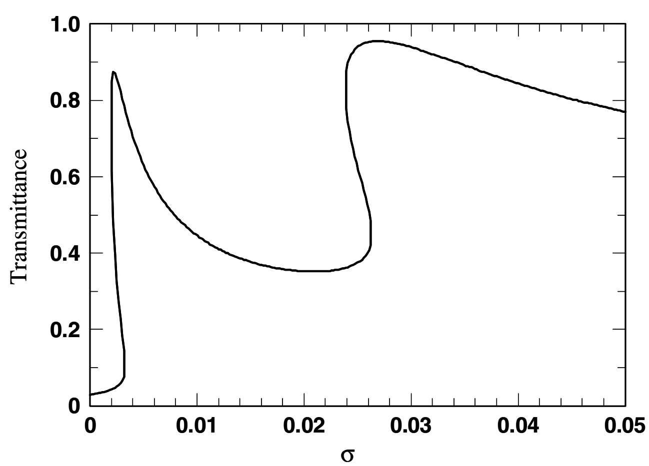

FIG. 2. Transmittance vs control parameter $\sigma = \chi^{(3)}|E^{(\text{input})}|^2$. The input field is tuned at $\omega_0 = \omega_{pe}$ in the band gap near the high frequency band edge.

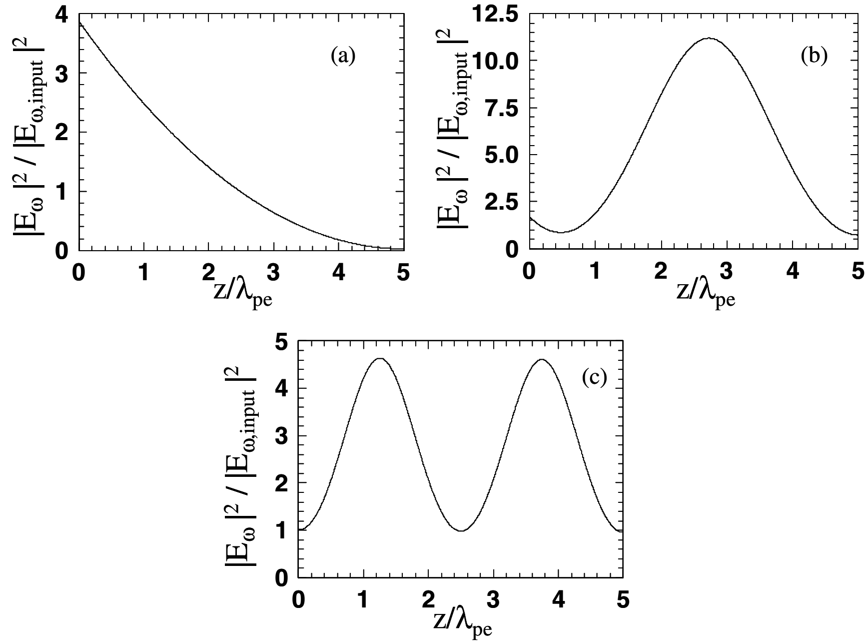

FIG. 3. Field localization in the cavity for different values of the control parameter: (a) $\sigma = 0$; (b) $\sigma = 0.0039$; (c) $\sigma = 0.027$.

ditions that apply to Eq. (2) are those valid in the case of normal incidence in a magnetic material. Equation (2) has been numerical integrated using an explicit method in conjunction with a shooting procedure [14]. In Fig. 2 we show that transmission of the FP etalon as function of the control parameter $\sigma = \chi^{(3)}|E^{(\text{input})}|^2$, where $E^{(\text{input})}$ is the input field. The input field is tuned at $\omega_0 = \omega_{pe}$, i.e., inside the band gap near the high frequency band edge. At $\omega_0 = \omega_{pe}$ the refractive index, the extinction coefficient, and the magnetic permeability are $n \cong 9.4 \times 10^{-3}$, $\beta = 9 \times 10^{-3}$, and $\mu = 3.6 \times 10^{-1} + i3 \times 10^{-4}$, respectively. The figure shows bistable behavior that is typical of distributed feedback structures with a cubic nonlinearity [1,15]. In Figs. 3 we calculate the field localization over the FP cavity for different values of the control parameter σ. In the linear case [$\sigma = 0$, Fig. 3(a)], the field is evanescent, consistent with its tuning inside the band gap. For $\sigma = 0.0039$ the field becomes localized in the form of a single bright-soliton envelope, similar to that reported in Reference [1]. For $\sigma = 0.027$ a two-peaked, localized, bright-soliton state is excited.

The FP etalon also supports dark solitons. These states manifest themselves when the carrier frequency is tuned inside the gap, but now near the low frequency band edge. In Fig. 4 we show the transmission as a function of the control parameter σ, for an input field tuned at $\omega_0 = 0.81\omega_{pe}$. In this case, the transmission shows multistable behavior. By increasing the value of the control parameter up to $\sigma = 5$, three stable branches are found. The first branch is located in the range $0 \leq \sigma \leq 1.5$, and it corresponds to evanescent-type solutions as those shown in Fig. 5(a). The second branch is in the range $1.5 \leq \sigma \leq 2.61$, and the corresponding solutions are of the type shown in Fig. 5(b). Finally, for $\sigma \geq 2.61$ dark soliton-type solutions are excited as shown in Fig. 5(c). The excitation of dark solitons is somewhat surprising because, as discussed in the introduction, their appearance in the gap has to our knowledge never been predicted [16]. At $\omega_0 = 0.81\omega_{pe}$ the refractive index, the extinction coefficient, and the magnetic permeability are $n \cong -1.16 \times 10^{-3}$, $\beta \cong 1.13 \times 10^{-1}$, and $\mu \cong 2.45 \times 10^{-2} + i5.41 \times 10^{-4}$, respectively.

Our calculations suggest that when $\omega_{pm}/\omega_{pe} < 1$ and $\chi^{(3)} > 0$, bright solitons are excited near the high fre-

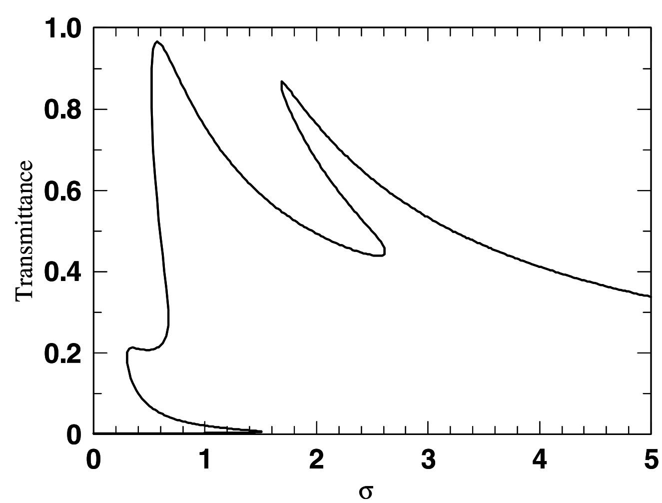

FIG. 4. Transmittance vs control parameter $\sigma = \chi^{(3)}|E^{(\text{input})}|^2$. The input field is tuned at $\omega_0 = 0.81\omega_{pe}$ in the band gap near the low frequency band edge.

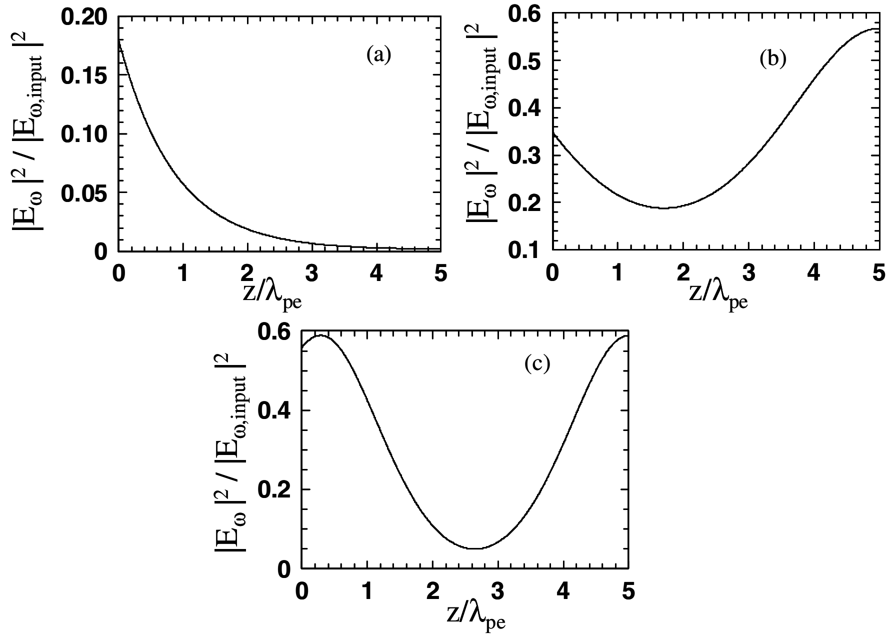

FIG. 5. Field localization in the cavity for different values of the control parameter: (a) $\sigma = 0$; (b) $\sigma = 1.6$; (c) $\sigma = 2.7$.

quency band edge, where $n > 0$, and that dark solitons are excited near the low frequency band edge where $n < 0$. On the contrary, in the case $\omega_{pm}/\omega_{pe} > 1$ and $\chi^{(3)} < 0$, bright solitons are excited near the low frequency band edge and dark solitons are excited near the high frequency band edge.

In conclusion, using a numerical approach we have predicted the existence of a new class of bright and dark gap solitons that are supported by NIMs. Our results suggest that NIMs could find further application in all-optical switching devices and all-optical buffering, for example.

[1] W. Chen and D. L. Mills, Phys. Rev. Lett. **58**, 160 (1987).
[2] C. M. de Sterke and J. E. Sipe in *Progress in Optics XXXIII*, edited by E. Wolf (Elsevier, Amsterdam, 1994), Chap. 3; D. N. Christodoulides and R. I. Joseph, Phys. Rev. Lett. **62**, 1746 (1989); D. L. Mills and S. E. Trulinger, Phys. Rev. B **36**, 947 (1987); J. E. Sipe and H. Winful, Opt. Lett. **13**, 132 (1998).
[3] B. J. Eggleton, et al., Phys. Rev. Lett. **76**, 1627 (1996).
[4] C. Conti, S. Trillo, and G. Assanto, Opt. Lett. **22**, 445 (1997); T. Peschel *et al.*, Phys. Rev. E **55**, 4730 (1997); C. Conti, S. Trillo, and G. Assanto , Opt. Express **3**, 389 (1998).
[5] S. John and N. Akozbek, Phys. Rev. Lett. **71**, 1168 (1993); S. F. Mingalev and Y. S. Kivshar, Phys. Rev. Lett. **86**, 5474 (2001); Ping Xie and Zhao-Qing Zhang, Phys. Rev. Lett. **91**, 213904 (2003).
[6] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[7] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[8] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[9] S. O'Brien *et al.*, Phys. Rev. B **69**, 241101 (R) (2004).
[10] A. D. Kim, W. L. Kath, and C. G. Goedde, Opt. Lett. **21**, 465 (1996).
[11] M. Scalora, S. Singh, and C. M. Bowden, Phys. Rev. Lett. **70**, 1248 (1993).
[12] A.V. Yulin and D. Skryabin, Phys. Rev. A **67**, 023611 (2003).
[13] R.W. Ziolkowski and E. Heyman, Phys. Rev. E **64**, 056625 (2001).
[14] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in C*, (Cambridge Univ. Press, Cambridge, 1988).
[15] B. Acklin *et al.*, Appl. Phys. Lett. **63**, 2177 (1993).
[16] Here we refer to a "true" in-gap dark soliton, i.e., a dark gap soliton excited by a single pump with no external feedback mechanism to reinject the output field back into the structure. This external feedback scheme indeed can lead to delocalized modes excited by two counter-propagating pumps. In this regard, it is well known that in the linear regime a multilayer structure pumped on both sides can give rise to unusual delocalized states by simply choosing a proper phase link between the incident pumps; see M. Centini *et al.*, Phys. Rev. E **67**, 036617 (2003). Therefore, in the nonlinear regime dark in-gap solitons or delocalized modes can be excited using either external feedback or two counter-propagating pumps that may cause destructive interference and a local minimum inside the structure. A very good example of external feedback that can give rise to delocalized states in the gap of a multilayer structure is discussed in H. Alatas *et al.*, J. Nonlinear Opt. Phys. Mater. **13**, 259 (2004).