Electromagnetic $N \to \Delta$ Transition and Neutron Form Factors

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The $C2/M1$ ratio of the electromagnetic $N \to \Delta(1232)$ transition, which is important for determining the geometric shape of the nucleon, is shown to be related to the neutron elastic form factor ratio G_C^n/G_M^n. The proposed relation holds with good accuracy for the entire range of momentum transfers where data are available.

The regularities seen in the spectrum of excited states of a physical system are usually due to an underlying symmetry. This is also the case in subnuclear physics, in particular, in baryon physics. There, SU(3) flavor symmetry allows grouping the known baryons into singlets, octets, and decuplets [1]. Furthermore, SU(6) spin-flavor symmetry unites the spin $1/2$, flavor octet baryons (2×8 states), among them the familiar proton and neutron, and the spin $3/2$, flavor decuplet baryons (4×10 states) into a common 56-dimensional supermultiplet [2,3]. These symmetries explain why the masses, electromagnetic moments, and other properties of baryons belonging to the same multiplet follow a regular pattern. They arise mainly because octet and decuplet baryons are composed of the same spin $1/2$, flavor triplet quarks merely coupled to different total spin and flavor.

The lowest mass member of the baryon flavor decuplet, called $\Delta(1232)$, with spin $3/2$ and isospin $3/2$ occupies a prominent place in baryon spectroscopy not only because it has of all nucleon resonances the highest production cross section, but also because its properties are closely related to those of the nucleon. The Δ resonance is the lowest lying excited state of the nucleon $N(939)$ with the same quark content as the ground state. When produced in an electromagnetic process, such as electron-nucleon scattering (Fig. 1), parity invariance and angular momentum conservation restrict the $N \to \Delta$ excitation to magnetic dipole ($M1$), electric quadrupole ($E2$), and charge (or Coulomb) quadrupole ($C2$) transitions.

At low momentum transfers the $N \to \Delta$ excitation is predominantly an $M1$ transition involving the spin and isospin flip of a single quark. The quadrupole amplitudes are only about $1/40$ of the dominant magnetic dipole amplitude. Despite their smallness, the $C2$ and $E2$ multipoles have been the focus of many recent experimental [4–7] and theoretical works [8–12]. They are nonzero only if the geometric shape of the nucleon deviates from spherical symmetry [13]. From the corresponding quadrupole transition form factors information on the spatial shape of the nucleon's charge distribution can be obtained.

The purpose of this Letter is to show that the ratio of the $N \to \Delta$ charge quadrupole over magnetic dipole form factors, called the $C2/M1$ ratio, follows in good approximation the same curve as the ratio of the elastic neutron charge over magnetic form factors G_C^n/G_M^n for the entire range of momentum transfers where data are available. This has not been noticed before.

Because the N and Δ belong to the same 56-dimensional ground state multiplet of the SU(6) spin-flavor group their properties are related. In particular, the electromagnetic $N \to \Delta$ transition form factors are related to the electromagnetic elastic form factors of the nucleon [14]. This remains true even if the symmetry is broken. The experimentally observed breaking of SU(6) symmetry is not a fundamental objection against its usefulness. If the relevant symmetry breaking mechanisms are included in the theory the resulting approximate symmetry leads to relations that are often very well satisfied in nature [15].

The SU(6) relation between the $N \to \Delta$ magnetic dipole transition form factor $G_{M1}^{N \to \Delta}(Q^2)$ and the elastic neutron magnetic form factor $G_M^n(Q^2)$ has been known for some time [3]

$$G_{M1}^{N \to \Delta}(Q^2) = -\sqrt{2} G_M^n(Q^2). \tag{1}$$

Here, Q is the four-momentum transfer of the virtual photon. At $Q^2 = 0$, both form factors are normalized to their magnetic dipole moments $\mu_{N \to \Delta}$ and μ_n
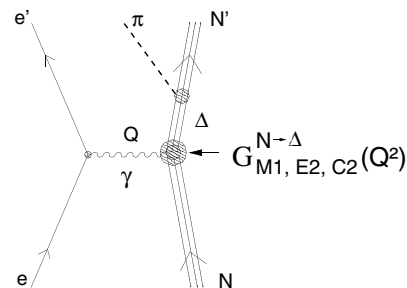

FIG. 1. The excitation of the Δ resonance by a virtual photon γ of momentum Q is described by the three electromagnetic transition form factors $G_{M1}^{N \to \Delta}(Q^2)$, $G_{E2}^{N \to \Delta}(Q^2)$, and $G_{C2}^{N \to \Delta}(Q^2)$. They can be determined by measuring the angular distribution of the decay pions in coincidence with the scattered electron.

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$$\mu_{N\to\Delta} = -\sqrt{2}\mu_n. \qquad (2)$$

These relations also hold when second order SU(6) symmetry breaking operators are included [15], and have also been derived in the quark model with two-quark currents [16,17]. They are violated only by three-quark currents [18] or third order SU(6) symmetry breaking operators [19]. The latter are suppressed by a factor $1/N_c^2$ with respect to the leading term [20] so that these relations are valid in good approximation.

The other relation between the $N\to\Delta$ charge quadrupole transition form factor $G_{C2}^{N\to\Delta}(Q^2)$ and the elastic neutron charge form factor $G_C^n(Q^2)$

$$G_{C2}^{N\to\Delta}(Q^2) = -\frac{3\sqrt{2}}{Q^2}G_C^n(Q^2) \qquad (3)$$

was unknown until quite recently [16,21]. If SU(6) symmetry were exact both $G_C^n(Q^2)$ and $G_{C2}^{N\to\Delta}(Q^2)$ would be zero. Spin-dependent two-quark terms in the charge density break SU(6) symmetry [22] and lead to nonzero form factors which are related as in Eq. (3).

In the $Q\to 0$ limit, Eq. (3) reduces to a relation [23] between the $N\to\Delta$ transition quadrupole moment $Q_{N\to\Delta}$ and the neutron charge radius r_n^2

$$Q_{N\to\Delta} = \frac{1}{\sqrt{2}}r_n^2, \qquad (4)$$

which is in good agreement with recent extractions of $Q_{N\to\Delta}$ from the data [9,24]. This relation and its generalization to finite momentum transfers in Eq. (3) are of more general validity because they also hold in a theory [12], which includes spin-dependent three-quark terms in the charge density, and for an arbitrary odd number of colors $N_c > 1$. From Eq. (4) we learn that the small deviation of r_n^2 from zero and the deviation of the nucleon's geometric shape from spherical symmetry as manifested in a nonzero $Q_{N\to\Delta}$ are closely related aspects of nucleon structure. Both phenomena have their origin in a nonspherical cloud of quark-antiquark pairs in the nucleon [13]. These pair degrees of freedom are effectively described by two-quark and three-quark currents [23,25].

Experimental results are often given for the $C2/M1$ ratio, which is defined in terms of the $N\to\Delta$ transition form factors times a kinematical factor [26,27]

$$\frac{C2}{M1}(Q^2) = :\frac{|\mathbf{q}|M_N}{6}\frac{G_{C2}^{N\to\Delta}(Q^2)}{G_{M1}^{N\to\Delta}(Q^2)}, \qquad (5)$$

where M_N is the nucleon mass and $|\mathbf{q}|$ is the three-momentum transfer of the virtual photon in the γN center of mass frame [28].

Inserting the above form factor relations [Eq. (1) and (3)], the $C2/M1$ ratio can be expressed as the product of G_C^n/G_M^n and a factor

$$\frac{C2}{M1}(Q^2) = \frac{|\mathbf{q}|}{Q}\frac{M_N}{2Q}\frac{G_C^n(Q^2)}{G_M^n(Q^2)} = :\mathcal{R}_n(Q^2). \qquad (6)$$

We abbreviate this product as $\mathcal{R}_n(Q^2)$. Thus, the inelastic $N\to\Delta$ and the elastic neutron form factor ratios are related. The theoretical uncertainty of this relation is mainly due to third order SU(6) symmetry breaking terms (three-quark currents) omitted in Eq. (1). We estimate it to be of order $1/N_c^2$ or 10% (slightly increasing the predicted $C2/M1$ ratio).

To check whether Eq. (6) is satisfied by the data, we calculated the ratio $\mathcal{R}_n(\exp)$ using experimental results [29,30] for G_C^n/G_M^n in the range $Q^2 = 0$–0.45 GeV^2 and compared it with $C2/M1$ data [24,31–33] from pion-production experiments (see Table I). We found the agreement between both data sets to be astonishingly good [34,39]. In particular, in the real photon limit $Q\to 0$ we obtained

$$\frac{C2}{M1}(0) = -\frac{M_\Delta^2 - M_N^2}{2M_\Delta}\frac{M_N}{12}\frac{r_n^2}{\mu_n} = -0.031 \qquad (7)$$

in good agreement with the experimental $E2/M1$ ratio obtained from pion photoproduction by different groups [24,31,40,41]. This result explains the experimental value for the $C2/M1$ ratio in terms of the charge radius and the magnetic moment of the neutron. We understand therefore why $C2/M1(0) = -0.03$.

In the following, we will see that the range of validity of Eq. (6) is not confined to low Q^2 but extends to the highest momentum transfers for which both ratios have been measured. In order to show that it is valid at higher momentum transfers, I use recent G_C^n/G_M^n data between $Q^2 = 0.45$–1.45 GeV^2 from double polarization experi-

TABLE I. The ratio $\mathcal{R}_n(\exp)$ from the elastic neutron form factor data is compared with the ratio $C2/M1(\exp)$ extracted from pion-electroproduction experiments for $Q^2 = 0$–4 GeV^2. A two-parameter fit of the experimental data for $\mathcal{R}_n$ using Eq. (8) with $a = 0.9$ and $d = 2.8$ [34] is also listed.

Q^2	$\mathcal{R}_n(\exp)$	$C2/M1(\exp)$	$\mathcal{R}_n$
0.00	$-0.031(01)$ [16]	$-0.030(03)$ [24]	-0.031
		$-0.025(02)$ [31]	
0.15	$-0.050(11)$ [29]	$-0.055(04)$ [32]	-0.047
0.29	$-0.068(10)$ [29]	$-0.064(21)$ [33]	-0.054
0.45	$-0.053(06)$ [30]	$-0.075(15)$ [33]	-0.059
0.67	$-0.059(12)$ [35]	$-0.066(06)$ [6]	-0.064
1.13	$-0.059(05)$ [30]	$-0.079(09)$ [6]	-0.068
1.45	$-0.077(07)$ [30]	$-0.077(16)$ [6]	-0.069
1.80	-0.058 [36]	$-0.116(31)$ [6]	-0.070
2.80	-0.061 [36]	$-0.060(10)$ [37]	-0.070
3.25	$-0.066(30)$ [38]		-0.070
4.00	$-0.078(43)$ [38]	$-0.110(10)$ [37]	-0.069
12.00			-0.065
∞			-0.061

ments involving both electron and hadron spin polarization [30,35], calculate $\mathcal{R}_n(\text{exp})$, and compare it with $C2/M1$ data [6] at nearly the same momentum transfers (see Table I). Considering the experimental uncertainties of both experiments the agreement between $\mathcal{R}_n(\text{exp})$ and $C2/M1(\text{exp})$ is good.

At still higher momenta $Q^2 = 1.8$–4.0 GeV^2, I employ a recent fit to the experimental results for all four nucleon form factors [36] and the SLAC data [38] for the neutron elastic form factors, and calculate $\mathcal{R}_n(\text{exp})$. This is then compared with the electroproduction data $C2/M1(\text{exp})$ [6,37]. Table I shows that Eq. (6) is satisfied within the experimental uncertainty.

In order to interpolate between experimental values and to extrapolate to higher Q^2, I also calculate the ratio $\mathcal{R}_n$ of Eq. (6) (fourth column of Table I) using for the numerator a two-parameter fit [42] of the G_C^n data and for the denominator the dipole fit G_D for G_M^n, i.e.,

$$G_C^n(Q^2) = -\mu_n \frac{a\tau}{1+d\tau} G_D(Q^2), \quad G_M^n(Q^2) = \mu_n G_D(Q^2),$$

(8)

where $\tau = Q^2/(4M_N^2)$ and $G_D = (1 + Q^2/\Lambda^2)^{-2}$ with $\Lambda^2 = 0.71$ GeV^2. The $C2/M1$ ratio is then given in terms of the parameters a and d, which have been determined from the lowest moments of the experimental neutron charge form factor, namely, the neutron charge radius r_n^2, and the fourth moment r_n^4 (see Ref. [34]).

In Fig. 2 we plot $\mathcal{R}_n$ calculated from G_C^n/G_M^n data using Eq. (8) and compare with $C2/M1$ data from various pion-electroproduction experiments [6,32,33,37,43]. The solid and dashed-dotted lines correspond to two different determinations of the parameter d. Note the approximate constancy of the ratio $\mathcal{R}_n$ which is mirrored by the approximate constancy of the $C2/M1$ data over a wide
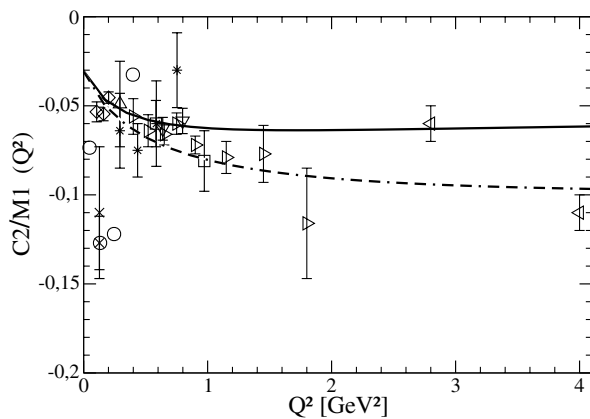

FIG. 2. The ratio $\mathcal{R}_n$ of Eq. (6) calculated from a two-parameter fit of elastic neutron form factor data according to Eq. (8). Solid curve for $a = 0.9$ and $d = 2.8$, and dashed-dotted curve for $a = 0.9$ and $d = 1.75$ [34]. This is compared with experimental results for the $C2/M1$ ratio extracted from pion-electroproduction cross sections [6,32,33,37,43].

range of momentum transfers. From Table I and Fig. 2 we conclude that the equality of the inelastic and elastic form factor ratios predicted by our Eq. (6) is obeyed by the data for momentum transfers between 0 and 4 GeV^2. This means that the quark-antiquark degrees of freedom, which give rise to a nonzero r_n^2 and $Q_{N\to\Delta}$, also determine the corresponding form factors at higher Q^2. It would be interesting to test the predicted constancy of this ratio at even higher momentum transfers. Work in this direction is in progress [44].

Finally, we extrapolate our result to $Q^2 \to \infty$ and check whether $\mathcal{R}_n(Q^2)$ is consistent with the perturbative QCD prediction for the asymptotic behavior of the $C2/M1(Q^2)$ ratio. From Eq. (6) I obtain using Eq. (8)

$$\mathcal{R}_n(Q^2 \to \infty) = \frac{1}{4}\frac{M_N}{M_\Delta}\left(-\frac{a}{d}\right) = -0.061.$$

(9)

Thus, we see that the $C2/M1$ ratio asymptotically approaches a small negative constant determined by the neutron structure parameters a and d. This is in qualitative agreement with expectations from perturbative QCD [8] modulo logarithmic corrections.

Having gained some confidence in the validity of Eq. (3) from low to high Q^2, we can Fourier transform it into coordinate space [45]. The resulting quadrupole transition charge density $\rho_{C2}^{N\to\Delta}(r)$ might be useful for future studies of the geometrical shape of the nucleon.

In summary, recent measurements of the elastic neutron form factor ratio G_C^n/G_M^n and the $C2/M1$ ratio in the electromagnetic $N \to \Delta$ transition show a remarkable agreement in sign and magnitude. This is true not only at $Q^2 = 0$ where $C2/M1$ is determined by the neutron charge radius and magnetic moment but for the entire range of four-momentum transfers where data are available. In addition, the asymptotic $C2/M1$ ratio predicted on the basis of the G_C^n/G_M^n ratio approaches a small negative constant in agreement with perturbative QCD.

According to our theory, both ratios are related due to the underlying spin-flavor symmetry and its breaking by spin-dependent two- and three-quark currents.

The main conclusion of this paper is the observation that the two data sets, which hitherto were thought to be quite independent of each other, satisfy the proposed relation Eq. (6) within experimental uncertainties. This finding suggests that one can gain information concerning the geometric shape of the nucleon not only from the inelastic electron scattering cross section, but also from the elastic neutron form factor data. Conversely, one can determine the elastic neutron charge form factor from the $N \to \Delta$ charge quadrupole form factor extracted from pion-electroproduction data.

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