CP Violation from Five-Dimensional QED

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It is shown that QED in (1 + 4)-dimensional space-time, with the fifth dimension compactified on a circle, is, in general, a *CP* violating theory. Depending on the fermionic boundary conditions, *CP* violation may be either explicit (through the Scherk-Schwarz mechanism) or spontaneous (via the Hosotani mechanism). The fifth component of the gauge field acquires (at the one-loop level) a nonzero vacuum expectation value which, in the presence of two fermionic fields, leads to spontaneous *CP* violation when the boundary conditions are *CP* symmetric. Phenomenological consequences are illustrated by a calculation of the electric dipole moment for the fermionic zero modes.

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Introduction.—The physics of grand unified theories has been plagued by fundamental difficulties to accommodate different mass scales within a single theory, the so-called hierarchy problem. For a long time supersymmetric models had the commendable feature of being able to solve this problem. More recently, nonsupersymmetric higher-dimensional models were proposed [1,2], which solve the hierarchy problem provided that an appropriate space-time geometry is realized.

Though in the original models only gravity was present outside a 4-dimensional slice of the compactified space, this is not an inescapable restriction. In fact, models where all fields propagate throughout the compactified space-time are natural and phenomenologically viable [3,4]. In this Letter we consider quantum electrodynamics (QED) in 5 dimensions (5D) focusing on the possibility that it naturally generates small but nontrivial *CP*-violating effects. For earlier attempts to obtain *CP* violation within extra-dimensional extensions of the standard model (SM) of electroweak interactions, see Refs. [5,6].

The model.—We consider an Abelian model in 5D, with coordinates x^M, $M = 0, \ldots, 4$, and $x^4 = y$ compactified on a circle of radius L. We assume the presence of two fermionic fields ($\psi_{1,2}$) interacting with the U(1) gauge field A_M according to the Lagrangian

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{MN}^2 + \sum_{i=1,2}\bar{\psi}_i(i\gamma^M D_M - M_i)\psi_i + \mathcal{L}_{gf}, \tag{1}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ and $D_M = \partial_M + ie_5 q_i A_M$, q_i denotes the charge of ψ_i in units of e_5, and $\mathcal{L}_{\text{gf}}$ stands for a gauge-fixing term. We assume that the gauge fields are periodic in y, but we allow the fermions to obey "twisted" boundary conditions (BC):

$$\psi_i(x^\mu, y + L) = T[\psi_i(x^\mu, y)] \equiv e^{i\alpha_i}\psi_i(x^\mu, y), \tag{2}$$

where x^μ, $\mu = 0, \ldots, 3$, denote the coordinates of the 4D Minkowski space-time ($\mathcal{M}_4$) and T is the twist operator. We also assume that the fermionic mass parameters M_i are positive and choose a convention where the Dirac matrices γ^M in 5D are the usual ones for $M \neq 4$ while $\gamma_{M=4} = i\gamma_5$; our choice of metric is $\mathrm{diag}(1, -1, -1, -1, -1)$.

The action is invariant under the local U(1) transformation:

$$\begin{aligned} \psi_i(x, y) &\to e^{-ie_5 q_i \Lambda(x,y)}\psi_i(x, y), \\ A_M(x, y) &\to A_M(x, y) + \partial_M\Lambda(x, y). \end{aligned} \tag{3}$$

In addition $\mathcal{L}_{\text{QED}}$ is symmetric under the 5D *CP* transformations [7]:

$$x^M \to \epsilon^M x^M, \qquad A^M \to -\epsilon^M A^M, \qquad \psi_i \to \eta_i\gamma^0\gamma^2\psi_i^\star, \tag{4}$$

where $|\eta_i| = 1$, $\epsilon^{0,4} = -\epsilon^{1,2,3} = +1$ (no sum over M).

Expanding the fields in Fourier series leads to an infinite tower of modes propagating in $\mathcal{M}_4$,

$$\begin{aligned} \psi_i(x, y) &= \frac{1}{\sqrt{L}}\sum_{n=-\infty}^{\infty}\psi_{i,n}(x)e^{i\bar{\omega}_{i,n}y}, \\ A^M(x, y) &= \frac{1}{\sqrt{L}}\left[\sum_{n=-\infty}^{\infty}A_n^M(x)e^{i\omega_n y} - a\delta_4^M\right], \end{aligned} \tag{5}$$

where $\omega_n = 2\pi n/L$, $\bar{\omega}_{i,n} = \omega_n + \alpha_i/L$, and $[A_n^M(x)]^\star = A_{-n}^M(x)$. The modes associated with $A_{M=4}$ become 4D scalars, which raises the interesting possibility that A_4 may acquire a nonzero vacuum expectation value; this, in fact, is known to occur [8]. In this case (4) suggests that this is also a sign of spontaneous *CP* violation [6], an expectation that is indeed confirmed, as we see below.

For $\alpha_i \neq 0, \pm\pi$ the BC (2) are not symmetric under CP; this is an *additional* (explicit) source of CP violation (present even if $\langle A_4 \rangle = 0$). Since the twist operator T does not commute with CP, this type of CP violation provides an example of the Scherk-Schwarz mechanism [9].

Let us first focus on the fermionic piece of the Lagrangian (1). Integrating over the y coordinate we find

$$\begin{aligned}\mathcal{L}_\psi = {} & \sum_{i,n} \bar{\psi}_{i,n}[i\gamma^\mu \partial_\mu - M_i + i\gamma_5 \mu_{i,n}]\psi_{i,n} \\ & - e\sum_{i,l,n} q_i \bar{\psi}_{i,l}(\not{A}_{l-n} + iA^4_{l-n}\gamma_5)\psi_{i,n}, \end{aligned} \tag{6}$$

where $\mu_{i,n} \equiv [2\pi n + (\alpha_i + eq_i La)]/L$, with $e \equiv e_5/\sqrt{L}$ the 4D gauge coupling. In order to diagonalize the fermion mass term we define the angles $\theta_{i,n}$ by

$$\tan(2\theta_{i,n}) = \mu_{i,n}/M_i; \qquad |\theta_{i,n}| \leq \pi/4, \tag{7}$$

and replace [10] $\psi_{i,n} \to \exp(i\gamma_5 \theta_{i,n})\psi_{i,n}$. From this we find that the physical fermion masses are $m_{i,n} = (M_i^2 + \mu_{i,n}^2)^{1/2}$, while the interactions with the gauge fields read

$$\begin{aligned} \mathcal{L}_{\varphi\psi} &= e\sum_{i,k} q_i \Big\{ \sum_{l\neq k} A^4_{k-l} \bar{\psi}_{i,k} \Gamma^{(s)}_{i,kl} \psi_{i,l} - \varphi \bar{\psi}_{i,k} \Gamma^{(s)}_{i,kk} \psi_{i,k} \Big\}, \\ \mathcal{L}_{A\psi} &= -e\sum_{i,k} q_i \Big\{ \bar{\psi}_{i,k} \not{A} \psi_{i,k} + \sum_{l\neq k} \bar{\psi}_{i,k} \Gamma^{(v)}_{i,kl} \not{A}_{k-l} \psi_{i,l} \Big\}, \end{aligned} \tag{8}$$

where $\varphi \equiv -A^4_{n=0}$, $A^\mu \equiv A^\mu_{n=0}$, $\not{A}_n = \gamma_\mu A^\mu_n$, and

$$\Gamma^{(s)}_{i,kl} \equiv -i\gamma_5 e^{i\gamma_5(\theta_{i,k}+\theta_{i,l})}, \qquad \Gamma^{(v)}_{i,kl} \equiv e^{i\gamma_5(\theta_{i,k}-\theta_{i,l})}. \tag{9}$$

A_μ clearly corresponds to the 4D photon, while φ is a new, physical, low-energy degree of freedom. Though the φ Yukawa couplings (as those of $A^N_{n\neq 0}$) appear to be CP violating, we show that is in general not the case.

The 5D gauge transformation (3) in terms of Kaluza-Klein (KK) modes implies $A^4_k \to A^4_k + i\omega_k \Lambda_k$, where $\Lambda(x,y) = L^{-1/2}\sum_{n=-\infty}^{+\infty} \Lambda_n(x) e^{i\omega_n y}$, which shows that, while $A^4_{k\neq 0}$ can be removed by an appropriate gauge choice, φ is a gauge singlet [11]; because of this, even though its mass m_φ vanishes at tree level, it receives calculable finite corrections at higher orders in perturbation theory.

It is worth noting that even if by a choice of gauge A_4 is a function of x only, there still remains a residual y-dependent discrete gauge freedom:

$$A_4 \to A_4 + \frac{2\pi n_i}{e_5 q_i L}, \quad \psi_i \to e^{-i(2\pi n_i/L)y}\psi_i, \quad n_i = 0, \pm 1, \ldots, \tag{10}$$

provided q_1/q_2 is a rational number. In this case there exist some discrete constant values of φ [$= 2\pi n_i/(e_5 q_i L)$] that can be removed completely. Note also that $\alpha_i + e_5 q_i A_4$ is invariant under (10); this will be relevant when we discuss the one-loop effective potential for $\langle \varphi \rangle$.

The effective potential.—The above discussion raises the possibility that φ will acquire a nonvanishing vacuum expectation value $a \equiv \langle \varphi \rangle$. In order to determine the conditions under which this occurs, we evaluate the corresponding effective potential to one loop. We adopt dimensional regularization for the d^4p integral together with a summation over the infinite tower of KK modes. After dropping an irrelevant constant the contribution from one fermion equals

$$\begin{aligned} V(M;\omega) = {} & [x^2 Li_3(z) + 3x Li_4(z) + 3Li_5(z) \\ & + \text{H.c.}]/(2\pi^4 L^4), \end{aligned} \tag{11}$$

where $x \equiv LM$, $\omega = (\alpha + eq_\psi La)/L$, $z = \exp(iL\omega - x)$, and $Li_n(x)$ is the polylogarithm function. Note that $V(M;\omega) = V(M;-\omega)$ as a consequence of the Hermiticity. For a theory with fermions the total effective potential reads

$$V_{\text{eff}}(a) = \sum_{i=1,2} V(M_i;\omega_i). \tag{12}$$

This is periodic in a only when q_1/q_2 is a rational number n_1/n_2 [the period then equals $2\pi n_1/(eq_1 L)$], as a result of the residual gauge invariance (10).

If we had just one fermionic field ψ the minimum of V is at $L\omega = \pi(2l+1)$ for integer l. Since $\alpha = \alpha \text{mod} 2\pi$ we can choose the minimum $\alpha + eq_\psi La = \pi$. We can eliminate α from (2) by the following field redefinition:

$$\psi' = e^{-i\alpha y/L}\psi, \qquad A'_M = A_M + \alpha\delta_{M,4}/(e_5 q_\psi), \tag{13}$$

so that ψ' and A' are periodic in y with period L. We then expand around the vacuum $e_5 q_\psi \langle A'_M \rangle = (\pi/L)\delta_{M4}$ by shifting the gauge field $e_5 q_\psi A'_M \to e_5 q_\psi A'_M + (\pi/L)\delta_{M4}$ and again redefine the fermion fields, so that the effect of this shift disappears from the Lagrangian density: $\chi = \exp(i\pi y/R)\psi$ which is antiperiodic in y. This shows that the original theory is equivalent to one where the gauge field has a vanishing vacuum expectation value (no spontaneous CP violation) and the BC are CP invariant; consequently, this theory does not generate CP violation. However, in (8) we have noted the presence of CP-violating couplings of φ, A^N_n even if only one fermion is present; this therefore deserves further explanation.

At the minimum of V, $\mu_n = \pi(2n+1)/L$ and $m_n = m_{-n-1}$, so that any unitary transformation U acting on the (ψ_n, ψ_{-n-1}) subspace leaves the kinetic terms invariant and allows the generalized CP transformation:

$$\psi_i \overset{CP}{\to} U_{ij} C \psi_j^*, \qquad i,j = n, -n-1, \tag{14}$$

where $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Choosing $U = \sigma^1$ (the usual Pauli matrix) one can see that the $\psi_{n,-n-1}$ couplings are invariant under (14).

The situation can be different if a second fermion is present. Then, following the steps described above for the case of a single fermion, it can be shown that we can assume the BC $\psi_1(y+L)=\psi_1(y)$, $\psi_2(y+L)=\exp(i\alpha)\psi_2(y)$. The condition for an extremum $\partial V_{\rm eff}/\partial a=0$ leads to a CP-conserving vacuum when the minimum of $V_{\rm eff}$ is at $\omega_i=0,\ \pi/L$. In general, however, the minimum is located elsewhere, thus allowing spontaneous CP violation: *at least two fermions are necessary to observe CP violation in 5D QED compactified on a circle.* In this case the KK modes of both fermions have CP-violating Yukawa couplings (8). In Fig. 1 we plot $V_{\rm eff}$ as a function of ea for various choices of α.

When $\alpha=0,\ \pi$ $V_{\rm eff}$ is symmetric in a (due to the symmetry of V under $\omega\to-\omega$). In this case the Lagangian and BC are CP symmetric. Under CP, $a\to-a$, and therefore this symmetry is a consequence of CP invariance and choosing any of the two degenerate vacua leads to spontaneous CP violation. For other (CP asymmetric) choices of α, $V_{\rm eff}$ is not symmetric in a and CP is explicitly violated.

Phenomenology.—The most striking consequence of CP violation in our model is the prediction of a nonzero fermionic electric dipole moment (EDM) d defined through the following effective $\gamma\bar{\psi}\psi$ vertex: $\langle p'|j^{\mu}_{\rm EM}|p\rangle=-(d/e)\bar{u}(p')\sigma^{\mu\nu}\gamma_5(p'-p)_\nu u(p)$, where p, p' are on shell and the limit $p'\to p$ is assumed. This nonzero EDM is generated already at the one-loop level (in the SM at least three loops are required). For the fermion ψ_i, the diagram involving φ yields

$$d_{i,0}=-\frac{(eq_i)^3c^{(+)}_{i,0}}{16\pi^2m_{i,0}}J^{(s)}(m^2_\varphi/m^2_{i,0},1), \tag{15}$$

while loops containing the A^N_n modes equal

$$\begin{aligned} d^{(v)}_{i,n}&=\frac{(eq_i)^3c^{(-)}_{i,n}}{4\pi^2}\frac{m_{i,n}}{m^2_{i,0}}J^{(v)}(x_{i,n},y_{i,n})(A^\mu_n),\\ d^{(s)}_{i,n}&=-\frac{(eq_i)^3c^{(+)}_{i,n}}{16\pi^2}\frac{m_{i,n}}{m^2_{i,0}}J^{(s)}(x_{i,n},y_{i,n})(A^4_n), \end{aligned} \tag{16}$$

where $c^{(\pm)}_{i,n}=\pm M_i(\mu_{i,n}\pm\mu_{i,0})/(m_{i,n}m_{i,0})$, $x_{i,n}=(\omega_n/m_{i,0})^2$, $y_{i,n}=(m_{i,n}/m_{i,0})^2$, and

$$\begin{aligned} J^{(s)}(x,y)&=1+\frac{x-y+1}{2}\ln\Big(\frac{y}{x}\Big)+\Big(\frac{2x}{\rho}-\rho\Big)\Theta,\\ J^{(v)}(x,y)&=-1+\frac{y-x}{2}\ln\Big(\frac{y}{x}\Big)+(\rho-\cot\Theta)\Theta \end{aligned} \tag{17}$$

with $\rho^2\equiv 4xy-(x+y-1)^2$ and $\tan\Theta\equiv\rho/(x+y-1)$.

The total EDM of the ith zero-mode fermion is then

$$\begin{aligned} d_i=d_{i,0}+\frac{(eq_i)^3}{16\pi^2m^2_{i,0}}\sum_{n\neq0}m_{i,n}[4c^{(-)}_{i,n}J^{(v)}(x_{i,n},y_{i,n})\\ -c^{(+)}_{i,n}J^{(s)}(x_{i,n},y_{i,n})]. \end{aligned} \tag{18}$$

Note that, for large n, $m_{i,n}c^{(\pm)}_{i,n}J^{(s/v)}(x_{i,n},y_{i,n})\sim 1/n+\mathcal{O}(1/n^2)$, so that (18) will be finite after symmetric summation over n. It follows that the EDM is finite and therefore insensitive to the cutoff of the 5D theory.

In Fig. 2 we plot the fermionic EDM for zero mode of the $i=1$ fermion as a function of the compactification scale L. We chose $\alpha=0$ (the case of spontaneous CP violation) and selected the positive vacuum expectation value of A_4 (see Fig. 1); the EDM, as a CP-odd quantity, would change sign if the other (negative) vacuum expectation value is chosen. The parameters have been adjusted in such a way that the model has mass scales and coupling constants of the same order as those in the SM. Note that the mass of the zero mode $m_{i=1,n=0}$ depends on L; for our choice of parameters it varies from $\sim$37 TeV for $L=0.1\ \mathrm{TeV}^{-1}$ to $\sim$1.5 TeV when $L=2.5\ \mathrm{TeV}^{-1}$; for these

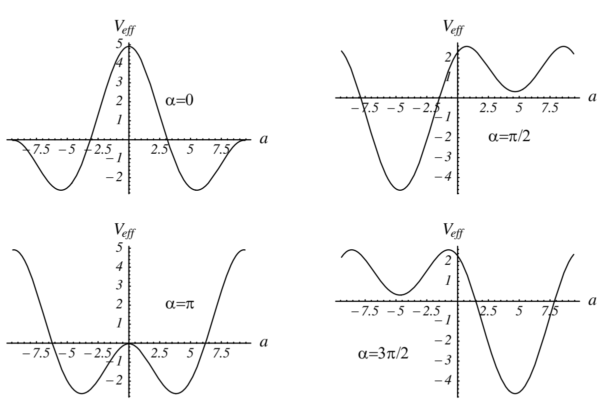

FIG. 1. The effective potential $V_{\rm eff}$ (in units of 10^{-3} TeV^4) as a function of a (in units of TeV) for $L^{-1}=0.3$ TeV, $M_1=0.2$ TeV, $M_2=0.005$ TeV, $q_1=2/3$, $q_2=-1/3$, and four choices of the twist angle $\alpha=0,\ \pi/2,\ \pi,\ 3\pi/2$.

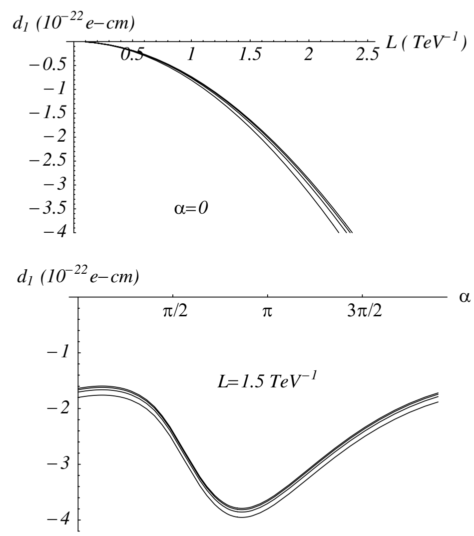

FIG. 2. Left: the fermionic EDM, $d_1(L)$ for $\psi_{i=1,n=0}$ for $\alpha=0$. Right: the same, as a function of α for $L=1.5\ \mathrm{TeV}^{-1}$. Curves correspond to the number of modes included in (18), from $|n|=1$ (bottom) to $|n|=4$ (top), indicating fast convergence. Note that the mass of the zero mode also varies with L. We used $e=\sqrt{4\pi\alpha_{\rm QED}}$ and the same parameters as in Fig. 1.

values, m_φ ranges from $\sim$27 to $\sim$1 GeV. The leading contribution to d_i comes from the φ exchange; the contribution of the nonzero modes is of opposite sign and smaller by a factor of $\mathcal{O}(5)$.

Conclusions.—We have shown that QED in $(1 + 4)$-dimensional space-time, with the fifth dimension compactified on a circle, leads to CP violation. Depending on fermionic boundary conditions, CP violation may be either explicit, or spontaneous via the Hosotani mechanism. The new possibility of CP breaking by fermionic, twisted boundary conditions has been emphasized and demonstrated explicitly by the derivation of CP-violating effective couplings. The fifth component of the gauge field acquires (at the one-loop level) a nonzero vacuum expectation value [12]. We have shown that in the presence of two fermionic fields this leads to spontaneous CP violation in the case of CP-symmetric boundary conditions. The one-loop effective potential for A_0^4 has been calculated, and its features have been discussed in the presence of two fermionic fields.

The most striking feature of the model considered here is the presence of the light scalar φ, which has CP-violating Yukawa couplings similar to those present in the scalar sector of the 2-Higgs-doublet model. The presence of CP-violating couplings leads to a nonzero EDM, which was calculated at the one-loop level for a zero-mode fermion. This effect can be used to test the mechanism for CP violation present in our model.

There are several other observables, originally developed to investigate extended Higgs sectors, which can also be used to detect the presence of a light scalar (regardless of whether its couplings conserve or violate CP) such as φ. For example, aside from the fermionic electric and magnetic dipole moments, one also has $\Gamma[\Upsilon \to \varphi\gamma]$ and $B[b \to \varphi s]$. Experimental constraints on all such quantities would impose some restrictions on the parameters of the model. We will present the results of such an investigation in a separate publication, where we will consider a more realistic non-Abelian theory.

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[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[2] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis *et al.*, Phys. Lett. B **436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, and J. March-Russell, Phys. Rev. D **63**, 064020 (2001).

[3] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).

[4] T. Appelquist, H. C. Cheng, and B. A. Dobrescu, Phys. Rev. D **64**, 035002 (2001).

[5] G. C. Branco, A. de Gouvea, and M. N. Rebelo, Phys. Lett. B **506**, 115 (2001); D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **87**, 211601 (2001); D. Chang, W. Y. Keung, and R. N. Mohapatra, Phys. Lett. B **515**, 431 (2001); C. S. Huang *et al.*, Eur. Phys. J. C **23**, 195 (2002); J. Kubo and H. Terao, Phys. Rev. D **66**, 116003 (2002); D. Dooling, D. A. Easson, and K. Kang, J. High Energy Phys. 07 (2002) 036; G. Burdman, Phys. Lett. B **590**, 86 (2004); D. Chang *et al.*, Phys. Rev. D **70**, 096010 (2004); R. Horvat, M. Krcmar, and B. Lakic, Phys. Rev. D **69**, 125011 (2004); K. R. Dienes, E. Dudas, and T. Gherghetta, in *Proceedings of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)*, edited by N. Graf, eConf C010630, P349 (2001); M. Chaichian and A. B. Kobakhidze, Phys. Rev. Lett. **87**, 171601 (2001); E. Ma, M. Raidal, and U. Sarkar, Phys. Lett. B **504**, 296 (2001); L. Di Lella *et al.*, Phys. Rev. D **62**, 125011 (2000); K. R. Dienes, E. Dudas, and T. Gherghetta, Phys. Rev. D **62**, 105023 (2000); S. Chang, S. Tazawa, and M. Yamaguchi, Phys. Rev. D **61**, 084005 (2000); S. Y. Khlebnikov and M. E. Shaposhnikov, Phys. Lett. B **203**, 121 (1988).

[6] N. Cosme, J.-M. Frère, and L. Lopez Honorez, Phys. Rev. D **68**, 096001 (2003); N. Cosme and J.-M. Frère, Phys. Rev. D **69**, 036003 (2004).

[7] K. Shimizu, Prog. Theor. Phys. **74**, 610 (1985); M. B. Gavela and R. I. Nepomechie, Classical Quantum Gravity **1**, L21 (1984).

[8] Y. Hosotani, Phys. Lett. **126B**, 309 (1983); **129**, 193 (1983).

[9] J. Scherk and J. H. Schwarz, Phys. Lett. **82B**, 60 (1979); Nucl. Phys. **B153**, 61 (1979).

[10] The chiral rotation of the fermions induces an $\tilde{F}_{\mu\nu}F^{\sigma\rho}$ term in the Lagrangian which can be ignored in the Abelian case considered here.

[11] This is a consequence of a compact x^4 direction; in an uncompactified space one could always choose the $A^4(x, y) = 0$ gauge. For compactification on the orbifold S^1/Z_2, φ disappears as a consequence of the requirement that A_4 be antisymmetric under Z_2. Though CP cannot be the spontaneously violated, the BC can still generate CP violation of the gauge fields to the fermions.

[12] The scalar $\varphi(x)$ is massless at tree-level massless, so our results provide an illustraion of the so-called Georgi-Pais theorem [H. Georgi and A. Pais, Phys. Rev. D **10**, 1246 (1974)] that precludes spontaneous symmetry breaking by radiative corrections unless the scalar potential has a flat direction at tree-level.