Exact Scale Invariance of Composite-Field Coupling Constants

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We show that the coupling-constant of a quantum-induced composite field is scale invariant due to its compositeness condition. It is first demonstrated in next-to-leading order in $1/N$ in typical models, and then we argue that it holds exactly.

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Composite fields induced by quantum fluctuations play important roles in wide areas of physics. The Cooper pair, or the order parameter in the Landau-Ginzberg theory of superconductivity, can be taken as such a composite of elementary fields in the system. The ideas are also successfully used to describe properties of hadron dynamics in the Nambu–Jona-Lasinio (NJL) model [1]. They are further applied to induced gauge theories [2], induced gravity [3], composite models of quarks, leptons, gauge bosons, and Higgs scalars [4,5], various collective motions in nuclei and solid states, brane-induced gravity and field theories [6], etc. The theories of the quantum-induced composites are not renormalizable in many cases. They can, however, be formulated as the special case of some renormalizable theory with the compositeness condition (CC) [7,8], which says that $Z = 0$, where Z is the renormalization constant of the to-be-composite field. For example, with CC, the Yukawa model for elementary fermions and bosons reduces to the NJL model with elementary fermions and composite bosons.

In spite of extensive studies in their long history, it is not clear what happens in the infinite cutoff limit, or equivalently, in the integral dimension limit of these nonrenormalizable theories [7–9]. In fact, in any of the known perturbative treatments, the induced composite coupling constants vanish in this limit. Here we do not adhere to this difficult problem. Instead we consider nonlimiting cases by fixing the number of spacetime dimensions $d = d_0 - 2\epsilon$ at some value close to but different from the integral number of physical dimensions d_0 (=4 or 6 below). We interpret that it simulates existing finite cutoffs in various physical systems in Nature. We adopt the minimal subtraction scheme, where the poles in ϵ are retained in the renormalization constants. Note that, in this scheme, the renormalization group is well defined even for the nonlimiting case $\epsilon \neq 0$.

In this Letter, we show the following fact: In a composite-field theory which is equivalent to some renormalizable field theory under CC, the induced coupling constant of the composite field is exactly scale invariant by virtue of the CC itself. We previously demonstrated it in some models at the leading order in $1/N$ where N is the number of the elementary matter species [10]. Here we first show it to next-to-leading order in $1/N$ and to leading order in ϵ with $g^2 \sim \epsilon/N$ in three typical models. Then we argue that the scale invariance holds exactly [11]. It is remarkable that the awkward nonrenormalizable theories bury such a high symmetry in their depth. The compositeness, i.e., the absence, of its elementary degree of freedom protects the coupling constants from flowing with the scale parameter.

Scalar composite in six dimensions.—We consider a system of $2N$ complex scalar fields $\chi_0^i = (\chi_{01}^i, \ldots, \chi_{0N}^i)$ ($i = 1, 2$) with mass m_0 in six dimensions:

$$\mathcal{L} = \Sigma_i(|\partial_\mu \chi_0^i|^2 - m_0^2|\chi_0^i|^2) - F|(\chi_0^1\chi_0^2)|^2, \quad (1)$$

where F is a coupling constant. This is one of the simplest models that realize our present idea. Naively we can see that the chain of χ_0^i loop diagrams gives rise to their composite pole in the total momentum square of the channel. It is systematically described in the following way. The Lagrangian (1) is equivalent to

$$\mathcal{L}' = \Sigma_i(|\partial_\mu \chi_0^i|^2 - m_0^2|\chi_0^i|^2) - |\Phi|^2/F + \Phi(\chi_0^1\chi_0^2) + \text{H.c.}, \quad (2)$$

where Φ is an auxiliary field. We compare this with the renormalizable model for χ_0^i and an elementary complex scalar ϕ_0 with mass M_0 and a coupling constant g_0:

$$\tilde{\mathcal{L}} = \Sigma_i(|\partial_\mu \chi_0^i|^2 - m_0^2|\chi_0^i|^2) + |\partial_\mu \phi_0|^2 - M_0^2|\phi_0|^2 + g_0\phi_0(\chi_0^1\tilde{\chi}_0^2) + \text{H.c.} \quad (3)$$

We renormalize χ_0^i, ϕ_0, m_0, M_0, and g_0 with the renormalization constants Z_1, Z_2, Z_3, Z_m, and Z_M and the renormalized quantities χ, ϕ, m, M, and g: $\chi_0^i = \sqrt{Z_2}\chi^i$, $\phi_0 = \sqrt{Z_3}\phi$, $\sqrt{Z_2}m_0 = \sqrt{Z_m}m$, $\sqrt{Z_3}M_0 = \sqrt{Z_M}M$, and

$$Z_2\sqrt{Z_3}g_0 = Z_1 g\mu^\epsilon, \quad (4)$$

where a mass parameter μ is introduced to make the coupling constant g dimensionless. We can directly see that (5) entirely coincides with (2) if

$$Z_3 = 0, \quad Z_1 \neq 0, \quad Z_2 \neq 0, \quad Z_m \neq 0, \quad Z_M \neq 0, \quad (5)$$

and if we identify $Z_2\Phi$ with $Z_1 g\mu^\epsilon\phi$ and F with

$Z_1^2 g^2 \mu^{2\epsilon}/Z_2^2 Z_M M^2$. Thus we can calculate the physical quantities in the system with (1) at nonvanishing ϵ via well understood Lagrangian (3) with the condition (5), which is called "compositeness condition." Equation (4) indicates that the CC (5) is equivalent to

$$g_0 \to \infty, \quad Z_1 \neq 0, \quad Z_2 \neq 0, \quad Z_m \neq 0, \quad Z_M \neq 0. \tag{6}$$

After calculations, we get [with $I = 1/6(4\pi)^3\epsilon$]

$$Z_1 = 1, \qquad Z_2 = 1 + N^{-1}\ln(1 - Ng^2 I), \tag{7}$$

$$Z_3 = 1 - (N+2)g^2 I - 2N^{-1}(1 - Ng^2 I)\ln(1 - Ng^2 I) \tag{8}$$

to next-to-leading order in $1/N$ and leading order in ϵ with $g^2 \sim \epsilon/N$. Then, the renormalization group beta function is calculated as

$$\beta \equiv \mu \partial g/\partial \mu = -\epsilon g + (N+2)g^3/6(4\pi)^3. \tag{9}$$

The differential equation (9) is solved to give the running coupling constant

$$\begin{aligned} g^2 &= [(N+2)/6(4\pi)^3\epsilon + \mu^{2\epsilon}/g_0^2]^{-1} \\ &= [a - (N+2)\ln\mu^2/6(4\pi)^3]^{-1} + O(\epsilon), \end{aligned} \tag{10}$$

where the integration constant is chosen in accordance with (4), and $a = \lim_{\epsilon\to 0}\{(N+2)I + 1/g_0^2\}$.

With CC (6), the μ-dependent part in (10) disappears, and (10) reduces to the scale invariant form

$$g^2 = 6(4\pi)^3\epsilon/(N+2). \tag{11}$$

In fact, (11) is the solution of CC (5) with Z_3 in (8), and it implies $\beta = 0$ within the present approximation. Thus, here, the compositeness implies the scale invariance of the induced coupling constant.

Nambu–Jona-Lasinio model in four dimensions.—We consider a system of N fermions $\psi_0 = (\psi_{01}, \ldots, \psi_{0N})$ in four dimensions with the Lagrangian

$$\mathcal{L} = \bar{\psi}_0 i\partial\!\!\!/ \psi_0 - F|\bar{\psi}_{0L}\psi_{0R}|^2, \tag{12}$$

where F is a coupling constant. (Note that the notations are renewed model by model.) The composite pole due to the chain of ψ_0 loops is systematically described in the following way. The Lagrangian (12) is equivalent to

$$\mathcal{L}' = \bar{\psi}_0 i\partial\!\!\!/ \psi_0 + \bar{\psi}_{0L}\Phi\psi_{0R} + \text{H.c.} - F^{-1}|\Phi|^2, \tag{13}$$

where Φ is an auxiliary field. We compare this with the Yukawa model for ψ_0 and an elementary boson ϕ_0 with mass M_0 and coupling constants g_0 and λ_0:

$$\begin{aligned} \tilde{\mathcal{L}} &= \bar{\psi}_0 i\partial\!\!\!/ \psi_0 + g_0(\bar{\psi}_{0L}\phi_0\psi_{0R} + \text{H.c.}) + |\partial_\mu\phi_0|^2 \\ &\quad - M_0^2|\phi_0|^2 - \lambda_0|\phi_0|^4. \end{aligned} \tag{14}$$

We renormalize ψ_0, ϕ_0, M_0, g_0, and λ_0 with the renormalization constants Z_1, Z_2, Z_3, Z_4, and Z_M and the renormalized quantities ψ, ϕ, M, g, and λ: $\psi_0 = \sqrt{Z_2}\psi$, $\phi_0 = \sqrt{Z_3}\phi$, $\sqrt{Z_3}M_0 = \sqrt{Z_M}M$, and

$$Z_2\sqrt{Z_3}g_0 = Z_1 g\mu^\epsilon, \qquad Z_3^2\lambda_0 = Z_4\lambda\mu^{2\epsilon}, \tag{15}$$

where the coupling constants g and λ are made dimensionless with a mass parameter μ. We can see that (14) entirely coincides with (13) if we have the CC

$$Z_3 = 0, \quad Z_4 = 0, \quad Z_1 \neq 0, \quad Z_2 \neq 0, \quad Z_M \neq 0, \tag{16}$$

and we identify F with $Z_1^2 g^2\mu^{2\epsilon}/Z_2^2 Z_M M^2$ and $Z_2\Phi$ with $Z_1 g\mu^\epsilon\phi$. Thus we can calculate the physical quantities in the system with (12) at nonvanishing ϵ via well understood Lagrangian (14) with CC (16). Equation (15) indicates that the CC (16) is equivalent to

$$g_0 \to \infty, \qquad Z_1 \neq 0, \qquad Z_2 \neq 0, \qquad Z_M \neq 0 \tag{17}$$

with arbitrary λ_0.

After calculations, we get [with $I = 1/(4\pi)^2\epsilon$]

$$Z_1 = 1, \qquad Z_2 = 1 + \ln(1 - Ng^2 I), \tag{18}$$

$$\begin{aligned} Z_3 &= 1 - (N+1)g^2 I - N^{-1}(1 - Ng^2 I)\ln(1 - Ng^2 I), \\ Z_4 &= 1 - (N-8)g^2 I/\lambda - 20(\lambda - g^2)^2 I/\lambda(1 - Ng^2 I) \\ &\quad - (N\lambda)^{-1}(20\lambda - 18g^2 - 2Ng^4 I)\ln(1 - Ng^2 I) \end{aligned} \tag{19}$$

to next-to-leading order in $1/N$ and leading order in ϵ with $g^2 \sim \epsilon/N$. Then the renormalization group beta functions are calculated as

$$\begin{aligned} \beta_g &\equiv \mu\partial g/\partial\mu = -\epsilon g + (N+1)g^3/(4\pi)^2. \\ \beta_\lambda &\equiv \mu\partial\lambda/\partial\mu = -2\epsilon\lambda + (4N\lambda g^2 - 2Ng^4)/(4\pi)^2 \\ &\quad + (40\lambda^2 - 40\lambda g^2 + 20g^4)/(4\pi)^2. \end{aligned} \tag{20}$$

The coupled differential equation (20) is solved to give the running coupling constants

$$\begin{aligned} g^2 &= 1/[(N+1)I + \mu^{2\epsilon}/g_0^2], \\ \lambda &= \frac{(N-8)I + \lambda_0\mu^{2\epsilon}/g_0^4}{[(N+1)I + \mu^{2\epsilon}/g_0^2]^2} - \frac{20(\lambda_0 - g_0^2)^2 I\mu^{2\epsilon}}{(Ng_0^2 I + \mu^{2\epsilon})^3} \\ &\quad - \frac{18(\lambda_0 - g_0^2)\mu^{2\epsilon}}{N(Ng_0^2 I + \mu^{2\epsilon})^2}\ln(1 + Ng_0^2\mu^{-2\epsilon}I), \end{aligned} \tag{21}$$

where the integration constants are chosen with (15).

With CC (17), the μ-dependent parts in (21) disappear, and (21) reduces to the scale invariant form

$$g^2 = (4\pi)^2\epsilon/(N+1), \qquad \lambda = (4\pi)^2\epsilon/(N+10). \tag{22}$$

In fact, (22) is the solution of CC (16) with Z_3 and Z_4 in (19) [8], and it implies $\beta_g = 0$ and $\beta_\lambda = 0$ within the present approximation. Thus, here, the compositeness implies the scale invariance of the induced coupling constants.

Induced gauge theory in four dimensions.—We consider the strong coupling limit $F \to \infty$ of the system of N $SU(N_c)$-N_c-plet fermions $\psi_0 = (\psi_{01}, \ldots, \psi_{0N})$ with mass m_0 in four dimensions:

$$\mathcal{L} = \bar{\psi}_0(i\not\partial - m_0)\psi_0 - F(\bar{\psi}_0 T^a \gamma_\mu \psi_0)^2, \qquad (23)$$

where T^a is the generator matrix of the fundamental representation of $SU(N_c)$, and γ_μ is the Dirac matrix. (Note that the notations are renewed model by model.) The composite pole due to the chain of ψ_0 loops is systematically described in the following way. The Lagrangian (23) is equivalent to

$$\mathcal{L}' = \bar{\psi}_0(i\not\partial - m_0)\psi_0 + \bar{\psi}_0 T^a \not\Phi^a \psi_0 - F^{-1}|\Phi^a_\mu|^2, \quad (24)$$

where Φ^a_μ is an auxiliary vector field in the adjoint representation of $SU(N_c)$. We compare this with the $SU(N_c)$ gauge theory for ψ_0 and the elementary gauge boson $G^a_{0\mu}$ with the gauge coupling constant g_0,

$$\tilde{\mathcal{L}} = \bar{\psi}_0(i\not\partial - m_0)\psi_0 + g_0\bar{\psi}_0 T^a \not G^a_0 \psi_0 - \tfrac{1}{4}(G^a_{0\mu\nu})^2, \quad (25)$$

where $G^a_{0\mu\nu}$ is the field strength of $G^a_{0\mu}$. We renormalize ψ_0, $G^a_{0\mu}$, M_0, and g_0 with the renormalization constants Z_1, Z_2, Z_3, and Z_m and the renormalized quantities ψ, G^a_μ, M, and g: $\psi_0 = \sqrt{Z_2}\psi$, $G^a_{0\mu} = \sqrt{Z_3}G^a_\mu$, $Z_2 m_0 = Z_m m$, and

$$Z_2\sqrt{Z_3}g_0 = Z_1 g \mu^\epsilon, \qquad (26)$$

where the coupling constant g is made dimensionless with a mass parameter μ. We can see that (25) entirely coincides with (24) if we have the CC

$$Z_3 = 0, \qquad Z_1 \neq 0, \qquad Z_2 \neq 0, \qquad Z_m = 0, \qquad (27)$$

and we take $Z_2\Phi^a_\mu = Z_1 g \mu^\epsilon G^a_\mu$ and $F \to \infty$. Thus we can calculate the physical quantities in the system with (23) at nonvanishing ϵ via well understood Lagrangian (25) with CC (27). Note that, for a consistent quantum description, we should and we can introduce the gauge fixing term and the Faddeev-Popov term without changing physical contents. We denote the gauge parameter by α. Equation (26) indicates that the CC is equivalent to

$$g_0 \to \infty, \qquad Z_1 \neq 0, \qquad Z_2 \neq 0, \qquad Z_m = 0, \qquad (28)$$

in terms of the bare parameters of (25).

After calculations, we obtain [with $I = 1/(4\pi)^2\epsilon$]

$$Z_1 = 1 + \frac{9N_c}{8N}\ln\left(1 - \frac{2Ng^2I}{3}\right) - \alpha g^2 I\frac{3N_c^2 - 2}{4N_c},$$
$$Z_2 = 1 - \alpha g^2 I(N_c^2 - 1)/2N_c, \qquad (29)$$

$$Z_3 = 1 - \frac{(2N - 11N_c)g^2I}{3} - \frac{\alpha N_c g^2 I}{2}\left(1 - \frac{2Ng^2I}{3}\right) + \frac{9N_c}{4N}\left(1 - \frac{2Ng^2I}{3}\right)\ln\left(1 - \frac{2Ng^2I}{3}\right) \qquad (30)$$

to next-to-leading order in $1/N$ and leading order in ϵ with $g^2 \sim \epsilon/N$. Then, the renormalization group beta function is calculated as

$$\beta \equiv \mu\partial g/\partial\mu = -\epsilon g + (2N - 11N_c)g^3/3(4\pi)^2. \quad (31)$$

The differential equation (31) is solved to give the running coupling constant

$$g^2 = [(2N - 11N_c)/3(4\pi)^2\epsilon + \mu^{2\epsilon}/g_0^2]^{-1}$$
$$\{= [a - (2N - 11N_c)\ln\mu^2/3(4\pi)^2]^{-1} + O(\epsilon)\}, \qquad (32)$$

where the integration constant is chosen with (26), and $a = \lim_{\epsilon\to 0}\{(2N - 11N_c)I/3 + 1/g_0^2\}$.

With CC (28), the μ-dependent part in (32) disappears, and (32) reduces to the scale invariant form

$$g^2 = 3(4\pi)^2\epsilon/(2N - 11N_c). \qquad (33)$$

In fact, (33) is the solution of CC (27) with Z_3 in (30), and it implies $\beta = 0$ within the present approximation. Thus, here again, the compositeness implies the scale invariance of the induced coupling constant.

We argue that the scale invariance of the composite-field coupling constants persists in all orders. The μ dependences of g's and λ in (10), (21), and (32) due to the differential equations (9), (20), and (31) originally come from the relations (4), (15), and (26). The solutions of (9), (20), and (31) are given by the algebraic solutions of Eqs. (4), (15), and (26) with Z's in (7), (8), (18), (19), (29), and (30) inserted. The μ dependences of the coupling constants as the algebraic solutions of Eqs. (4), (15), and (26) arise through the factors μ^ϵ and $\mu^{2\epsilon}$ on the right-hand side of (4), (15), and (26). With CC (6), (17), and (28), the μ-dependent parts disappear from Eqs. (4), (15), and (26) in all orders. Therefore the coupling constants g's and λ with CC are independent of the scale μ in all orders. The scale invariance of the composite-field coupling constants holds exactly.

It holds in any order as far as the expansion is proper as in the case of the $1/N$ expansion. However, the coupling-constant expansions and loop expansions are improper under CC, because CC at the lowest two orders implies that infinite higher diagrams have the same order of magnitude, and we cannot see the CC based scale invariance at any particular order in these expansions. In general, the CC can have an isolated solution that is not a limit with respect to some particular perturbative parameter. The CC based scale invariance persists even in this nonperturbative case. The composite coupling constants are entirely on the fixed point from the infrared to the ultraviolet region. They neither are asymptotically free nor blow up at some finite scale. The composite-field coupling constant has no Landau pole. So far we considered the cases where the induced coupling constants are dimensionless. It is straightforward to extend the present argument to the cases of induced coupling constants with positive mass dimensions, i.e., the composite theories equivalent to super-renormalizable theories with CC. The induced composite-field coupling constants would scale with its canonical dimensions.

The scaling properties would provide a powerful clue to discriminate between compositeness and elementariness phenomenologically. For example, if the weak bosons and Higgs scalar are quantum-induced composites [4,5], the coupling constants should be scale invariant. On the other hand, the photon and gluons cannot be quantum-induced composites in spite of several theoretical suggestions [2,4], since the running of coupling constants is supported by experiments. The elementariness of the gluon is consistent with the related theoretical indication by complementarity that asymptotically free gauge bosons cannot be a quantum-induced composite [12]. The scaling behavior would also be realized in various phenomena based on collective motions in condensed matters. When fundamental fermions are coupled to gravity, the dimensionless part of the induced composite scalar sector is known to exhibit conformal symmetry, reflecting asymptotic conformal invariance of the fermion sector in the ultraviolet region [13]. It would be an interesting challenge to inquire about the interrelations between this conformal symmetry and the scale invariance demonstrated here. The idea of the quantum-induced composite is used explicitly or implicitly in many areas from particle to cosmological physics. We expect that the scaling properties presented here would have chances to be realized in many of such systems and would elucidate varieties of phenomena in Nature.

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