Algebroid Yang-Mills Theories

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A framework for constructing new kinds of gauge theories is suggested. Essentially it consists in replacing Lie algebras by Lie or Courant algebroids. Besides presenting novel topological theories defined in arbitrary spacetime dimensions, we show that equipping Lie algebroids E with a fiber metric having sufficiently many E-Killing vectors leads to an astonishingly mild deformation of ordinary Yang-Mills theories: Additional fields turn out to carry no propagating modes. Instead, they serve as moduli parameters gluing together in part different Yang-Mills theories. This leads to a symmetry enhancement at critical points of these fields, as is also typical for String effective field theories.

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Yang-Mills (YM) theories and Lie group symmetries are part and parcel of present day fundamental physics. Both of these concepts are "nondeformable" under very mild assumptions [1]. The advent of supersymmetry, possible after changing the perspective on symmetries, is an example of the fruitfulness of enlarging that framework. In this Letter we suggest a possibly similar broadening, which in its essence replaces Lie groups by Lie groupoids in the context of Yang-Mills theories; for trivial bundles this reduces to replacing the structural Lie algebra of a YM theory by a Lie algebroid. In part, this generalization is related to rather old attempts [2] for constructing so-called "nonlinear gauge theories"; the recent mathematical understanding of Lie algebroids and groupoids [3,4], however, provides new tools and a new focus for approaching such a generalization.

In the theories under discussion generically one encounters structure functions in the symmetry algebra, typical for gravitational theories; but there will exist a finite dimensional object underlying the infinite dimensional space of symmetries: infinitesimally the symmetries are generated by sections in a Lie algebroid. Two spacetime dimensions already provide an example where these concepts have been realized successfully in terms of Poisson sigma models (PSMs) [5,6], which, on the physical side, permit one to unify gravitational and YM-gauge theories [7]. Using the PSM, as well as Chern-Simons (CS) theory defined in $d = 3$, as a guideline, we leave behind low dimensions in this Letter and, besides suggesting also possibly interesting topological models, permit theories with propagating degrees of freedom.

We briefly recall some mathematical background [8]: A Lie algebroid consists of a vector bundle $E \to M$, a bundle map $\rho : E \to TM$, and Lie-algebra brackets $[\cdot, \cdot]$ on $\Gamma(E)$ satisfying the Leibniz rule $[s_1, fs_2] = f[s_1, s_2] + (\rho(s_1)f)s_2$. For M a point this reduces to an ordinary Lie-algebra $\mathfrak{g}$, for $\rho \equiv 0$ to a bundle of Lie algebras, M then being a parameter space of Lie-algebra deformations. Given a Poisson manifold (M, Π), $\Pi^{ij} \equiv \{X^i, X^j\}$, one obtains a less obvious Lie algebroid by means of $E := T^*M$, $\rho(\alpha) := \alpha_i \Pi^{ij} \partial_j$, and $[\mathrm{d}f, \mathrm{d}g] := \mathrm{d}\{f, g\}$.

For later use we note that the image of ρ is integrable so that M is foliated into orbits. Moreover, because of the Leibniz rule, the bracket reduces to a fiberwise Lie-algebra structure for elements in the kernel of ρ, which is isomorphic for any two points in the same orbit.

In local coordinates $(X^i)_{i=1}^{\dim M}$ and frame $(b_I)_{I=1}^{\mathrm{rank} E}$ the Lie algebroid data are encoded in structural functions $\rho^i_I(X)$, $C^I_{JK}(X)$, where $\rho(b_I) \equiv \rho^i_I \partial_i$, $[b_I, b_J] \equiv C^K_{IJ} b_K$.

Guided by the other obvious example of a Lie algebroid, $E = TM$ with $\rho = id$, one may introduce differential geometrical notions on Lie algebroids. With b^I denoting the dual basis in E^*, the Leibniz extension of ${}^E\mathrm{d}X^i = b^I \rho^i_I(X)$ and ${}^E\mathrm{d}b^I = -\frac{1}{2} C^I_{JK}(X) b^J \wedge b^K$ defines a generalization of the de Rham differential on the space of E forms $\Gamma(\Lambda^\cdot E^*) \equiv \Omega^\cdot_E(M)$; ${}^E\mathrm{d}^2 = 0$ entails all the differential compatibility conditions to be satisfied by the structural functions introduced above. Also one may be interested in differentiation along sections of E: On E tensors this may be done by means of a generalized E-Lie derivative. For $s_1, s_2 \in \Gamma(E)$, e.g., ${}^E L_{s_1} s_2 := [s_1, s_2]$ while on $\Omega^\cdot_E(M)$ one uses ${}^E L_s = {}^E\mathrm{d}\iota_s + \iota_s {}^E\mathrm{d}$. For sections $\psi \in \Gamma(V)$, V being any other vector bundle over M, one introduces "an E-covariant" derivative ${}^E\nabla : \Gamma(V) \to \Gamma(E^*) \otimes \Gamma(V)$ by a straightforward extrapolation of the axioms of a standard covariant derivative.

Besides a Lie algebra, the definition of a YM action requires a nondegenerate ad-invariant scalar product. The obvious generalization would be a fiber metric ${}^E g$ with ${}^E L_s {}^E g = 0$, $\forall\ s \in \Gamma(E)$. However, for $\rho \not\equiv 0$ this requires Courant algebroids (CAs): As before, $\rho : E \to TM$, and there is a bracket for sections satisfying the Leibniz property with respect to itself as well as multiplication by $C^\infty(M)$. In addition, there is ${}^E g$ with $\rho(s_1)\, {}^E g(s_2, s_3) = {}^E g([s_1, s_2], s_3) + {}^E g(s_2, [s_1, s_3])$, $\forall\ s_i \in \Gamma(E)$. If the bracket were required to be antisymmetric, E would become a Lie algebroid and, as one may check, the existence of ${}^E g$ would require $\rho \equiv 0$. The final axiom of a CA, circumventing this rather trivial case, is ${}^E g([s, s], s') := \rho(s')\, {}^E g(s, s)$. Although it is possible to define a Courant algebroid YM (CAYM) theory, and we do so at the end, we show that the better understood Lie algebroids are sufficient for our purposes.

To develop a framework for the fields of a YM-type theory we first draw on the relation between the PSM and the CS gauge theory. We argue that, viewed from the correct perspective, they are essentially the same; one needs only to exchange the target Lie algebroid E, which is T^*M, M Poisson, in the PSM and $\mathfrak{g}$ in the CS theory [11]. Let Σ denote the spacetime under consideration. Then in both cases the fields of the theory are vector bundle morphisms $a : T\Sigma \to E$ [12]. Such maps are specified by a base map $\mathcal{X} : \Sigma \to M$ and a section $A \in \Omega^1(\Sigma, \mathcal{X}^*E)$. In local terms, $\mathcal{X}$ corresponds to scalar fields $X^i(x)$ and $A = A^I \otimes \bar{b}_I$, where $(\bar{b}_I)$ denotes the basis in $\mathcal{X}^*E$ induced by (b_I) in E, and A^I is a collection of 1-forms on Σ. In the Poisson case, $b_I \sim dX^i$, and we recover the fundamental fields (X^i, A_i) used to define the theory. In the CS theory $\mathcal{X}$ contains no information, mapping all of Σ to a single point, and $\mathcal{X}^*E \cong E = \mathfrak{g}$, so that A becomes the Lie-algebra valued connection 1-form.

Next we turn to the field equations. The transpose of a vector bundle morphism a is a map between sections: $a^T : \Omega^\cdot_E(M) \to \Omega^\cdot(\Sigma)$. For example, $a^T(X^i) = \mathcal{X}^*X^i \cong X^i(x)$, $a^T(b^I) = A^I$. The vector bundle morphism a is also a morphism of Lie algebroids, iff the operator $\mathcal{F} := da^T - a^T\,{}^E d$ vanishes identically on $\Omega^\cdot_E(M)$. Specializing

$$F^i := \mathcal{F}(X^i) = dX^i - \rho^i_I A^I, \tag{1}$$

$$F^I := \mathcal{F}(b^I) = dA^I + \tfrac{1}{2} C^I_{JK} A^J \wedge A^K \tag{2}$$

to $E \cong \mathfrak{g}$ and $E \cong T^*M$ (with $\rho^j_I \sim \Pi^{ij}$, $C^I_{JK} \sim \Pi^{jk}{}_{,i}$), one recovers the respective field equations from $\mathcal{F} = 0$.

Finally, two solutions a, a' to $\mathcal{F} = 0$ are gauge equivalent, iff they are homotopic, $a \sim a'$. Infinitesimally this implies $\delta_\varepsilon X^i = \rho^i_I \varepsilon^I$, $\delta_\varepsilon A^I \approx d\varepsilon^I + C^I_{JK} A^J \varepsilon^K =: \delta^{(0)}_\varepsilon A^I$ with $\varepsilon \in \Omega^0(\Sigma, \mathcal{X}^*E)$, where $\approx$ is chosen so as to stress the on-shell character of the equation (resulting from an on-shell concept). This is readily recognized as the gauge symmetries of the PSM and CS upon specialization.

We now address the question whether one can find a topological theory for any dimension d of Σ and any choice of the target Lie algebroid E such that the morphism property $\mathcal{F} = 0$ is contained in the field equations and the homotopy in the gauge symmetries. Introducing auxiliary $(d-1)$-forms B_i and $(d-2)$-forms B_I,

$$S_{\text{LABF}} := \int_\Sigma B_i \wedge F^i + B_I \wedge F^I, \tag{3}$$

obviously leads to the desired field equations and—no more so trivial, relying heavily on ${}^E d^2 = 0$—it is invariant with respect to the above gauge symmetries for $\delta_\varepsilon B_I := C^J_{KI} B_J \varepsilon^K$, $\delta_\varepsilon B_i := -\varepsilon^I(\rho^j_{I,i} B_j + C^K_{IJ,i} B_K \wedge A^J)$.

For $E = \mathfrak{g}$ this action reduces to that of a non-Abelian BF theory, for $E = T^*M$, M Poisson, it was suggested already in [14]. It is topological due to further gauge symmetries on the B fields, following from Bianchi-type identities for the "curvatures" F^i and F_I. They can be obtained most easily by applying the obvious relation $d \circ \mathcal{F} = -\mathcal{F} \circ {}^E d$ to X^i and b^I, respectively. In the first case this gives $dF^i \equiv -\rho^i_I F^I + \rho^i_{I,j} A^I \wedge F^j$, leading to the independent gauge symmetry (besides $\delta_\lambda X^i = 0 = \delta_\lambda A^I$),

$$\delta_\lambda B_i = d\lambda_i + \rho^j_{I,i} A^I \wedge \lambda_j, \quad \delta_\lambda B_I = (-1)^{d+1} \rho^i_I \lambda_i. \tag{4}$$

Likewise, $dF^I + C^I_{JK} A^J \wedge F^K \equiv \frac{1}{2} C^I_{JK,i} A^J \wedge A^K \wedge F^i$ gives $\delta_\mu B_I = d\mu_I - C^J_{KI} A^K \wedge \mu_J$, $\delta_\mu B_i = \frac{1}{2} C^I_{JK,i} A^J \wedge \mu_I \wedge A^K$ as an invariance of (3) for any $\mu \in \Omega^{d-3}(\Sigma, \mathcal{X}^*E^*)$.

To obtain a general framework for constructing action functionals for a field $a: T\Sigma \to E$, reducing to ordinary YM theory for $E = \mathfrak{g}$, we need to address the meaning of (1) and (2) as well as what the off-shell gauge transformations are [15]. While F^i and $\delta_\varepsilon X^i$ have a well-defined meaning, e.g., $\Phi := F^i \otimes \partial_i \in \Omega^1(\Sigma, \mathcal{X}^*TM)$, corresponding to the bundle map $\phi = \mathcal{X}_* - \rho \circ a: T\Sigma \to TM$, this is the case for neither F^I nor $\delta^{(0)}_\varepsilon A^I$; despite their above naive use, both of them are frame dependent.

This can be cured by introducing a connection ∇ on E. The combination $F^I + \Gamma^I_{iJ} F^i \wedge A^J =: F^I_{(\Gamma)}$, where $\nabla b_I \equiv \Gamma^J_{iI} dX^i \otimes b_J$, transforms correctly under a change of frame. In explicitly covariant terms this yields

$$F^I_{(\Gamma)} = (DA)^I - \tfrac{1}{2} {}^E T^I_{JK} A^J \wedge A^K, \tag{5}$$

where D is the canonical exterior covariant derivative on $\Omega^\cdot(\Sigma, \mathcal{X}^*E)$ induced by ∇ and ${}^E T$ is the E torsion of ${}^E\nabla_s := \nabla_{\rho(s)}$. Now $F = F^I_{(\Gamma)} \otimes b_I \in \Omega^2(\Sigma, \mathcal{X}^*E)$, a 2-form on spacetime with values in what replaces the Lie algebra, also comes from a bundle map $f : \Lambda^2 T\Sigma \to E$.

Covariant A-gauge symmetries have the form $\delta_\varepsilon A^I = \delta^{(0)}_\varepsilon A^I + \lambda^I_i F^i$ for some ε-dependent choice of λ^I_i. One option is to use the above (or any other) connection, setting $\lambda^I_i = \Gamma^I_{iJ} \varepsilon^J$. Another, qualitatively different possibility is $\lambda^I_i = -\varepsilon^I_{,i}$, where ε^I now is viewed as a pullback of $\varepsilon^I(x, X)$ by $\mathcal{X}$ [16]. This has a nice geometrical interpretation: Replacing E by the exterior sum Lie algebroid $\bar{E} = T\Sigma \boxplus E$ over $\Sigma \times M$, ${}^{\bar{E}}d = d + {}^E d$, and using the graph $\bar{a} = id \boxplus a$, the gauge symmetries are generated by sections ε of $\bar{E}$ with values in E fibers by means of a left action of $\bar{E}$ on itself: $\delta_\varepsilon \bar{a}^T = \bar{a}^T \circ {}^{\bar{E}} L_\varepsilon$. Locally we can always choose a flat connection Γ or an X-independent prolongation of $\varepsilon \in \Omega^0(\Sigma, \mathcal{X}^*E)$; then in both cases one reobtains $\delta^{(0)}_\varepsilon A^I$. But for nonflat bundles E this is not possible globally.

Now we are able to obtain explicitly covariant, globally defined action functionals. We first reconsider topological theories from this enhanced perspective. The covariant analogue of (3) would be $\int_\Sigma b_i \wedge F^i + B_I \wedge F^I_{(\Gamma)}$, where $b \in \Omega^{d-1}(\Sigma, \mathcal{X}^*T^*M)$, $B \in \Omega^{d-2}(\Sigma, \mathcal{X}^*E^*)$. But whatever the choice for Γ is, in any local patch the redefinition $B_i := b_i + \Gamma^I_{iJ} B_I \wedge A^J$ makes the connection disappear, bringing the action into the form (3). The globally well-defined form of the transformations of the

Lagrange multiplier fields also depend on λ_i^I. We display one of them, $\delta B_I = -C_{IJ}^K B_K \varepsilon^J - \rho_I^i B_J \lambda_i^J$, as it is needed also in the context of (8) below. Retrospectively, the previous considerations can be justified and backed up by an underlying global construction.

For particular choices of E there may exist topological actions producing Lie algebroid morphisms from $T\Sigma$ to E up to homotopy without any auxiliary fields. But $d = \dim\Sigma$ depends on the choice of E: For E a quadratic Lie-algebra $\mathfrak{g}$, one has the CS theory in $d = 3$, for $E = T^*M$, M Poisson, one has the PSM in $d = 2$. The analogy between the two models goes even further: The CS theory can be regarded as being induced on the boundary $\Sigma = \partial\tilde{\Sigma}$ of a spacetime $\tilde{\Sigma}$ with an extra dimension, using an $F \wedge F$ action; here an ad-invariant metric on $E = \mathfrak{g}$ is needed to contract the Lie-algebra indices of $F = F^I \otimes b_I$. On the other hand, for $E = T^*M$, no metric is needed to contract F^i with $F_{i(\Gamma)}$. This now gives a 3-form, Γ drops out if ∇ is chosen torsion-free, and indeed

$$S_{\mathrm{PSM}} = \int_{\tilde{\Sigma}} F^i \wedge F_i, \qquad (6)$$

the Poisson sigma model induced on the two-dimensional boundary $\Sigma = \partial\tilde{\Sigma}$. Whereas the choice $E = \mathfrak{g}$ requires $d = 3$ for a CS theory, for $E = T^*M$, M Poisson a "Poisson-CS theory" naturally lives in two dimensions and coincides with the PSM. We note as an aside that similar to the Pontrijagin class $\langle F, {}^\wedge F\rangle$, also $F_i \wedge F^i$ corresponds to a characteristic class, one associated with Poisson fibrations [13].

If one does not permit WZ contributions to the action [17] nor auxiliary metrics, the PSM is the universal purely bosonic topological theory in two dimensions [14]. Correspondingly, in $d = 2$ the LABF theory (3) must be a particular PSM. Indeed, generalizing the old observation that ordinary BF theories are PSMs for $M = \mathfrak{g}^*$, in two dimensions one recovers the LABF theory from the PSM for the particular choice $M = E^*$. The dual of a Lie algebroid is canonically a Poisson manifold, locally $\{b_I, b_J\} = C_{IJ}^K b_K$, $\{X^i, X^j\} = 0$, and $\{b_I, X^i\} = \rho_I^i$. A^I and B_i collect into the 1-form fields of the PSM, X^i, B_I into the scalar ones, and also the gauge symmetries (4) are included in this description.

There exists a likewise universal [18] topological action in three dimensions, reducing to the CS theory for M a point, which one might call the Courant sigma model (CSM). Using the description [19] of Courant algebroids, the action can be obtained most easily by the Alexandrov-Kontsevich-Schwartz-Zaboronsky (AKSZ) approach [20,21]:

$$S_{\mathrm{CSM}} = \int_\Sigma B_i \wedge F^i + \frac{1}{2}\left\langle A, {}^\wedge dA + \frac{1}{3}[A, {}^\wedge A]\right\rangle, \qquad (7)$$

where $[\cdot,\cdot]$ and $\langle\cdot,\cdot\rangle$ denote bracket and fiber metric in a Courant algebroid, respectively, and F^i agrees with (1). The LABF theory in $d = 3$ is a particular CSM, where the Courant algebroid is $E \oplus E^*$ with anchor $\rho \oplus 0$, canonical fiber metric, and bracket $[s \oplus u, s' \oplus u'] = [s, s'] \oplus ({}^E L_s u' - \iota_{s'} {}^E \mathrm{d}u)$; $A^I b_I \oplus B_I b^I$ combine into the A field of (7).

We now turn to nontopological Lie algebroid theories, generalizing standard YM-gauge theories. Our main proposal in this context is the action

$$S_{\mathrm{LAYM}} = \int_\Sigma B_i \wedge F^i - \frac{1}{2}\langle F, {}^\wedge * F\rangle. \qquad (8)$$

Here the fiber metric $\langle\cdot,\cdot\rangle \equiv {}^E g(\cdot,\cdot)$ is assumed to admit a (possibly overcomplete) basis of sections ψ_A satisfying ${}^E L_{\psi_A} {}^E g = 0$. Then (8) is invariant under gauge transformations (of the Lie type) with respect to $\varepsilon^I = \varepsilon^A(x)\psi_A^I(X(x))$ for arbitrary $\varepsilon^A(x)$; the symmetries are a module of $C^\infty(\Sigma)$ [22], but no more of all of $C^\infty(\Sigma \times M)$ as in the topological theories. (This reminds one of a similar relation between Kac Moody algebras and current algebras governing topological models [23].)

Because of the presence of the first term, we are permitted to drop any explicit Γ dependence in (8). Indeed, Eq. (8) results from the explicitly covariant LABF theory upon addition of a term like ${}^E g^{IJ} B_I \wedge *B_J$, shifting b_i before eliminating B_I. Likewise, in the construction of (7) an orthonormal frame was used, and, as written, it is not explicitly covariant; but again the presence of the first term makes it invariant with respect to a change of frame. Covariance is a very useful tool in the discussion of (8), already when checking the above mentioned gauge invariance. Also it permits one to put ${}^E g_{IJ}$ constant or to bring the Lie algebroid structural functions into some particularly simple form.

Somewhat miraculously, contrary to first appearance, the action (8) contains no other propagating degrees of freedom as those present in *ordinary* YM-gauge theories. This may be seen as follows: The B_i-field equation $F^i = 0$ restricts the image of the map $\mathcal{X} : \Sigma \to M$ to lie in an orbit of E in M. Then one may use the gauge symmetries to reduce $\mathcal{X}$ to the respective homotopy classes $[\mathcal{X}]$. In the trivial class we may put $X(x) = X_0 = \mathrm{const.}$ (Nontrivial classes correspond to a new kind of YM instanton sectors.) Returning to $F^i = 0$, A now is forced to lie in the kernel of ρ at $X_0 \in M$. On the other hand, variation of S_{LAYM} along directions of A in $\ker\rho$ contain no B field and reduce to the ordinary YM-field equations for $\mathfrak{g} = \ker\rho|_{X_0}$. Moreover, the residual ε-gauge symmetry is readily recognized as the usual non-Abelian one of $\mathfrak{g}$.

It remains to show that B_i contains no propagating modes. This is somewhat more subtle and will be detailed elsewhere. (It may be illustrative for the reader, however, to check the particularly simple cases $E = TM$ and E a bundle of Lie algebras.) Essentially it works as follows: the A variation determines B_i up to parts in the conormal bundle of the respective orbit (in terms of X_0 and the $\mathfrak{g}$-connection A). On the other hand, the variation with respect to X yields a field equation containing $\mathrm{d}B_i$. To kill the remaining parts of B, up to possibly global modes when Σ has nontrivial topology, one needs an additional symmetry. This is provided by a relict of (4): Eq. (8) is invariant under a B_i transformation for any λ taking values in the conormal bundle of an orbit.

We have $\dim\mathfrak{g} = \mathrm{rank}E - \dim\mathcal{O}$, where $\mathcal{O}$ denotes the orbit through X_0. Correspondingly, at singular leaves one obtains a higher dimensional structural group of the respective YM theory in comparison with points X_0 in nearby leaves. At such exceptional orbits (they are of measure zero in M), it is also not clear if one should eliminate all of the B_i modes as mentioned above, since this in part corresponds to λ's of lower dimensional support on M. Classically there is no interaction between the LAYM theories defined over different leaves of the foliation. On the quantum level the situation may be more intricate for what concerns singular leaves of the foliation, as we see in the Kontsevich formula [24]. In fact, in $d = 2$ the action (8) becomes a particular almost topological PSM [5].

The above strategy used to obtain LAYM theories may be extended to Courant algebroids. Instead of the PSM one then starts with the CSM (7), and as before uses its symmetries and the left-hand side of its field equations in arbitrary dimensions. In this way one obtains a CABF theory, which becomes a CAYM theory by adding quadratic terms in the Lagrange multipliers. If one uses only the multiplier for the 2-form field strength, the given scalar product is sufficient for gauge invariance. Again one obtains YM theories, but now also gauge theories for 2-form gauge fields can be constructed. This is particularly interesting in view of their presence in string effective field theories, but also their and CAs believed relation to gerbes. Higher degree gauge fields will arise when climbing up in the ladder of dimensions, at the price of more complicated algebroids.

In this Letter we showed, however, that in all dimensions we can fruitfully restrict to Lie algebroids. The nonexistence of invariant metrics for nontrivial ρ can be circumvented by the weaker condition for the existence of E-Killing vectors; also, in the context of gravitational theories, a likewise problem was circumvented by introducing an E-Riemannian foliation on the target [9]. Lie algebroids are well understood [3,4], recently even the necessary and sufficient conditions for integrating them to Lie groupoids have been clarified [25], so that a generalization of, e.g., Wilson loops comes into reach.

The above tools permit one to construct also other Lie algebroid theories. For example, in some cases, one can add a term quadratic in B_i to (8) not spoiling gauge invariance. Physically this corresponds to adding Higgs fields and/or giving mass to some of the gauge fields.

In two spacetime dimensions the PSM unifies gravitational and YM fields into a common framework, and supersymmetrization is obtained by using graded Poisson manifolds as a target [26]. It also remains to be seen how far this picture can be extended to higher dimensions.

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