Vortex-Lattice Dynamics in Rotating Spinor Bose-Einstein Condensates

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We observe interlaced square vortex lattices in rotating dilute-gas spinor Bose-Einstein condensates (BEC). After preparing a hexagonal vortex lattice in a one-component BEC in an internal atomic state $|1\rangle$, we coherently transfer a fraction of the superfluid to a different state $|2\rangle$. The subsequent evolution of this pseudo-spin-1/2 superfluid towards a state of offset square lattices involves an intriguing interplay of phase-separation and -mixing dynamics, both macroscopically and on the length scale of the vortex cores, and a stage of vortex turbulence. The stability of the square structure is proved by its response to applied shear perturbations. An interference technique shows the spatial offset between the two vortex lattices. Vortex cores in either component are filled by fluid of the other component, such that the spin-1/2 order parameter forms a Skyrmion lattice.

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Some of the most intriguing phenomena of superfluidity are revealed when a superfluid is set rotating. Depending on the complexity of the order parameter a superfluid can carry angular momentum in different ways. A single-component superfluid exhibits quantized vortices that, in a dilute-gas single-component Bose-Einstein condensate (BEC), organize into a regular hexagonal Abrikosov lattice [1] [see Fig. 1(a)], whose static properties and long-wavelength excitations have been examined [2–5]. Such lattices are also formed in Type II superconductors in a magnetic field [1] and in superfluid H^4e [6]. Multicomponent superfluids, with their pseudospin order parameters, carry angular momentum in spin textures (Skyrmions) [7,8], which may again form lattices [9–11], such as in the superfluid phases of liquid ^{3}He [12] and in certain regimes of two-dimensional electron systems [13].

In this work we study rotating two-component superfluids in a magnetically trapped dilute-gas BEC [14] described by a pseudo-spin-1/2 order parameter. State $|1\rangle$ represents spin-down and state $|2\rangle$ represents spin-up, and the azimuthal component of the spin is fixed by the relative phase Φ between the state $|1\rangle$ and $|2\rangle$ wave functions. Both the large-scale density profiles and the vortex-lattice structure depend on the nature of intercomponent and intracomponent interactions. The interaction energy, determined by three scalar coupling constants g_{11}, g_{22}, and g_{12} and the population densities n_1 and n_2, is given by

$$E_{int} = \int d^3x[g_{11}n_1^2 + g_{22}n_2^2 + g_{12}n_1n_2]. \qquad (1)$$

For attractive intercomponent interaction ($g_{12} < 0$) it is energetically favorable if the spatial profiles of both components coincide, and a hexagonal vortex lattice is expected, as in the one-component case. For repulsive intercomponent interaction ($g_{12} > 0$), realized in our ^{87}Rb system, it is energetically favorable for the two

components to reduce spatial overlap. In a static BEC this may be achieved by macroscopic phase separation, whereas in rotating two-component BEC the presence of the vortex lattice allows for more subtle separation effects, such as interlacing the lattices. Vortices in either component may be filled by the other component, and the pseudo-spin-1/2 order parameter forms a Skyrmion lattice. Studies by Mueller and Ho [9] and Kasamatsu et al. [10] predict, among other structures, interlaced square
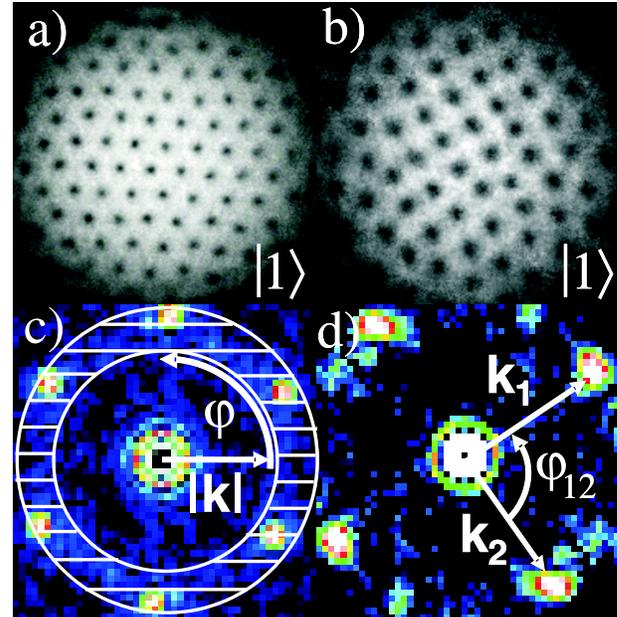

FIG. 1 (color online). (a) Hexagonal vortex lattice in a one-component BEC. (b) Square lattice, viewed in the $|1\rangle$ component of a two-component BEC. (c) Reciprocal ($k-$) space of the hexagonal lattice, obtained by 2D Fourier transform of (a), showing six peaks spaced by 60°. (d) $k-$ space of the square lattice. The reciprocal lattice vectors k_1 and k_2 enclose an angle $\varphi_{12} = 90°$.

lattices over a wide range of parameters. Presumably this is because in a square lattice, unlike in a triangular lattice, each vortex in component $|1\rangle$ can have all its nearest-neighbor vortices be in component $|2\rangle$, and vice versa [15].

Under suitable conditions we indeed observe a spatial separation of vortices in the two components, followed by the formation of an ordered square lattice structure, as shown in Fig. 1(b). Its formation and decay dynamics, as well as its static and dynamic properties are examined in this work.

Our ^{87}Rb two-state system [14] consists of two magnetically trappable hyperfine-Zeeman levels of the ^{87}Rb atom—$|F = 1, m_F = -1\rangle$, henceforth called $|1\rangle$, and $|F = 2, m_F = 1\rangle$, henceforth called $|2\rangle$. Spatial overlap of the two states is realized in a harmonic, axially symmetric magnetic trap with oscillation frequencies $\{\omega_\rho, \omega_z\} = 2\pi\{7.7, 4.9\}$ Hz. An electromagnetic coupling between the two states can be achieved via a two-photon transition, involving a microwave photon at $\sim$6.833 GHz and a radio frequency photon at $\sim$1 MHz. A short-pulse application of this coupling drive yields a desired, spatially uniform amount of population transfer between the two states, instantaneous with respect to the external dynamics of the two states. In the pseudo-spin-1/2 picture the effect of the coupling drive is to cause spin rotations [14]. Inelastic atomic collisions limit the lifetime of the $|2\rangle$ population to a few seconds. The decay does not cause $|2\rangle$ atoms to convert back to the $|1\rangle$ state but rather to leave the trap altogether.

To study the rotational properties of this two-superfluid system, we initially create regular hexagonal vortex lattices in BECs in state $|1\rangle$, as described in earlier work [4,5,16]. Here we start with near-pure condensates containing $(3.5–4) \times 10^6$ atoms, rotating at a rate $\Omega \approx 0.75 \times \omega_\rho$ about the z axis. A "transfer pulse" of the coupling drive then transfers a fraction of the population into state $|2\rangle$, in this work 80–85% as discussed below. After a variable wait time we take two images of the system. A nondestructive phase-contrast image is taken either of one component alone or of both components simultaneously [14], along an axis perpendicular to the rotation axis ("side view") while the system is still trapped. In order to optically resolve the vortex structures we expand either the $|1\rangle$ or $|2\rangle$ component of the condensate by a factor of 9, to a diameter of $\sim$600 μm, before a second, destructive image is taken along the rotation axis ("top view"). The other component is removed at the beginning of the expansion. The details of expansion and imaging have been described in Ref. [5].

In Fig. 2 we analyze the formation and decay dynamics of the square lattice structure. Figures 2(a) and 2(b) show time sequences of top-view images of the expanded $|1\rangle$ and $|2\rangle$ states, taken in different experimental runs. The time evolution after the transfer pulse involves several stages. For the first $\sim$0.1–0.25 s surprisingly little dy-
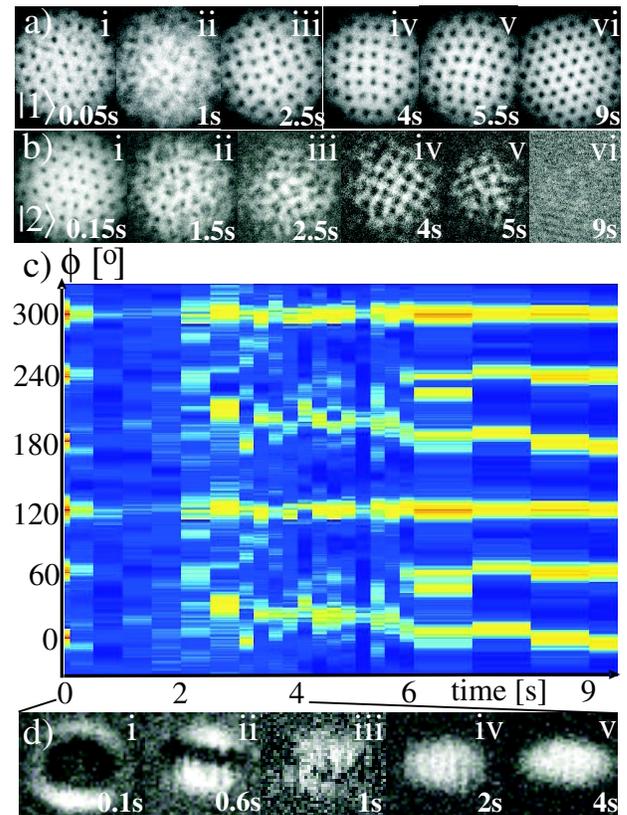
FIG. 2 (color online).   (a) Time sequence of images of state $|1\rangle$, after $\sim$80% population transfer to $|2\rangle$, showing evolution from a hexagonal lattice over a turbulent stage to a square structure and back to a hexagonal lattice. (b) Also state $|2\rangle$ forms a square lattice (iv) before its decay. (c) Detailed time evolution in reciprocal space. Intensity in an annulus along the φ coordinate [defined in Fig. 1(c)] is shown on the ordinate. The initial 6-peak structure of the hexagonal lattice vanishes quickly because of turbulence. From 3–5.5 s square lattices give rise to a four-peak structure. Around 6–9 s a transition back to hexagonal lattices occurs. (d) Two-color side-view images of the initial turbulent evolution. State $|1\rangle$ ($|2\rangle$) appears bright (dark) on gray background. The fine filament structures in (ii-v) are due to mutual filling of vortex cores.

namics is visible and certainly no structural transition in the vortex lattice is seen in either component. From $\sim$0.25–2 s a turbulent stage evolves in both components in which vortex visibility degrades significantly, shown in Fig. 2(a)(ii) and Fig. 2(b)(ii). As we will show, this turbulence is directly linked with the transition from overlapping hexagonal vortex lattices to interlaced square lattices. From 2–3 s square domains emerge from the turbulent state, and defects propagate out of the lattice [Fig. 2(a)(iii)]. From 3–5.5 s stable square lattices are observed in both components [Fig. 2(a)(iv) and Fig. 2(b)(iv)]. At this stage, around 4 s, despite the large (80–85%) initial population transfer to state $|2\rangle$, the number of $|2\rangle$ atoms has decreased to only 1.5×10^5, while state $|1\rangle$ contains $(5–7) \times 10^5$ atoms. As the $|2\rangle$ state population continues to decay, the vortex-lattice

planes bend [Fig. 2(a)(v)] and a transition back to a hexagonal lattice in state $|1\rangle$ takes place [Fig. 2(a)(vi)]. During the transition from square lattices to a hexagonal lattice no turbulence occurs.

A more quantitative analysis of these dynamics is possible in reciprocal space. Figure 2(c) shows the time evolution of the intensity within an annulus in reciprocal space, defined in Fig. 1(a), which contains the reciprocal lattice peaks. Initially six peaks are visible, separated by 60°, forming the reciprocal lattice of a hexagonal vortex-lattice. Because of turbulence these peaks vanish between ~200 ms and 2 s. After 2–3 s a four-peak structure appears, that is stable for a period of ~2.5 s. The observed ratio of reciprocal lattice vector lengths [defined in Fig. 1(d)] $k_1/k_2 = 0.98(2)$ and the angle $\varphi_{12} = 95(3)°$ between k_1 and k_2 clearly identify a square lattice. At ~5.5 s the appearance of two additional peaks signals the onset of a transition back to the hexagonal state, which is completed by ~9 s.

To examine the origin of the initial turbulence, we show in Fig. 2(d) a time sequence of in-trap side-view images, where the $|1\rangle$ ($|2\rangle$) state appears bright (dark). As visible in Fig. 2(d)(i), a macroscopic component separation from the initially homogeneous two-component superposition to a ball-shell structure takes place within 50–100 ms [14]. During this period of dramatic axial separation, both the individual vortices and the overall vortex-lattice remain remarkably quiet, as viewed along the rotation axis [see Fig. 2(a)(i) and Fig. 2(b)(i)]. Around 600 ms, fine filament structures appear at the intercomponent boundary [Fig. 2(d)(ii)], coincident with the full development of vortex turbulence seen in top-view images [Fig. 2(a)(ii)]. The filament structures distort and fill out the whole BEC as the vortex turbulence peaks at around 1 s [Fig. 2(d)(iii)], and straighten up at around 2 s [Fig. 2(d)(iv)] coincident with the restoration of vortex visibility in top-view images. Subsequently the filaments become less visible as state $|2\rangle$ decays [Fig. 2(d)(v)]. We interpret these structures as vortex cores in either component being filled by fluid of the other component, forming a Skyrmion lattice [8]. Filled vortex cores grow in size above the resolution limit of our side-view images and become observable, while empty vortices in single-component BECs in equilibrium are well below this resolution limit and are not observed in-trap [17]. The initial *macroscopic*, predominantly axial, phase separation has thus evolved into a *microscopic* separation which gives rise to the observed turbulence.

To study the stability of the square lattice, we excite shear perturbations (Tkachenko modes) [18,19] in the square lattice by focusing a resonant laser beam onto the center of the condensate [3]. As shown in Fig. 3, the perturbation relaxes to the equilibrium square configuration within 500–800 ms after excitation. However, in contrast to single-component triangular lattices [3], no clear oscillation is observed. Two effects may contribute: The excitation may be overdamped because of ill-defined
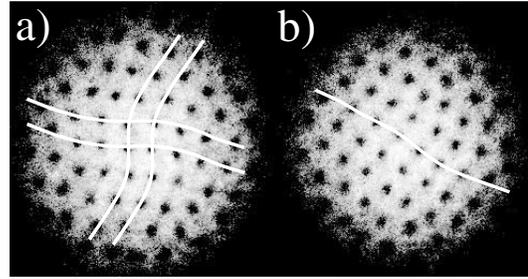
FIG. 3. (a) Tkachenko excitation in the square lattice, observed 250 ms after beginning of excitation, and (b) after relaxation back to a square lattice structure (at 850 ms).

boundary conditions caused by imperfections in the outer region of the square lattice, where the $|2\rangle$ state population has already decayed. The oscillation may also be masked by random lattice excitations of comparable amplitude that cannot be completely removed in the short time between formation and decay of the square lattice. However, the return of the lattice to its square configuration is sufficient to demonstrate the stability of the square structure in the two-component system.

So far we have presented evidence that both components separately form regular and stable square lattices. Filament structures indirectly indicate a microscopic-scale spatial separation of vortices. In the following, we employ an interference technique between the two superfluids to more concisely address two questions: Are the vortex lattices really offset from each other? Do the vortices really only separate from each other during the turbulent stage, after a surprising delay of 200 ms? Figures 4(a)(i) and (iii) show results of a simple simulation of interference between two square vortex-lattice wave functions, spatially offset by $1/2(\vec{a} + \vec{b})$, where $\vec{a}$, $\vec{b}$ are the two basis vectors of the square lattice. Two values ($\Phi = 0$ and $\Phi = 45°$) of the relative quantum phase Φ between the two wave functions [20] are considered. A population ratio of $N_{|2\rangle}:N_{|1\rangle} = 20:80$ was assumed, as in the experiment at 4 s. Evidently two very dissimilar patterns are seen, consisting either ($\Phi \sim 0$) of dark patches in between two vortices, surrounded by a bright square structure, or ($\Phi \sim 45°$) of staircaselike vortex arrangements separated by bright stripes. We checked that the qualitative appearance of such simulations changes drastically when altering the spatial offset from $1/2(\vec{a} + \vec{b})$.

To observe such structures experimentally, we apply a $\pi/2$ "interference" pulse to the two-component system just before expansion for the top-view image. This pulse results in a phase-coherent transfer of population between the two states, thus creating interference. If the $\pi/2$ pulse is applied under conditions when regular square lattices are expected in both components, the observed images, Fig. 4(a)(ii) and (iv), agree qualitatively with the simulations, clearly demonstrating a spatial offset close to $1/2(\vec{a} + \vec{b})$ between the vortex lattices.
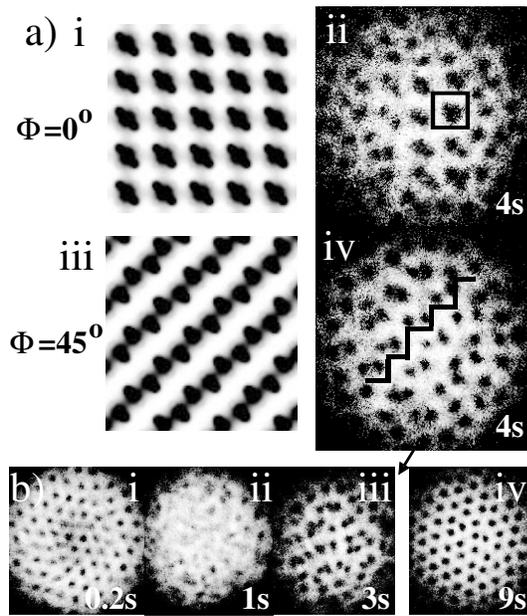
FIG. 4.   (a) Vortex-lattice interference. (i) and (iii) Simulations for different values of the relative phase Φ between the $|1\rangle$ and $|2\rangle$ state wave functions. (ii) and (iv) Experimental results. (b) Time evolution of the interference patterns shows no vortex offset before the turbulence (i) but patterns characteristic of offset vortices afterwards [b(iii)],[a(ii)],[a(iv)].

The qualitative agreement between experimental results and simulation indicates that vortex-lattice interference may be employed to examine the separation process of the vortex lattices. When varying the wait time between the transfer pulse and the $\pi/2$ interference pulse, we observe the following qualitative features [Fig. 4(b)]: Until turbulence occurs (after 200–300 ms) there are no indications for separation of the vortex lattices. Figure 4(b)(i) shows a hexagonal lattice with good vortex contrast, very similar to images of each single component at this time. This confirms that during the initial 200–300 ms of dramatic macroscopic component separation, vortices in the two components continue to form identical and overlapped lattice structures, when viewed along the rotation axis. Only after this surprisingly long delay, and after the turbulent stage, do interference patterns show a grouping characteristic of offset vortices [Fig. 4(b)(iii)]. This observation is further proof of the direct link between the turbulent period and the microscopic separation of the vortex lattices.

In conclusion, we observe a new vortex-lattice structure in rotating two-component BEC. Each component carries a square vortex lattice, and the lattices are interlaced. Vortices in both components are filled by fluid of the other component, forming a Skyrmion lattice. This structure is stable, as evidenced by relaxation of applied shear excitations back to the square structure. An intriguing turbulent stage accompanies the transition from overlapped hexagonal lattices to interlaced square lattices. Vortex-lattice interference is used to study the offset dynamics of the two lattices.

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[20] The relative phase Φ is well defined and measurable but is relevant only in the presence of coupling between the states. Directly after population transfer it is zero and uniform across the sample, but with each experimental run it quickly acquires an arbitrary value, e.g., because of slight irreproducibility in the trapping potentials.