Are Brazil Nuts Attractive?

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We present event-driven simulation results for single and multiple intruders in a vertically vibrated granular bed. Under our vibratory conditions, the mean vertical position of a single intruder is governed primarily by a buoyancylike effect. Multiple intruders also exhibit buoyancy governed behavior; however, multiple neutrally buoyant intruders cluster spontaneously and undergo horizontal segregation. These effects can be understood by considering the dynamics of two neutrally buoyant intruders. We have measured an attractive force between such intruders which has a range of five intruder diameters, and we provide a mechanistic explanation for the origins of this force.

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Granular materials are ubiquitous in nature and exhibit a wide range of nontrivial dynamical behavior [1]. Many industrial processes rely on the mixing or separation of these materials [2]. Understanding the behavior of such systems is an important scientific and technological challenge both to physicists and to engineers.

The simplest granular mixture is that of a single large intruder in a bed of identical host particles. Under vertical vibration the intruder may rise to the top of the bed, the Brazil nut effect (BNE) [3]. Conversely, under different conditions the intruder may sink in the bed, the reverse BNE [4]. These two effects have been attributed to a range of microscopic mechanisms depending on the specific details of the system, such as the size and density of the intruder, and the vibratory conditions [5–7].

A related problem to the BNE is that of multiple intruders in a granular bed. Recently, there has been much interest in trying to accurately predict when a granular mixture will segregate under vibration. Segregation “phase diagrams” have been proposed based on experiments [8], simulations [9,10], and kinetic theories [11]. However, to date, there is still no consensus on when and how even idealized granular mixtures will separate.

One key question which is central to understanding granular segregation is this: Does a collection of intruder particles segregate because the intruders are attracted to each other, or do they segregate because they congregate in a particular region of space? Insight into this question can be gained by comparing the behavior of single and multiple intruder systems. To this end, we have performed event-driven simulations for the simplest idealized mixture of particles: the particles undergo inelastic collisions; the particles are contained in a box subjected to vertical oscillations of the base in the presence of gravity, so that they form a fluidized bed. Our simulations neglect many microscopic details pertinent to experiments, such as friction. However, these approximations are close to those made in granular kinetic theories, allowing a comparison between these theories and simulation.

For the conditions we have chosen, our simulations show that for both single and multiple intruders the BNE is dominated by a buoyancylike effect. For a mixture of neutrally buoyant particles we observe that multiple intruders tend to cluster, and for large numbers of intruders there is a novel sideways segregation. All our simulations lead us to conclude that there is an effective attractive interaction between neutrally buoyant intruders when the system is subjected to vertical oscillations. In the case of two intruders, we show that the effective attraction is present for separations up to five intruder diameters, and we propose a mechanism for the interaction.

We have developed a two-dimensional event-driven (ED) code [12] to simulate the motion of vertically vibrated hard disks under gravity. The simulation uses $N_1 = 1000$ disks of species 1 with diameter $d_1 = 2.0$ mm and density $\rho_1 = 1.0\ \mathrm{g\,mm^{-2}}$; mixed with the 1000 disks are N_2 disks of species 2, with diameter d_2 and density ρ_2. The particles are constrained to move in the x-z plane, with gravity acting in the negative z direction. The coefficient of restitution for all particle-particle collisions is chosen to be $e = 0.9$. The particles inhabit a rectangular domain with dimensions $L_x \times L_z$ with $L_x = 200$ mm and $L_z = 400$ mm. The simulation data presented here are for relatively shallow beds, with up to a maximum depth of 26 particle diameters. Periodic boundary conditions were imposed in the x direction. The base oscillates sinusoidally with amplitude $A = 1.0$ mm and frequency $\nu = 30$ Hz; the maximum dimensionless acceleration is $\Gamma = A\omega^2/g = 3.6$.

The classic Brazil nut problem corresponds to $N_2 = 1$. Simulations were run for several values of d_2/d_1, but we present results here for $d_2/d_1 = 3$. The time-averaged vertical position of the intruder particle, $\langle z_2 \rangle$, and the host particles, $\langle z_1 \rangle$, were generated. These averages were taken after 6000 base oscillations to ensure that the initial conditions had no effect on the results. In Fig. 1 we plot the ratio $\langle z_2 \rangle / \langle z_1 \rangle$ as a function of the ratio ρ_2/ρ_1.

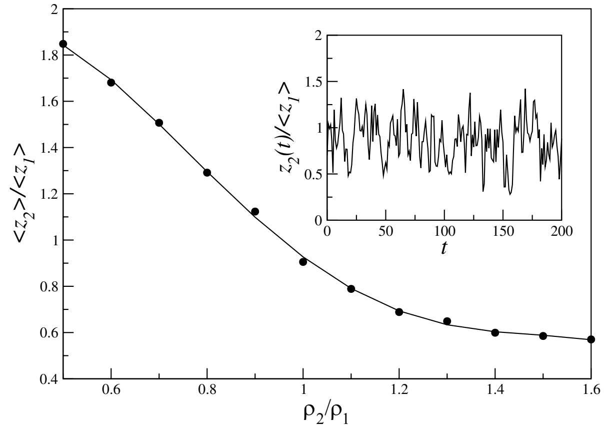

FIG. 1. The ratio $\langle z_2\rangle/\langle z_1\rangle$ as a function of the density ratio ρ_2/ρ_1 for $d_2/d_1 = 3.0$. The inset shows the time dependence of $z_2/\langle z_1\rangle$ which reveals that the intruder moves freely through a large part of the bed. The distribution of $z_2/\langle z_1\rangle$ is a Gaussian to a good approximation. Note that if the intruder were to sit at the very top of the bed $z_2/\langle z_1\rangle \approx 2.3$.

We see from Fig. 1 that the value of $\langle z_2\rangle/\langle z_1\rangle$ for $\rho_2/\rho_1 = 1.0$ is roughly 1, which indicates that the intruder particle sits near the middle of the bed on the average. Only for significantly larger (or smaller) values of ρ_2/ρ_1 does the intruder sink to the bottom (or rise to the top) of the bed. Under our vibratory conditions, the sinking or rising of the intruder is determined primarily by buoyancy. This conclusion is consistent with the experimental results of Huerta and Ruiz-Suárez [6] who showed that for fluidized beds where granular convection is suppressed, buoyancy controls vertical segregation. The height, $z_2(t)$, of the single intruder fluctuates with time, t; the amplitude of the fluctuations is greatest when $\rho_2 \simeq \rho_1$. The quantity $z_2(t)$ for equal densities is plotted as an inset to Fig. 1. It is clear that the intruder is not exclusively found at a particular height in the bed; instead it moves extensively in the bed from near the top to near the bottom.

What happens when there are several intruder particles? Do they behave as a collection of independent single intruders, or do they exhibit collective behavior? When buoyancy effects are strong for a single intruder it rises to the top or sinks to the bottom; multiple intruders acting independently should behave in the same way. If the densities are nearly equal and there are no collective effects we expect the intruders to fluctuate between the top and bottom of the bed, thereby forming a near uniform mixture. Strong deviations from uniformity would indicate the presence of collective behavior.

Simulations were run with $N_2 = 11$. In Fig. 2 we show typical configurations of the particles in the bed after all transients have died. In Fig. 2(a) the ratio $\rho_2/\rho_1 = 0.6$ and the intruders float to the top; in Fig. 2(c) $\rho_2/\rho_1 = 1.6$ and the intruders sink to the bottom. In Fig. 2(b), in contrast, $\rho_2/\rho_1 = 1.0$ and the intruders cluster together.

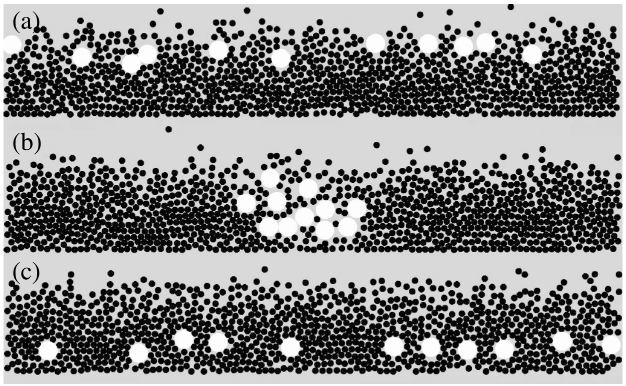

FIG. 2. Three typical configurations of a bed with $N_2 = 11$ for different values of $\rho_2/\rho_1 = 0.6$ (a), 1.0 (b), and 1.6 (c).

Once the cluster forms it remains together indefinitely, moving like a swarm of bees. This is powerful evidence of collective behavior of the intruders.

The number of intruders was increased to $N_2 = 111$, so that the area occupied by the intruders is equal to that of the host particles. In Fig. 3 we show typical configurations of the particles in the bed after all transients have died. In Fig. 3(a) the ratio $\rho_2/\rho_1 = 0.6$ and the intruders float to the top; in Fig. 3(c) $\rho_2/\rho_1 = 1.6$ and the intruders sink to the bottom. In Fig. 3(b), where $\rho_2/\rho_1 = 1.0$, the intruders cluster together as if there were two phases which have phase separated. This type of horizontal segregation is present in the range ρ_2/ρ_1 from 1.0 to 1.25. We have also varied ν and A and have observed similar behavior for a range of Γ up to approximately 5.5. Horizontal instabilities have been seen previously in

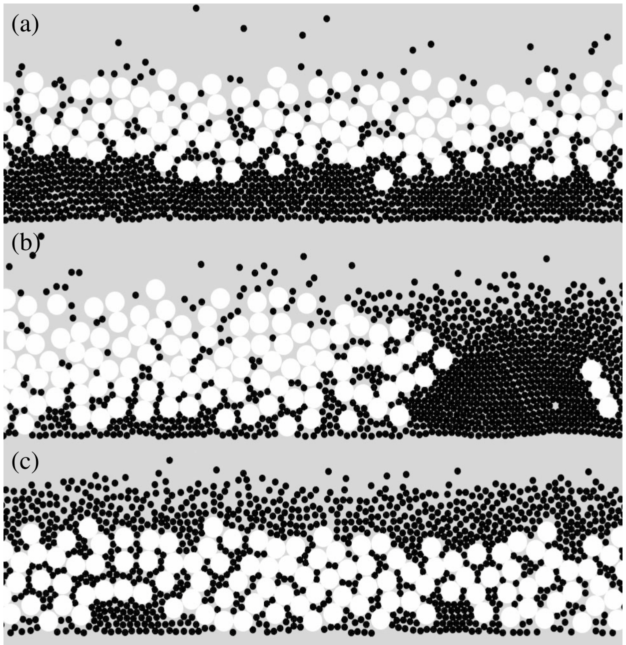

FIG. 3. Three typical configurations of a bed with $N_2 = 111$ for different values of $\rho_2/\rho_1 = 0.6$ (a), 1.0 (b), and 1.6 (c).

granular mixtures [10], but under very different conditions to those examined here.

We have shown that multiple intruder particles with densities $\rho_2 \approx \rho_1$ exhibit collective behavior. Do two intruder particles interact in a similar fashion? We ran the simulation for $N_2 = 2$ measuring the separation of the two intruder particles' centers at the same phase in the cycle, where the base is lowest. From these values we generated a probability distribution $p(r)dr$ of finding the intruders separated by a distance between r and $r + dr$. The statistics were improved by making long runs and averaging over different initial starting configurations of the bed. The results are shown in Fig. 4 as a bold line. For separations greater than five intruder-particle diameters the probability is roughly constant, which is to be expected if there is no attraction between intruders. However, for smaller separations the probability $p(r)dr$ increases by nearly 2 orders of magnitude, indicating the presence of an effective attraction. The oscillations in $p(r)$ for the smallest separations are pair correlation effects mediated by the host particles. While the definition of $p(r)$ does not explicitly take into account the geometry of the bed, it clearly provides evidence of an effective attraction between intruders.

In Fig. 4 we also show data for $\rho_2/\rho_1 = 0.6$ as a thin solid line. Here there is a more uniform probability, which indicates any effective interaction between the intruders is much weaker, if present at all. In addition, in Fig. 4 we show data $\rho_2/\rho_1 = 1.6$ as a dashed line. In this case there appears to be a weak, short-range repulsion.

The effective attraction between neutrally buoyant intruders appears to extend up to a separation of five intruder diameters. The host particles must be mediating the attraction via their collisions with the intruders and so, by measuring the momentum changes experienced by the intruders in such collisions it is possible to determine the magnitude of the attractive force.

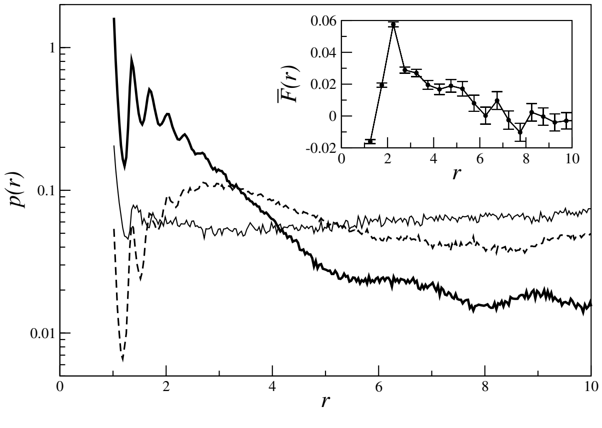

FIG. 4. The probability density $p(r)$ of separation as a function of r (r is in units of intruder-particle diameters). Three density ratios are shown: $\rho_2/\rho_1 = 0.6$ (thin solid line), $\rho_2/\rho_1 = 1.0$ (bold line), $\rho_2/\rho_1 = 1.6$ (dashed line). The inset shows the average attractive force (in units of the intruder's weight) as a function of intruder separation for density ratio $\rho_2/\rho_1 = 1.0$.

We ran the ED simulation with $N_2 = 2$ and $\rho_2/\rho_1 = 1.0$ as before, recording the details of all collisions suffered by one of the two intruders. For each of these collisions, we extract the momentum change of the intruder, $\Delta\mathbf{p}$, and the unit vector from this intruder to the other, $\hat{\mathbf{r}}$. For all collisions which occur for separations between r and $r + dr$, we calculate a quantity $\Delta P(r)$ by summing the components $\Delta\mathbf{p} \cdot \hat{\mathbf{r}}$. An approximation to the magnitude of the attractive force between the intruders, $\overline{F}(r)$, can be obtained by dividing $\Delta P(r)$ by the total time spent at this separation. This is shown in the inset of Fig. 4. The data used to estimate $\overline{F}(r)$ is subject to very large fluctuations; it is possible to produce a meaningful estimate only by taking long time averages. The standard error obtained from these data is also shown in Fig. 4. The range of $\overline{F}(r)$ is found to be comparable to the effective attraction suggested by the probability distribution, $p(r)$. Note that this force is much longer range than would be expected for forces of geometric origin [13].

We have demonstrated that there is an attractive interaction between two intruders when $\rho_2/\rho_1 = 1.0$. It is our belief that this force is responsible for the clustering and sideways segregation present in multiple intruder systems described above. When there is strong two particle attraction the corresponding multiple intruder system exhibits clustering [Fig. 2(b)] or sideways segregation [Fig. 3(b)]. In contrast, when there is no attraction between the two intruders the corresponding multiple intruder system does not exhibit clustering [Figs. 2(a) and 2(c)] or sideways segregation [Figs. 3(a) and 3(c)].

Let us now speculate on the possible causes of the attraction. We have measured in a similar way $F(r)$ at different phases of the cycle and have identified three separate regimes. As the bed is thrown by the base, the bed starts to dilate and the intruders experience a repulsive force, caused by the host particles being forced between them. During the subsequent period of free flight, there is little force between intruders. However, as the bed begins to land, the intruders now experience an attractive force. Figure 5 is a schematic representation of the dilated bed. Above and between the intruders, region A, there are relatively few host particles. As the bed collides with the base, the intruders experience more collisions with host particles from region B than from region A, leading to an attraction. Because of the variation in bed dilation over a cycle, the attractive force is slightly larger in magnitude than the repulsive force. Over many cycles, this asymmetry in the forces tends to ratchet the intruders together.

This begs the question as to what causes the low density of host particles in region A. Several explanations are possible. The presence of the intruders may restrict the flow of host particles from regions C to A in the upward part of the cycle. Alternatively, collisions between the more massive intruders and the host particles in region A

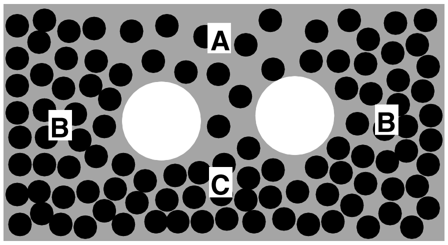

FIG. 5. Schematic diagram showing a typical bed configuration just before collision with the base. Three distinct regions of the bed are identified: A, B, and C. In region A the density of host particles is low but their kinetic energies are high. In regions B and C the density of host particles is much higher.

result in the host particles being more energetic near the intruders than elsewhere, thus leading to a lower density. It is possible that a combination of these two mechanisms is responsible for lowering the density.

As a test of this ratcheting mechanism, we have carried out simulations in which the sinusoidally vibrating base is replaced by a thermalizing lower boundary [14]. To make a direct comparison between these two systems, we choose the temperature of the base such that the time-averaged kinetic energy of the bed is the same in both cases. We find that with the thermalizing boundary there is no tendency for the intruders to cluster. We have also varied the temperature of the lower boundary but do not observe clustering for any temperature.

The ratcheting mechanism also explains why, in the sinusoidally vibrated system, the interaction depends on the vertical position of the intruders in the bed. The condition for an attractive force is that there is a high density of host particles in region B and a low density of host particles in the region between intruders (see Fig. 5). This condition is not met when the intruder floats to the top as here, region B has a low density of host particles, nor is it obeyed when the intruder sinks to the bottom as the region between intruders now has a high density of host particles.

Finally, this mechanism can be generalized to multiple intruder systems. The presence of more intruders in close proximity will create a larger region of dilated host particles above and between the intruders, similar to region A in Fig. 5, which will tend to bind the intruders together more strongly than in the case of only two. This is evident from the simulations of 11 intruders where, once a cluster has formed, it remains indefinitely over the time scale of the simulation.

Note that our findings are not compatible with the assumptions and predictions of present kinetic theories [11]. Specifically, the simulated system exhibits strongly correlated dynamics, two-dimensional density variation, and sensitivity to the driving mechanism.

In summary, we have carried out ED simulations of the BNE for single and multiple intruders in a sinusoidally vibrated bed of smaller host particles. We find that the mean position of a single intruder is governed by buoyancy-type effects and that a system of many neutrally buoyant intruders exhibits strong collective behavior that results in clustering or sideways segregation. We have traced this behavior to the existence of a weak attractive force between two neutrally buoyant intruders and propose a possible mechanism for the attraction.

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