Goldstone-Mode Relaxation in a Quantized Hall Ferromagnet

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We report on a study of the spin relaxation of a strongly correlated two-dimensional electron gas in the $\nu = 2\kappa + 1$ quantum Hall regime. As the initial state we consider a coherent deviation of the spin system from the **B** direction and investigate a breakdown of this Goldstone-mode (GM) state due to the spin-orbit coupling and smooth disorder. The relaxation is considered in terms of annihilation processes in the system of spin waves. The problem is solved at an arbitrary value of the deviation. We predict that the GM relaxation occurs nonexponentially with time.

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In spite of much recent interest in the spin relaxation (SR) in a two-dimensional electron gas, only a few of works have been devoted to the SR in the quantized Hall regime proper [1–4]. Meanwhile, exactly these quantum Hall conditions enable one to study a Goldstone mode (GM), which is a remarkable collective state representing quantum precession of a macroscopically large spin **S** around the **B** direction. We study the case of a *quantum Hall "ferromagnet"* (QHF); i.e., the filling factor is $\nu = \mathcal{N}/N_\phi \simeq 2\kappa + 1$, where $\mathcal{N}$ and $N_\phi = L^2/2\pi l_B^2$ are the numbers of electrons and magnetic flux quanta (l_B is the magnetic length). In the solution to the first order in the ratio $r_c = (e^2/\varepsilon l_B)/\hbar\omega_c$ considered to be small ($e^2/\varepsilon l_B$ is the characteristic Coulomb energy; ω_c is the cyclotron frequency) we get the ground state with $\kappa - 1$ low Landau levels fully occupied and with the κth level filled only by spin-up electrons aligned along **B**. The total spin is $S_0 = N_\phi/2$. This system provides a rare case where a relaxation problem may be solved analytically for any value of the initial deviation (not necessarily small). The SR process is found to occur nonexponentially.

Up to now only a bare handful of experimental results relevant to the SR in a QHF were obtained: indirectly, in Ref. [1] the linewidths of the electron spin resonance (ESR) were measured; and then directly, the photoluminescence dynamics of spin-up and spin-down states was studied [2]. Although these experiments do not concern exactly the GM relaxation, the measured times, 5–10 ns, should also be characteristic of the GM decay. These are rather long, exceeding by 1–2 orders the single-electron SR times observed in GaAs quantum wells [5].

In our problem the relevant SR time is actually not a spin dephasing time but the time of Zeeman energy relaxation due to the spin-flip process. Indeed, any spin flip means actually a reduction of the Zeeman energy $\epsilon_Z \Delta S_z$ ($\vec{B} \parallel \hat{z}$, $\Delta S_z = S_z - S_0$ is the S_z component deviation from the equilibrium value S_0, $\epsilon_Z = |g|\mu_B B$ is the Zeeman energy of one spin-flipped electron, $g \approx -0.44$). The spin-flip mechanism which makes the relaxation irreversible has thereby to provide the energy transform. In this Letter, we report on the study where the spin-orbit (SO) coupling is considered as the cause mixing different spin states, and the disorder [to be more precise, a smooth random potential (SRP)] as the condition of irreversibility. The proposed SR mechanism is thereby *purely electronic*. Energy conservation leads to an intermediate state with spatially modulated electron density and reduced Zeeman energy (at the expense of exchange energy). Namely, in terms of spin waves (spin excitons), the SR presents an annihilation process diminishing the number of spin excitons (SEs). The elementary event is a SE "coalescence" transforming the Zeeman energy $2\epsilon_Z$ of two "cold" SEs (with negligible momenta) into the energy of a single SE. The latter has a pronounced momentum $\mathbf{q}^*$ and the energy consisting of both the Zeeman part ϵ_Z and the kinetic part $\sim q^{*2}$.

This comparatively slow process is only the first step of the relaxation. The second one is a fast "cooling" of the intermediate state (with momentum q^*). The electron-phonon interaction (not considered directly) *does not change the spin state but provides a rapid thermalization* occurring much faster than the SR. (Thermodynamic relaxation times are estimated to be $\lesssim 1$ ns.)

An important feature of the present work consists of applying a *microscopic* approach to the study of the motion of the *classical* QHF spin. Indeed, this differs from the current continuum-field technique (see the pioneering work [6] and Ref. [7]) based on the generalization of the nonlinear σ model. However, the presently used method of the excitonic representation [3,8] has some advantages. Operating virtually in terms of many-exciton quantum-mechanical states, it can describe the dynamics of latent processes of spin-exciton (spin-wave) condensation. Namely, the GM relaxation is presented as a decay of the condensate of "zero" spin SEs (having momenta strictly equal to zero) with a simultaneous transformation to the "nonzero-exciton" condensate. After the development stage, the latter also dies out with time.

Other key points of the calculation are as follows: we use, as a bare one-electron basis, the set of states where the SO coupling is immediately (to the lowest order) taken into account; in terms of these states we present

the SRP as a secondary-quantized operator and employ the Fermi golden rule to find transition rates. Matrix elements are calculated between many-exciton states.

So, the starting point is the initial state $|i\rangle = (\hat{S}_-)^N|0\rangle$, where $|0\rangle$ stands for the QHF ground state and $\hat{S}_- = \sum_j \hat{\sigma}_-^{(j)}$ is the lowering spin operator. [j labels electrons; $\hat{\sigma}_\pm = (\hat{\sigma}_x \pm i\hat{\sigma}_y)/2$, where $\hat{\sigma}_{x,y,z}$ are the Pauli matrices.] The number N is assumed to be macroscopically large: $0 \ll N < N_\phi$. The spin numbers of the $|i\rangle$ state are $S = S_0 = N_\phi/2$ and $S_z = N_\phi/2 - N$. The total Hamiltonian has the form $H_{\rm tot} = \sum_j H_1^{(j)} + H_{\rm int}$. The $H_{\rm int}$ operator presents here the Coulomb interaction part of the Hamiltonian, and H_1 is the single-electron part:

$$H_1 = \hbar^2\hat{\mathbf{q}}^2/2m_e^* - \epsilon_Z\hat{\sigma}_z/2 + H_{\rm SO} + \varphi(\mathbf{r}), \qquad (1)$$

where $\hat{\mathbf{q}} = -i\nabla + e\mathbf{A}/c\hbar$ is a 2D operator and $\varphi(\mathbf{r})$ is the SRP field [$\mathbf{r}$ has components (x, y)]. The SO Hamiltonian is specified for the (001) GaAs plane,

$$H_{\rm SO} = \alpha(\hat{\mathbf{q}} \times \hat{\sigma})_z + \beta(\hat{q}_y\hat{\sigma}_y - \hat{q}_x\hat{\sigma}_x), \qquad (2)$$

and presents a combination of the Rashba term [9] (with the coefficient α) and the crystalline anisotropy term [10] (see also Ref. [3]). The parameters α and β are small: $\alpha, \beta \ll l_B\hbar\omega_c$ (really $\alpha < \beta \sim 10^{-7}$ K cm $< l_B\epsilon_Z$).

The relevant analysis shows that the SO interaction (2) alone does not provide a quantum fluctuation from the GM to any state with a nonuniform electron density. This fact generates a need to take into account the SRP which is assumed to be Gaussian, i.e., determined by the correlator $K(\mathbf{r}) = \langle\varphi(\mathbf{r})\varphi(0)\rangle$. If choosing $\langle\varphi(\mathbf{r})\rangle = 0$, then, in terms of the correlation length Λ and Landau level width Δ, the correlator is $K(\mathbf{r}) = \Delta^2\exp(-r^2/\Lambda^2)$. In a real situation $\Delta \approx 5$–10 K, and $\Lambda \sim 30$–50 nm. We consider that $\Lambda > l_B$ and besides

$$T \lesssim T^* \ll \epsilon_Z < \Delta \ll e^2/\varepsilon l_B < \hbar\omega_c. \qquad (3)$$

Here the first inequality counted from the right justifies the approximation of projection onto a single Landau level; the second one determines the QHF "rigidity" with respect to the SRP fluctuations. T is the temperature, which is actually assumed to be zero in the calculations (the value T^* is determined by the SRP and will be defined later). The two left inequalities (3) mean that the energy of metastable (cooled down) SEs is accumulated only in their Zeeman parts.

The SEs are lowest-energy eigenstates of the QHF. Before describing them by means of exciton-creation operators, we choose the single-electron basis which diagonalizes the first three terms in Eq. (1) to within the leading order in $u = \beta\sqrt{2}/l_B\hbar\omega_c$ and $v = \alpha\sqrt{2}/l_B\hbar\omega_c$:

$$\Psi_{pa} = \begin{pmatrix} \psi_{\kappa p} \\ v\sqrt{\kappa+1}\,\psi_{\kappa+1\,p} + iu\sqrt{\kappa}\,\psi_{\kappa-1\,p} \end{pmatrix},$$
$$\Psi_{pb} = \begin{pmatrix} -v\sqrt{\kappa}\,\psi_{\kappa-1\,p} + iu\sqrt{\kappa+1}\,\psi_{\kappa+1\,p} \\ \psi_{\kappa p} \end{pmatrix}. \qquad (4)$$

($\psi_{\kappa p}$ is the electron wave function in the Landau gauge.) Now we can define the exciton operator (cf. [3,8]):

$$\mathcal{Q}^\dagger_{ab\mathbf{q}} = \frac{1}{\sqrt{N_\phi}}\sum_p e^{-iq_x p l_B^2} b^\dagger_{p+q_y/2} a_{p-q_y/2}, \qquad (5)$$

where a_p and b_p are the Fermi annihilation operators corresponding to the states (4). The annihilation excitonic operator is $\mathcal{Q}_{ab\mathbf{q}} \equiv \mathcal{Q}^\dagger_{ba-\mathbf{q}}$ and we employ also the "shift" operators $\mathcal{A}^\dagger_{\mathbf{q}} = N_\phi^{-1/2}\mathcal{Q}^\dagger_{aa\mathbf{q}}$ and $\mathcal{B}^\dagger_{\mathbf{q}} = N_\phi^{-1/2}\mathcal{Q}^\dagger_{bb\mathbf{q}}$. In the following we drop the "spin-orbit" index ab in the definition (5). The commutation rules present a special Lie algebra:

$$[\mathcal{Q}_{\mathbf{q}_1}, \mathcal{Q}^\dagger_{\mathbf{q}_2}] = e^{i\theta_{12}}\mathcal{A}_{\mathbf{q}_1-\mathbf{q}_2} - e^{-i\theta_{12}}\mathcal{B}_{\mathbf{q}_1-\mathbf{q}_2},$$
$$e^{i\theta_{12}}[\mathcal{A}_{\mathbf{q}_1}, \mathcal{Q}_{\mathbf{q}_2}] = -e^{-i\theta_{12}}[\mathcal{B}_{\mathbf{q}_1}, \mathcal{Q}_{\mathbf{q}_2}] = N_\phi^{-1}\mathcal{Q}_{\mathbf{q}_1-\mathbf{q}_2}, \qquad (6)$$

where $\theta_{12} = l_B^2(\mathbf{q}_1 \times \mathbf{q}_2)_z/2$. Besides, evidently $[\mathcal{Q}_{\mathbf{q}_1}, \mathcal{Q}_{\mathbf{q}_2}] = [\mathcal{A}_{\mathbf{q}_1}, \mathcal{B}_{\mathbf{q}_2}] = 0$. In the ground state $|0\rangle$, we have $\mathcal{A}_{\mathbf{q}}|0\rangle = \delta_{\mathbf{q},0}|0\rangle$ and $\mathcal{B}_{\mathbf{q}}|0\rangle = 0$.

In the limit $\Delta \to 0$, $H_{\rm SO} \to 0$ and at $r_c \ll 1$ the state

$$|N; 1; \mathbf{q}\rangle = \mathcal{Q}^\dagger_{\mathbf{q}}(\mathcal{Q}^\dagger_0)^N|0\rangle \qquad (7)$$

is the eigenstate of the system studied. If $q \neq 0$, it has the spin numbers $S = N_\phi/2 - 1$ and $S_z = N_\phi/2 - 1 - N$ [see below the expressions (8) which should be used to calculate S and S_z] and the energy $(N+1)\epsilon_Z + \mathcal{E}_q$, where $\mathcal{E}_q$ is the exchange part of the SE energy. The small momentum approximation $ql_B \ll 1$ is quite sufficient for our problem; therefore $\mathcal{E}_q = (ql_B)^2/2M_{{\rm x},\kappa}$ (see the general expression, e.g., in Ref. [11]). Here $M_{{\rm x},\kappa}$ is the SE mass at $\nu = 2\kappa + 1$, in particular: $1/M_{{\rm x},0} = (e^2/\varepsilon l_B)\sqrt{\pi/8}$.

In terms of the excitonic representation, spin operators are invariant with respect to $H_{\rm SO}$:

$$\hat{S}_z = N_\phi(\mathcal{A}_0 - \mathcal{B}_0)/2, \qquad \hat{S}_- = N_\phi^{1/2}\mathcal{Q}^\dagger_0,$$
$$\hat{\mathbf{S}}^2 = N_\phi\mathcal{Q}^\dagger_0\mathcal{Q}_0 + \hat{S}_z^2 + \hat{S}_z. \qquad (8)$$

At the same time in the basis (2) the operators $\varphi(\mathbf{r})$ and $H_{\rm int}$ acquire corrections proportional to u and v. Calculating $\int\Psi^\dagger\varphi(\mathbf{r})\Psi d^2\mathbf{r}$, where $\Psi = \sum_p(a_p\Psi_{pa} + b_p\Psi_{pb})$, we get the terms responsible for a spin flip:

$$\hat{\varphi} = N_\phi^{1/2}l_B\sum_{\mathbf{q}}\overline{\varphi}(\mathbf{q})(iuq_+ - vq_-)\mathcal{Q}_{\mathbf{q}} + {\rm H.c.} \qquad (9)$$

(we consider $ql_B \ll 1$). Here $\overline{\varphi}(\mathbf{q})$ is the Fourier component [i.e., $\varphi = \sum_{\mathbf{q}}\overline{\varphi}(\mathbf{q})e^{i\mathbf{qr}}$], and $q_\pm = \mp i(q_x \pm iq_y)/\sqrt{2}$.

In spite of the existence of a formal operator equivalence $\mathcal{Q}^\dagger_0 \equiv \lim_{q\to 0}\mathcal{Q}^\dagger_{\mathbf{q}}$ we find that $|N; 1; 0\rangle$ and $\lim_{q\to 0}|N; 1; \mathbf{q}\rangle$ present *different* states [12]. Indeed, in these states the system has different spin numbers $S = N_\phi/2$ and $S = N_\phi/2 - 1$, respectively. ($S_z = N_\phi/2 - 1 - N$ is the same for both.) So, the excitation of a zero exciton ($\mathbf{q} \equiv 0$) corresponds to the $S_z \to S_z - 1$ transition without any change of the S number, whereas each "non-

zero'' SE changes both numbers by 1: $S \to S-1$, $S_z \to S_z - 1$. Let us introduce the notation $|N\rangle = (\mathcal{Q}_0^\dagger)^N|0\rangle$. The initial state $|i\rangle$ is actually a ''Goldstone condensate'' (GC) containing N zero excitons: $|i\rangle = |N\rangle$.

The state $|N\rangle$ is degenerate, and the GC breakdown is studied in terms of the transitions governed by the Fermi golden rule probability: $w_{fi} = (2\pi/\hbar)|\mathcal{M}_{fi}|^2\delta(E_f - E_i)$. In our case, the final state $|f\rangle$ is obviously the state where a part of the Zeeman energy has been converted into a SE kinetic energy. Since it is exactly the single-electron terms that constitute the perturbation responsible for the $\mathcal{M}_{fi}$ matrix element, we could find that such a transition is the $2X_0 \to X_{\mathbf{q}^*}$ process in the lowest order of the perturbative approach (we denote the zero exciton by X_0 and the nonzero one by $X_\mathbf{q}$). The final state for this transition is thereby $|f\rangle = |N-2;1;\mathbf{q}^*\rangle$. The value q^* is determined by the energy conservation equation $E_f = E_i$ which reads $2\epsilon_Z = \epsilon_Z + \mathcal{E}(q^*)$; i.e., $q^* = \sqrt{2M_{\mathrm{x},\kappa}\epsilon_Z}/l_B$.

The transition is determined by the operator (9), and corresponding matrix element $\mathcal{M}_{if} = \langle \mathbf{q}^*;1;N-2| \times \hat{\varphi}|N\rangle[R(N)R(N-2;1;\mathbf{q}^*)]^{-1/2}$ was actually calculated in Ref. [3]. [The SRP plays the same role as the phonon field studied there, and hence the physical processes described by $\mathcal{M}_{if}$ can be seen in more detail from, e.g., the diagram presented in Fig. 1(a) in the second paper of Ref. [3].] Here and in the following the notation $R(\cdots)$ stands for the norm of the state $|\cdots\rangle$ [3,8]. The result is $|\mathcal{M}_{if}|^2 = \frac{N(N-1)}{2N_\phi}(u^2+v^2)|q^* l_B \overline{\varphi}(\mathbf{q}^*)|^2$ (it has been used that $q^* l_B \ll 1$), and we obtain the $i \to f$ rate: $(2\pi/\hbar)\sum_\mathbf{q}|\mathcal{M}_{if}|^2\delta(q^2 l_B^2/2M_{\mathrm{x},\kappa} - \epsilon_Z) = N(N-1)/\tau N_\phi$ (for any $N \geq 1$), where

$$1/\tau = 8\pi^2(\alpha^2+\beta^2)M_{\mathrm{x},\kappa}^2\epsilon_Z\overline{K}(q^*)/\hbar^3\omega_c^2 l_B^4. \tag{10}$$

Here $\overline{K}$ stands for the Fourier component of the correlator: $\overline{K}(q) = L^2|\overline{\varphi}(q)|^2/4\pi^2$.

The thermodynamically unstable state $|f\rangle = |N-2;1,\mathbf{q}^*\rangle$ turns to a state $|N-2;1,\mathbf{q}_0\rangle$ in a time which is much shorter than τ (due to phonon emission), where q_0 takes the lowest possible nonzero value. Relevant values of q_0 are determined by the SRP field. Indeed the SE interaction with the SRP incorporates the energy $U_{\mathrm{x-SRP}} \sim q l_B^2\Delta/\Lambda$ (the nonzero SE possesses the dipole momentum $e l_B^2[\mathbf{q}\times\hat{z}]$ [11]). The $\mathcal{E}_\mathbf{q} \lesssim U_{\mathrm{x-SRP}}$ condition determines the inhomogeneous uncertainty of the nonzero SE momentum:

$$0 < q_0 \lesssim M_{\mathrm{x},\kappa}\Delta/\Lambda. \tag{11}$$

It is believed that $q_0 \ll q^*$, i.e., the ''quasizero'' kinetic energy $(q_0 l_B)^2/2M_{\mathrm{x},\kappa}$, restricted to the value $T^* = M_{\mathrm{x},\kappa}(\Delta l_B/\Lambda)^2$ is considered to be negligible in comparison with ϵ_Z [see the conditions (3)].

To solve the problem in a complete form, we obviously have to study the general state of the type

$$|N;M_1,M_2,\ldots,M_K\rangle = (\mathcal{Q}_{\mathbf{q}_{01}}^\dagger)^{M_1}(\mathcal{Q}_{\mathbf{q}_{02}}^\dagger)^{M_2}\cdots \times (\mathcal{Q}_{\mathbf{q}_{0K}}^\dagger)^{M_K}|N\rangle. \tag{12}$$

All the wave vectors q_{0k} are assumed to satisfy the condition (11). We will also use for this state a shorthand notation $|N;M\rangle$, where $M = \sum_k^K M_k$ is the total number of the nonzero SEs. In the framework of our approach the state (12) is an approximate eigenstate of a QHF having energy $(N+M)\epsilon_Z$ and spin numbers $S_z = N_\phi/2 - N - M$ and $S = N_\phi/2 - M$. These are calculated with help of Eqs. (6) and (8) [the value of S_z is the exact one, but $\hat{\mathbf{S}}^2|N;M\rangle = (N_\phi/2-M)^2(|N;M\rangle + |\tilde{\varepsilon}\rangle)$, where $R(\tilde{\varepsilon})$ is small compared with $R(N;M)$]. The state $|0;M\rangle$ can be treated as a ''thermodynamic condensate'' (TDC) which arises if M is larger than the critical number of nonzero SEs. The latter may be estimated (cf. Ref. [3]) and in our case (3) it is at least smaller than $N_\phi M_{\mathrm{x},\kappa}T$. Meanwhile, M is determined by the spin S of the system; therefore at a *given* $M = N_\phi/2 - S$ we find that below some threshold temperature the nonzero SEs necessarily form a TDC. For macroscopically large N and M, the state (12) hence features a coexistence of GC and TDC, describing a *microscopic nature* of the intermediate QHF state. Of course specific values $\mathbf{q}_{0k}$ as well as specific distribution given by M_k numbers have no physical meaning. The final results should depend only on M and N.

The rate dN/dt is determined by the $2X_0 \to X_{\mathbf{q}^*}(\to X_{\mathbf{q}_0})$ process (which presents a GC depletion with a simultaneous ''flow'' to TDC) and by the $X_0 + X_{\mathbf{q}_0} \to X_{\mathbf{q}^*}(\to X_{\mathbf{q}_0'})$ one. The rate dM/dt is also formed by the $2X_0 \to X_{\mathbf{q}^*}(\to X_{\mathbf{q}_0})$ transition (which provides a TDC evolution) and by the $X_{\mathbf{q}_0} + X_{\mathbf{q}_0'} \to X_{\mathbf{q}^*}(\to X_{\mathbf{q}_0''})$ one (determining a TDC depletion). [Values of q_0, q_0' and q_0'' belong to the region (11).] The equations are derived again with the help of the Fermi golden rule and Eq. (6) (with vanishing θ_{12} there):

$$dn/dt = -(2\mu_{nn} + \mu_{nm})/\tau, \quad \text{and} \quad dm/dt = (\mu_{nn} - \mu_{mm})/\tau, \tag{13}$$

where the notations $n = N/N_\phi$ and $m = M/N_\phi$ are used for ''reduced'' quantum numbers,

$$\mu_{nn} = \frac{|\langle M;N-2|\mathcal{Q}_{\mathbf{q}^*}\mathcal{Q}_{-\mathbf{q}^*}|N;M\rangle|^2}{R(N;M)R(N-2;M+1;\mathbf{q}^*)} = \frac{N^4 R(N-2;M+1;\mathbf{q}^*)}{N_\phi^2 R(N;M)}\left[1 + O\left(\frac{m}{nN_\phi}\right)\right],$$

$$\mu_{nm} = \sum_k \frac{|\langle M_1,\ldots,M_k-1,\ldots,M_K;N-1|\mathcal{Q}_{\mathbf{q}^*+\mathbf{q}_{0k}}\mathcal{Q}_{-\mathbf{q}^*}|N;M\rangle|^2}{R(N;M)R(N-1;M;\mathbf{q}^*)} = \frac{4M^2N^2R(N-1;M;\mathbf{q}^*)}{N_\phi^2 R(N;M)}\left[1 + O\left(\frac{K}{N_\phi}\right)\right],$$

and

$$\mu_{mm} = \sum_{k<i} \frac{|\langle M_1, \ldots, M_k - 1, \ldots, M_i - 1, \ldots, M_K; N| \mathcal{Q}_{\mathbf{q}^* + \mathbf{q}_{0k} + \mathbf{q}_{0i}} \mathcal{Q}_{-\mathbf{q}^*} |N; M\rangle|^2}{R(N; M) R(N; M-1; \mathbf{q}^*)} = \frac{2M^4 R(N; M-1; \mathbf{q}^*)}{N_\phi^2 R(N; M)} \left[1 + O\left(\frac{K}{N_\phi}\right)\right].$$

[$R(N; M+1; \mathbf{q}^*)$ is the norm of the $\mathcal{Q}^\dagger_{\mathbf{q}^*}|N; M\rangle$ state.] In this way we find that the norms appearing in these equations satisfy the conditions $R(N; M+1; \mathbf{q}^*)/R(N; M) = r$, $R(N+1; M)/R(N; M) = Nr_n$, and $R(N; M_1, \ldots, M_k + 1, \ldots, M_K)/R(N; M) = M_k r_m$, so that

$$\mu_{nn} = \frac{n^2 r}{r_n^2}, \qquad \mu_{nm} = \frac{4mnr}{r_n r_m}, \qquad \mu_{mm} = \frac{2m^2 r}{r_m^2}, \quad (14)$$

where r, r_n, and r_m are determined by the equations

$$\begin{aligned} 1 &= \frac{1 - n - 2m}{r_n} + O\left(\frac{m^3}{nK}\right), \\ 1 &= \frac{1 - 2n - 2m}{r_m} + \frac{n^2}{r_n^2} + O\left(m^2\right), \\ 1 &= \frac{1 - 2n - 2m}{r} + \frac{4mn}{r_n r_m} + \frac{n^2}{r_n^2} + \frac{2m^2}{r_m^2} + O\left(\frac{1}{N_\phi}\right). \end{aligned} \quad (15)$$

The last terms in Eq. (15) would depend just on the specific set of the M_k numbers. We can therefore calculate r_m and obtain the final result only in the $m \ll 1$ case. Meanwhile, the values of $m(t)$ are determined by the initial deviation $n(0)$: in particular, at $n(0) = 0.5$ we find that $\max(m) \approx 0.1$. Thus we should put $r = r_n = r_m = 1$ in μ_{mn} and μ_{mm}, but $r_n = 1 - n$ and $r = (1 - n)^2$ in μ_{nn}. This yields the results $n(t) = 1/[2n(0) \times (t/\tau)^2 + 2(t/\tau) + 1/n(0)]$ and $m = n(t)n(0)t/\tau$.

The time dependences are shown in Fig. 1. We stress once again the nonexponential behavior, which could be just an observable manifestation of the existence of the SE condensates. The gap between the dashed and dotted lines in the inset reflects the reduction of the spin modulus during the relaxation. One sees that it is rather small even at significant initial deviation from the ground symmetry. It is exactly this favorable circumstance that enabled us to find an approximate but analytical result.

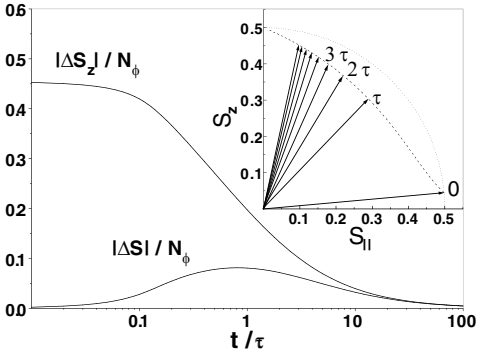

FIG. 1. Time dependences of $|\Delta S_z|/N_\phi = n(t) + m(t)$ and of $|\Delta S|/N_\phi = m(t)$ are shown in the main picture for $n(0) = |\Delta S_z(0)|/N_\phi = 0.455$. The vectors $\mathbf{S}(t)$ (normalized to N_ϕ) at equidistant moments of time are plotted in the inset with step τ. The dotted line is the arc with radius $S_0 = N_\phi/2$.

Let us discuss the time τ governing the GM breakdown. Earlier calculations based on the phonon mechanism of the relaxation give the SR times >1 μs [3], which are in essential discrepancy with the relevant measurements. Now we have studied a disorder relaxation channel and report that under realistic conditions the characteristic time (10) should be weakly dependent on magnetic field and is estimated to be $\tau \approx \tau_0 \exp(q^{*2}\Lambda^2/4) = \tau_0 \times \exp(\gamma B^{3/2})$, where $\tau_0 = 10$–100 ns and $\gamma \sim 0.01\ \mathrm{T}^{-3/2}$. This value brings the theory substantially closer to the available (but still indirect) experimental data [1,2].

The GM could probably be created by microwave pumping at the ESR frequency $\epsilon_Z/\hbar$. This should cause the QHF spin to "precess" without changing the $\mathbf{S}$ modulus. As to observing the SR with time, one can think that the optical technique [2] is relevant in the case.

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[12] Note that there is no continuum transition from the $\mathbf{q} \equiv 0$ and $q \to 0$ exciton states, since the former really means that $q \ll 1/L$], but the latter implies still that $q > 1/L$, namely, $q \sim q_0 \gg 1/L$ [see Eq. (11)].