Cavity Light Bullets: Three-Dimensional Localized Structures in a Nonlinear Optical Resonator

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We consider the paraxial model for a nonlinear resonator with a saturable absorber beyond the mean-field limit. For accessible parametric domains we observe total radiation confinement and the formation of 3D localized bright structures. Different from freely propagating light bullets, here the self-organization proceeds from the resonator feedback, combined with diffraction and nonlinearity. Such "cavity" light bullets can be independently excited and erased by appropriate pulses, and once created, they endlessly travel the cavity round-trip.

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The competition between transverse diffraction and nonlinearities in optical systems (resonators, systems with feedback mirror, or counterpropagating beams) leads to the formation of cavity solitons (CS) appearing as bright pulses on a homogeneous background [1,2]. The possibility of exploiting CS as self-organized micropixels in semiconductor vertical microresonators has been recently proved [3,4] after previous observations and predictions in various classes of macroscopic optical systems (see, e.g., [5,6]). Such phenomena occur when the single-longitudinal-mode approximation within the mean-field limit (MFL) is valid, so that in the propagation direction z (coinciding with the resonator axis), there is no spatial modulation of the field envelope. In this work we predict the analogous formation of solitonlike structures confined in the transverse plane *and* in the longitudinal direction, beyond the MFL. We call these structures cavity light bullets (CLBs). They appear as bright stable pulses, spontaneously stemming and self-organizing from the modulational instability (MI) of a homogeneous field profile, and traveling along the resonator with a definite period.

In the past, the temporal dynamics of a coherent field, linked to the competition of longitudinal modes far from the MFL, has been extensively studied [7,8]. Those approaches included the plane wave approximation and thus could not account for pattern formation. On the other hand, there are relatively few works dealing with pattern formation beyond the MFL in an optical resonator. An exact treatment has been proposed in [9] where, though, negligible field absorption is assumed, i.e., the field intensity is always constant on the z axis. Other approaches referring to different types of nonlinear media proceed from a model proposed in [10], where second-order dispersion is the main mechanism for longitudinal self-modulation of propagating pulses and the adaptation to resonator systems is heuristically provided, with a formalism similar to MFL models. There, the formation of 3D localized structures is shown [11,12]. Finally, recent theoretical studies have considered parametric resonators (χ^2 nonlinearities) where, yet, the mechanism of structure confinement is due to domain wall locking [13] rather than to pattern localization; a system and phenomenology where experimental confirmations are still missing.

On the other side, the formation of freely propagating light bullets in media with different types of nonlinearities (Kerr type, quadratic, saturable, etc.) is caused by the self-reshaping of initial pulses due to the balance between diffraction/dispersion and nonlinear processes, as extensively studied by various authors [14–16]. The CLBs presented in our work, conversely, are self-confined stable structures in the intracavity field profile, superimposed to a nonmodulated background, that spontaneously emerge from a modulational instability of stationary, transversely homogeneous solutions in presence of a plane wave input beam.

In this Letter, we analyze a well-established model for a resonator with a saturable absorber [17], extended to include diffraction in the transverse plane [18].

After casting the model, by means of a 3D linear stability analysis tool we recently developed for this purpose [19], we determine parametric ranges where conditions for self-confinement are expected to be best met. We then find conditions for radiation confinement and specifically report about cavity light bullets in the cited parametric domain. Finally, we show how CLB can be externally excited and displaced.

In the paraxial and slowly varying envelope approximation the equations governing the spatiotemporal dynamics of a two level saturable absorber in a nonlinear unidirectional ring resonator driven by a coherent field, can be cast as in [20]. After adiabatic elimination of the atomic variables (good cavity limit), we have

$$\frac{1}{c}\frac{\partial F}{\partial t}+\frac{\partial F}{\partial z}=\frac{-\alpha F(1-i\Delta)}{1+\Delta^2+|F|^2}+\frac{i}{2k_0}\nabla_\perp^2 F \qquad (1)$$

(and complex conjugate) with the boundary condition

$$F(x, y, 0, t) = TY_{inj} + RF(x, y, L, t)e^{-i\delta_0}, \qquad (2)$$

where Y_{inj} and F respectively denote the normalized envelope of the input beam (continuous wave and transversely homogeneous) and of the intracavity field; Δ is the scaled atomic detuning, δ_0 is the scaled cavity detuning, and α is the absorption coefficient per unit length at resonance. T and R are the transmission and reflection coefficients, respectively, of the entrance and exit mirror ($T + R = 1$); L is the medium length; x and y are the transverse Cartesian coordinates, and z the longitudinal coordinate, being $z = 0$ and $z = L$ the input and the output mirrors, respectively. Here, for simplicity, we assume that the active medium completely fills the cavity.

The transverse Laplacian $\nabla_\perp^2$ describes diffraction in the transverse plane (x, y) while $\partial F/\partial z$ accounts for the variation of the field envelope along the cavity axis.

We have recently performed a detailed study of the instabilities affecting the stationary, transversely homogeneous solutions giving rise to patterns modulated in all the three spatial dimensions [19]. Although the competition between transverse and longitudinal modes generally leads to a complex scenario, we have already found a parametric set that revealed the first phenomena of self-organization in space and time.

By fully exploiting the predictive power of the 3D stability analysis, we scanned the parameter space to determine the optimal parametric choice, where a sizeable branch of 3D localized structures could be achieved. On the basis of our studies, we steered our search by imposing a few heuristic conditions on the features of the homogeneous stationary curve and of the instability domains which are necessary for, or highly favor the occurrence of, structure self-confinement. They can be summarized as follows: (i) the curve should correspond to a bistable regime (S-shaped stationary curve); (ii) the unstable portion of the high intensity branch should be as extended as possible and at least a part of it should not coexist with the lower homogeneous branch; (iii) the low-intensity homogeneous branch (ideally the whole of it) should be stable. In addition to these criteria, the nonlinear coupling should be moderate in order to avoid chaotic multimode dynamics.

We thus performed systematic studies in the parameter space, identifying promising regions in the self-focusing regime. The set we report in detail is thus $T = 0.1$, $\delta_0 = -0.4$, $\Delta = -2$, and $\alpha L = 10$.

We remark that we found 3D pattern localization also in the self-defocusing regime, as it was predicted for 2D cavity solitons (see, for example, Fig. 4 in Ref. [21]) but, as in that case, the branch was reduced and we found in addition that their robustness was limited. This nevertheless is in favor of the CLB emerging from the same physical mechanisms leading to CS in 2D.

In Fig. 1 we show the stationary, transversely homogeneous curve by plotting the intensity of the intracavity field at the exit window $I = |F(L)|^2$ versus the input field intensity. In Fig. 1(b) the instability domain is shown and the threshold intensities of the unstable branch are evidenced for comparison to Fig. 1(a). In this parametric regime, the numerical integration of the dynamic equation shows that upon crossing the threshold B, the field profile shows the formation of longitudinal filaments [see Fig. 2(a)], showing strong 2D confinement and longitudinal modulations.

By lowering the input intensity field, these structures eventually break and shorten [see Fig. 2(b)]. The 3D structures that emerge at the steady state (i.e., after all transients have faded away) are confined in the transverse plane; their lengths (along z) appear to be constant, although each structure may have a different one [see Fig. 2(b)]. Besides, they circulate inside the resonator periodically with a period commensurable to the cavity round-trip. This represents thus a valid example of spontaneous self-organization of a complex optical system in 3D (and time), and these structures are what we call CLBs. From preliminary measurements, their minimal length at the steady state seems to decrease with the input field intensity until they disappear and the system precipitates to the homogeneous low-intensity solution; we have also observed that the shortening process is reversible in the sense that starting from an initial condition where one CLB at the steady state is present, and increasing the input field, it lengthens and for appropriate values can extend to a full filament. This is a good indication that the self-confinement is not influenced by the numerical grid

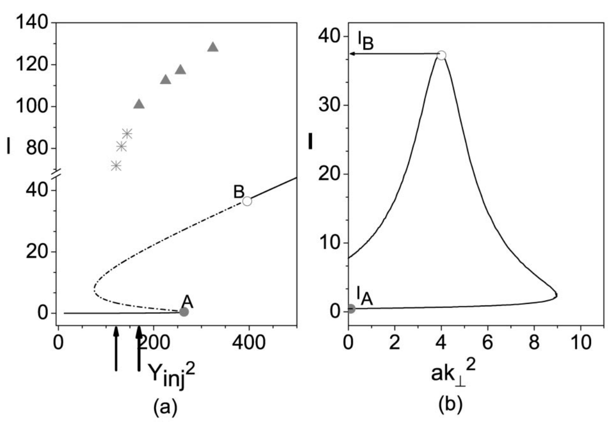

FIG. 1. (a) Steady state curve of the homogeneous solution and results of numerical simulations; continuous and dashed lines refer to stable and unstable portion of the homogeneous branch, respectively. Filled triangles denote the mean values of the maximum intracavity intensity for filaments; asterisks indicate the mean values of the maximum intracavity intensity for CLBs. The two arrows delimit the interval of Y_{inj}^2 where the CLB solutions coexist with the stationary stable homogeneous branch. (b) Instability domain. I_A and I_B are the ordinates of the points A and B in (a) marking the boundary of the unstable branch.

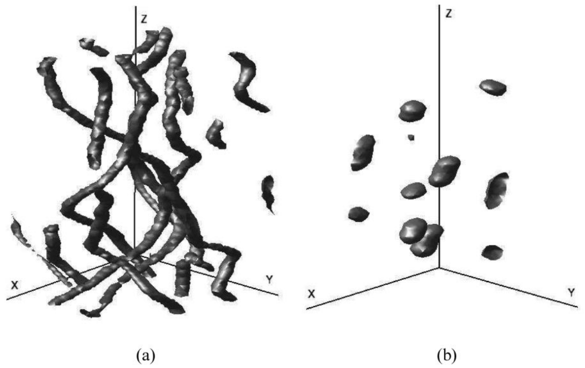

FIG. 2. Two examples of system self-organization. (a) 3D filaments in the intensity profile for $Y_{inj}^2 = 169.0$ can collapse in (b) 3D confined structures (CLBs) which make a cavity round-trip in a definite period for $Y_{inj}^2 = 132.2$. Here, x, y are scaled to $\sqrt{L/2k_0T}$ and z is scaled to L.

discretization. Moreover, we tested their robustness and stability versus different choices of the integration parameters (grid density, step sizes, etc.). Finally we introduced an additive white noise distributed in space and time to rule out possible metastable pattern localizations. For a wide range of noise intensities, the CLB length remains stable for all practically achievable computational times (3×10^4 the resonator round-trip time). When the noise intensity exceeds a threshold value, a global destabilization seems to occur and the filaments tend to reappear while CLBs dissolve into the newly formed global structures.

We remark that the computationally most demanding simulations have also been carried on in a simplified system with only one transverse dimension (x), after repeatedly checking the coherence of the pattern scenario between the fully 3D and the (z, x) models.

The range of CLBs' stability in the terms discussed above is plotted in Fig. 1(a) by the two arrows and is satisfactorily sizeable (almost 2 orders of magnitude larger than that reported in Fig. 4 of Ref. [19]).

Key features of self-confined structures, regardless of their dimensionality, are the possibilities of exciting them everywhere in the resonator and their freedom to drift in the device cross section, a fingerprint of the system's neutral modes [22]; we thus applied the procedures proposed in [23,24] to demonstrate the possibility of switching on and off a CLB at any transverse location of the resonator. We used short and narrow pulses superimposed (and coherent) to the holding beam whose intensity falls in the region of CLB stability [Fig. 1(a)].

For suitable ranges of the pulse width, duration, intensity, and phase, we managed to "write" (see Fig. 3) and "erase" the CLBs. The pulse duration must be considerably longer (≈ 10 times) than the cavity round-trip, but as the pulse duration grows, so does the length of the CLB at the steady state, until it reaches the full resonator's length,

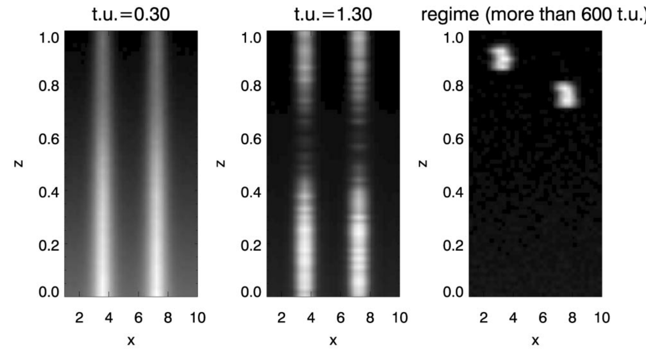

FIG. 3. Intracavity field intensity in one transverse dimension. The three frames show the writing process of two independent CLBs by means of two suitable Gaussian pulses. Here $Y_{inj}^2 = 121.0$. In the gray scale, higher values are associated with lighter ones. The time unit (t.u.) is L/cT.

and the stable structure emerging thereof becomes a straight bright channel in the cavity, confined just in the transverse plane, i.e., the analog of the 2D cavity soliton beyond the mean-field limit. We are still investigating the relation between pulses characteristics and CLB lengths, and an organized report on this property will be published separately.

We could show that it is possible to create several independent CLBs using addressing pulses aimed at any transverse location, provided they are separated at least by a critical distance on the order of the CLB diameter (see Fig. 3). In addition, by exploiting other MFL techniques [23,25], a CLB was caused to drift in the transverse plane by means of a phase gradient in the holding beam (see Fig. 4). With respect to the MFL case, the input phase gradient induces a complex variation of the transversal phase and intensity profile of the intracavity field in the propagation direction, as it can be expected. This certainly constitutes a complication in the control of the

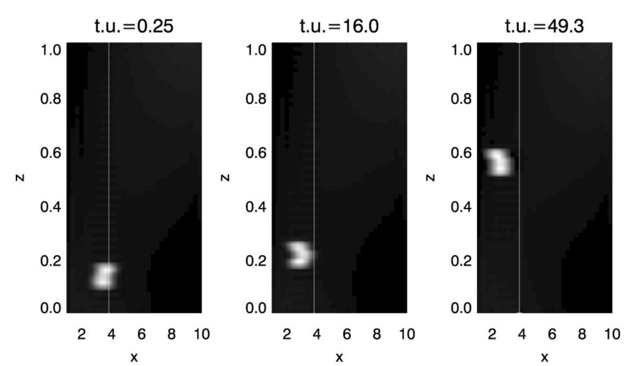

FIG. 4. A CLB drifts under the influence of a phase gradient in the holding beam. More precisely, the localized structure drifts following the gradient of the input phase profile. The intracavity intensity modulation, induced by the input phase gradient, can be appreciated in the background gray scale changes.

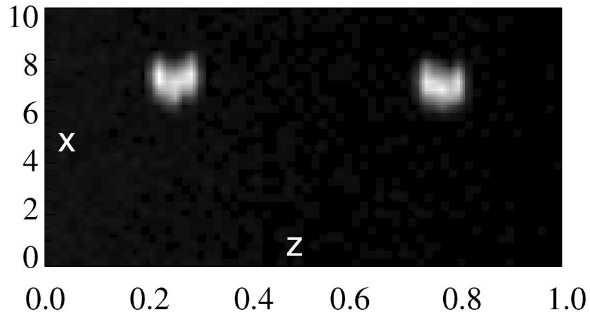

FIG. 5. A train of two independent CLBs travels through the resonator at the steady state.

structures: we are presently studying how the input gradient shape acts on the CLB drift properties and speeds (as we did with CS in [26]) and will report on this elsewhere. Figure 4 shows that CLBs exhibit neutral-mode dynamics, typical of self-confined structures in presence of translational symmetries.

Finally, starting from suitable initial conditions, we verified that it is possible to achieve a stable solution showing a "train" of two CLBs at the same transverse position (Fig. 5). They evolve with constant separation and length. Their separation is adjustable although a minimal value seems to exist. This shows a unique feature to 3D localized structures and opens the perspective to simultaneous serial and parallel optical encoding in the same device.

The properties of CLBs reported in this work are highly appealing for optical information processing: they allow to conceive self-clocked, self-organized, reconfigurable pixels arrays, which could encode all-optical information both serially in the form of longitudinal CLB trains and parallely (in the transversal arrays of CLB). With respect to manipulation of 2D arrays of CS (proposed, e.g., in [21,27]), one can figure out additional controls (e.g., transversely injected fields) to change the phase of CLB trains and thus manipulate information contents, or to increase the number of "input" channels, which can also be seen as logical gates operands.

On different grounds, CLB lend themselves to similar applications as the "standard" light bullets do, namely, stroboscopes for atomic/molecular dynamics. Although their time scales largely exceed the light bullet ones, the benefit unique to cavity-sustained structures is that they are a coherent state in the resonator that may repeatedly interact with, e.g., a quantum system (atom/condensate). In this case, one could fancy a "quantum stroboscope" provided the coherences in the system are long compared to the cavity round-trip.

As a final remark, although the present model is not immediately suited to describe the excitonic response of semiconductor devices [21] because of their slow carrier dynamics, we can note that as far as pattern formation is involved, it has been shown in literature how the qualitative pattern and soliton scenario in the MFL saturable absorber [25] is not radically different from the semiconductor case [21], although the self-confinement scales may significantly vary in a semiconductor device. The extension of the model to a MQW-based microresonator is presently a research mainstream in our group.

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